A branch-and-price algorithm to solve the integrated berth allocation and yard assignment problem in bulk ports

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1st Workshop on Large Scale Optimization

November 21, 2012

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Agenda

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Motivation

Figure: Vessels queueing at Newcastle port, Australia (queue hits 60 vessels as max.)

References

UMANG, N., M.BIERLAIRE, AND VACCA, I.

Exact and heuristic approach methods to solve berth allocation problem in bulk ports.

Tech. rep., TRANSP-OR, Ecole Polytechnique Federale De Lausanne, 2012.

ZHEN, L.

An integrated model for berth template and yard template planning in transshipment hubs.

Transportation Science 45 (2011), 483–504.

Berth Allocation Problem

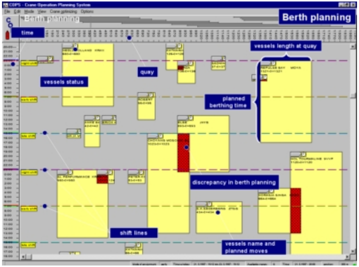
Figure: Lacon ltd.'s plan for extension of the Riga's port, Latvia

Yard Assignment Problem

Figure: Port of Weipa, Queensland, Australia

Objective ▸ Go to model

- Minimize Handling Time + Delay = Service Time
- Obtain berth schedule
- Obtain plan of a yard storage



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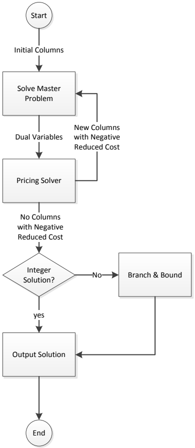
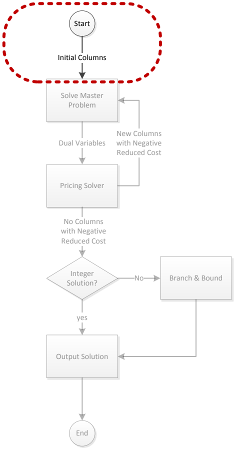
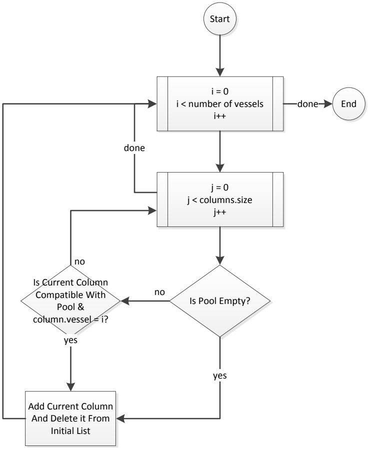
Framework



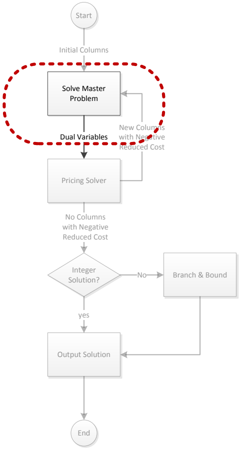
- Initial Solution
- Column Generation – Lower Bound
- Branch and Bound – Optimal Integer Solution

Initial Solution





Master Problem



Idea

- SP [Go to SP]
- relaxation of λ and μ
- Ω reduced to Ω_1

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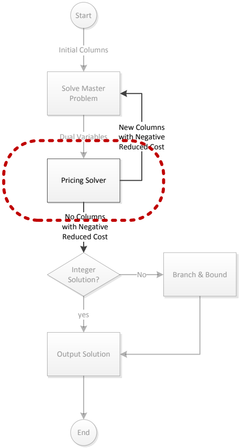
Sub-Problem

Go to model



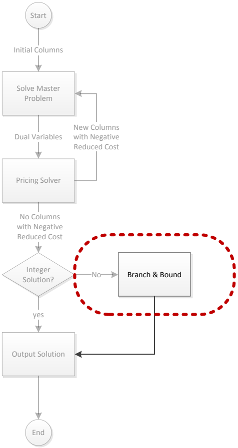
Idea

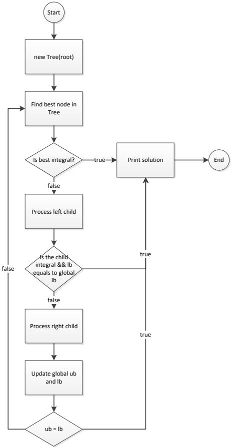
- run for each vessel separately
- get $\leq 40 \cdot n$ columns

Dual Variables

- $\alpha, \beta_{kt}, \gamma_{lt}, \delta_{lw}$

Branch and Bound





Improvement Methods – general + stabilization

General

- decomposition
- 40 columns/sub-problem/iteration

$$(c + s - a) - (\alpha + \sum_{k \in K} \sum_{t \in T} \beta_{kt} \cdot beta_{kt} + \sum_{l \in L} \sum_{t \in T} \gamma_{lt} \cdot gamma_{lt} + \sum_{l \in L} \sum_{w \in W} \delta_{lw} \cdot delta_{lw}) \leq 0 \quad (1)$$

Stabilization of duals

- assumption: duals oscilate
- solution: $\overline{\alpha_i} = \epsilon \cdot \overline{\alpha_i} + (1 - \epsilon) \cdot \alpha_i$

Improvement Methods – aggregation 1/2

row	section	time	column 1	column 2	column 3
1	1	1	1	0	0
2	1	2	1	0	0
3	1	3	1	0	0
4	1	4	0	0	0
5	1	5	0	0	0
6	1	6	0	0	0
7	2	1	0	0	0
8	2	2	0	0	0
9	2	3	0	1	0
10	2	4	0	1	0
11	2	5	0	1	0
12	2	6	0	1	0
13	3	1	0	0	0
14	3	2	0	0	1
15	3	3	0	1	1
16	3	4	0	1	1
17	3	5	0	1	0
18	3	6	0	1	0

Improvement Methods – aggregation 2/2

row	section	time	column 1	column 2	column 3
1	1	1	1	0	0
2	1	4	0	0	0
3	2	3	0	1	0
4	3	2	0	0	1
5	3	3	0	1	1

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Set Partitioning

Instance			RMP		
# vessels	# sections	congestion	time	value	Ω
10	10	no	1m 03s	202	121 863
		yes	3m 35s	219	149 163
	20	no	9m 35s	306	213 268
		yes	24m 08s	316	265 138
15	10	no	2m 27s	301	162 846
		yes	3m 52s	328	210 246
	20	no	27m 13s	450	284 086
		yes	1h 21m 15s	468	374 146
20	10	no	3m 36s	384	181 373
		yes	11m 36s	421	263 173
	20	no	40m 09s	567	312 515
		yes	–	–	467 935

Column Generation

Instance			Column Generation							
# vessels	# sections	congestion	time	# iter.	Ω_1	lb	ub	gap1	gap2	gap
10	10	no	2m 07s	4	587	197	206	2%	2%	4%
		yes	6m 28s	7	1 300	213	221	3%	1%	4%
	20	no	4m 49s	4	696	306	309	0%	1%	1%
		yes	18m 50s	8	1 454	311	316	2%	0%	2%
15	10	no	3m 25s	4	922	293	306	3%	2%	4%
		yes	26m 24s	7	2 123	314	328	4%	0%	4%
	20	no	13m 22s	5	1 308	447	452	1%	0%	1%
		yes	1h 7m 4s	12	3 270	457	471	2%	1%	3%
20	10	no	3m 43s	4	1 099	372	390	3%	2%	5%
		yes	38m 59s	9	3 077	396	435	6%	3%	9%
	20	no	16m 05s	5	1 780	562	572	1%	1%	2%
		yes	3h 27m 45s	18	4 736	583	625	–	–	7%

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Column Generation

Instance			Column Generation							
# vessels	# sections	congestion	time	# iter.	Ω_1	lb	ub	gap1	gap2	gap
10	10	no	2m 07s	4	587	197	206	2%	2%	4%
		yes	6m 28s	7	1 300	213	221	3%	1%	4%
	20	no	4m 49s	4	696	306	309	0%	1%	1%
		yes	18m 50s	8	1 454	311	316	2%	0%	2%
15	10	no	3m 25s	4	922	293	306	3%	2%	4%
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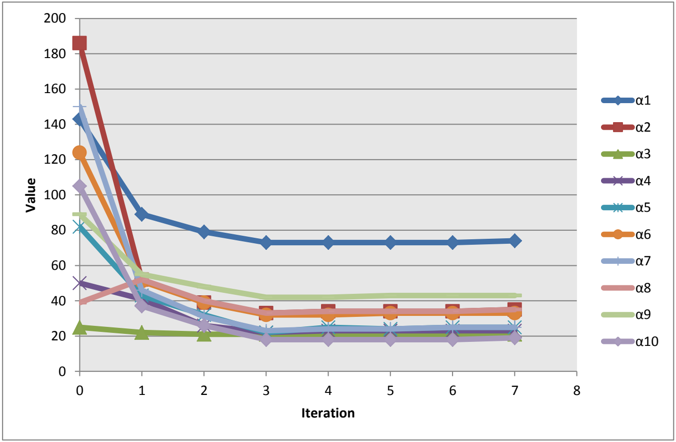
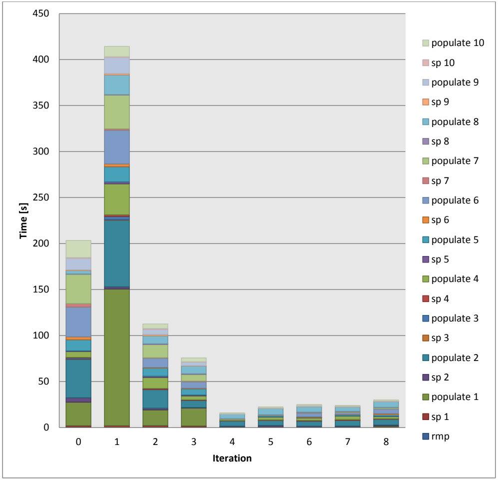
	20	no	13m 22s	5	1 308	447	452	1%	0%	1%
		yes	1h 7m 4s	12	3 270	457	471	2%	1%	3%
20	10	no	3m 43s	4	1 099	372	390	3%	2%	5%
		yes	38m 59s	9	3 077	396	435	6%	3%	9%
	20	no	16m 05s	5	1 780	562	572	1%	1%	2%
		yes	3h 27m 45s	18	4 736	583	625	–	–	7%

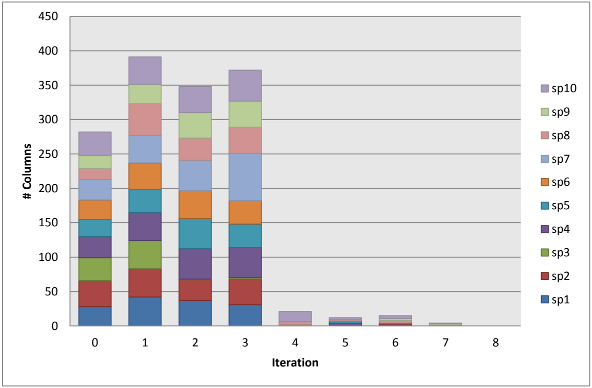
Column Generation – Duals



Column Generation – Time Consumption



Column Generation – Pool



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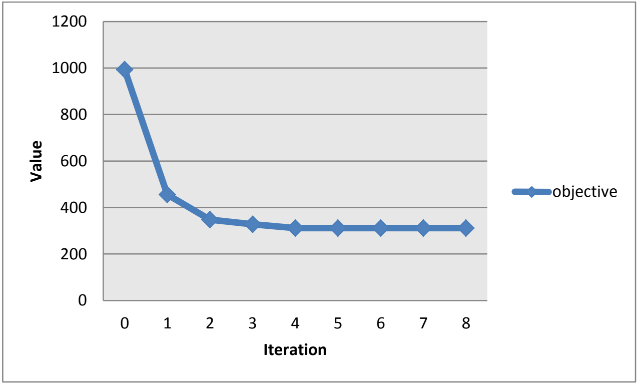
Column Generation – Objective Fce Evolution



Stabilized Method

		Uncongested			Congested		
ϵ	Δ	iter.	time	pool size	iter.	time	pool size
1.00	0.05	15	19m 15s	357	35	1h 7m 12s	560
	0.10	14	12m 31s	390	37	50m 17s	565
	0.20	11	10m 59s	397	26	44m 53s	624
0.75	0.05	10	8m 43s	403	21	45m 41s	642
	0.10	12	8m 38s	400	21	40m 11s	642
	0.20	9	7m 37s	407	21	39m 8s	642
0.50	0.05	7	4m 27s	397	11	23m 15s	724
	0.10	7	3m 58s	397	11	21m 2s	724
	0.20	7	3m 51s	397	11	19m 29s	724
0.25	0.05	6	3m 32s	434	8	15m 49s	916
	0.10	6	3m 22s	434	8	12m 14s	916
	0.20	6	3m 22s	434	8	10m 41s	916

Stabilized Method

		Uncongested			Congested		
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	0.20	11	10m 59s	397	26	44m 53s	624
	0.05	10	8m 43s	403	21	45m 41s	642
0.75	0.10	12	8m 38s	400	21	40m 11s	642
	0.20	9	7m 37s	407	21	39m 8s	642
	0.05	7	4m 27s	397	11	23m 15s	724
0.50	0.10	7	3m 58s	397	11	21m 2s	724
	0.20	7	3m 51s	397	11	19m 29s	724
	0.05	6	3m 32s	434	8	15m 49s	916
0.25	0.10	6	3m 22s	434	8	12m 14s	916
	0.20	6	3m 22s	434	8	10m 41s	916

Methods Comparison

	Uncongested			Congested		
	iter.	time	pool size	iter.	time	pool size
without	2	9s	122	3	13s	132
ARMP	2	9s	122	4	17s	226
ARMP+SRMP1	8	42s	85	10	59s	117
ARMP+SRMP2	3	14s	79	3	16s	159

	Uncongested			Congested		
	iter.	time	pool size	iter.	time	pool size
without	4	2m 18s	587	7	6m 18s	1300
ARMP	5	2m 29s	608	9	6m 51s	1546
ARMP+SRMP1	13	9m 56s	429	26	28m 51s	1107
ARMP+SRMP2	5	2m 59s	450	11	8m 3s	968

Methods Comparison

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	iter.	time	pool size	iter.	time	pool size
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ARMP+SRMP2	3	14s	79	3	16s	159

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ARMP+SRMP2	5	2m 59s	450	11	8m 3s	968

Aggregated Method

Instance			Column Generation							
# vessels	# sections	congestion	time	# iter.	Ω_1	lb	ub	gap1	gap2	gap
10	10	no	2m 36s	5	608	197	206	2%	2%	4%
		yes	7m 53s	9	1 546	213	221	3%	1%	4%
	20	no	5m 10s	4	726	306	310	0%	1%	1%
		yes	22m 09s	14	2 054	311	316	2%	0%	2%
15	10	no	3m 23s	5	1 106	293	304	3%	1%	4%
		yes	22m 02s	7	2 482	314	338	4%	3%	7%
	20	no	15m 06s	6	1 533	447	455	1%	1%	2%
		yes	1h 5m 54s	15	3 362	457	473	2%	1%	3%
20	10	no	3m 58s	7	1 330	372	389	3%	1%	4%
		yes	34m 24s	8	3 847	396	425	6%	1%	7%
	20	no	14m 24s	6	1 921	562	568	1%	0%	1%
		yes	4h 00m 00s	17	5 406	583	622	–	–	6%

Aggregated Method

Instance			Column Generation							
# vessels	# sections	congestion	time	# iter.	Ω_1	lb	ub	gap1	gap2	gap
10	10	no	2m 36s	5	608	197	206	2%	2%	4%
		yes	7m 53s	9	1 546	213	221	3%	1%	4%
	20	no	5m 10s	4	726	306	310	0%	1%	1%
		yes	22m 09s	14	2 054	311	316	2%	0%	2%
15	10	no	3m 23s	5	1 106	293	304	3%	1%	4%
		yes	22m 02s	7	2 482	314	338	4%	3%	7%
	20	no	15m 06s	6	1 533	447	455	1%	1%	2%
		yes	1h 5m 54s	15	3 362	457	473	2%	1%	3%
20	10	no	3m 58s	7	1 330	372	389	3%	1%	4%
		yes	34m 24s	8	3 847	396	425	6%	1%	7%
	20	no	14m 24s	6	1 921	562	568	1%	0%	1%
		yes	4h 00m 00s	17	5 406	583	622	–	–	6%

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Branch and Bound

Instance			Branch & Bound				
# vessels	# sections	congestion	time	value	# of nodes	# of μ	# of λ
10	10	no	1m 03s	202	13	6	0
		yes	3m 35s	219	15	7	0
	20	no	9m 35s	306	3	1	0
		yes	–	–	–	–	–
15	10	no	2m 27s	301	35	17	0
		yes	–	–	–	–	–
	20	no	27m 13s	450	15	7	0
		yes	–	–	–	–	–
20	10	no	–	–	–	–	–
		yes	–	–	–	–	–
	20	no	–	–	–	–	–
		yes	–	–	–	–	–

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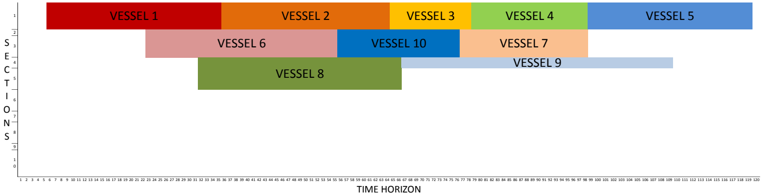
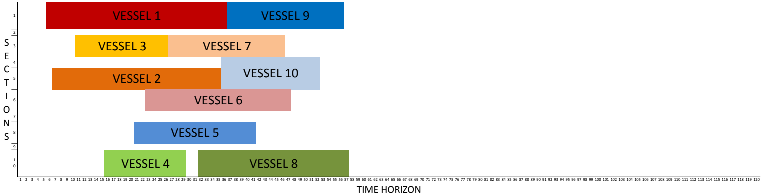
Initial vs. Optimal





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Conclusion

- proven re-formulation
- "fast" master problem
- "poor" sub-problem
- good (small number of nodes) and bad (time) performance of Branch and Bound
- fast convergence of Column Generation (number of iterations)
- tight bounds (upper and lower) of Column Generation
- dual variables not oscilating
- "hidden" power of aggregated method (although failed for Ω)

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Future Work

Model:

- simultaneous loading and unloading
- several cargo on the vessel
- uncertainity, disruptions
- (continous quay)

Branch and Price:

- better heuristic for initial solution
- heuristic/dynamic programming to solve sub-problem
- heuristic to find integral (sub-optimal) solution or Branch and Cut to find optimal solution, instead of Branch and Bound

Thank you for your attention.

Model Go back I

$$\min \sum_i (m_i - A_i + c_i) \tag{2}$$

$$\text{s.t.} \quad m_i - A_i \geq 0 \qquad \forall i \in N \tag{3}$$

$$\sum_{k \in M} (s_k^j b_k) + B(1 - y_{ij}) \geq \sum_{k \in M} (s_k^i b_k) + L_i \qquad \forall i, j \in N, i \neq j \tag{4}$$

$$m_j + B(1 - z_{ij}) \geq m_i + c_i \qquad \forall i \in N, \forall j \in N, i \neq j \tag{5}$$

$$y_{ij} + y_{ji} + z_{ij} + z_{ji} \geq 1 \qquad \forall i \in N, \forall j \in N, i \neq j \tag{6}$$

$$\sum_{k \in M} s_k^i = 1 \qquad \forall i \in N \tag{7}$$

$$\sum_{k \in M} (s_k^i b_k) + L_i \leq L \qquad \forall i \in N \tag{8}$$

Model Go back II

$$\sum_{p \in M} (\delta_{i\ell k} s^i_\ell) = x_{ik} \qquad \forall i \in N, \forall k \in M \qquad (9)$$

$$(d_k - D_i) x_{ik} \geq 0 \qquad \forall i \in N, \forall k \in M \qquad (10)$$

$$c_i \geq h^w_{ik} \rho_{i\ell k} Q_i - B(1 - s^i_\ell) \qquad \forall i \in N, \forall l \in M, \forall k \in M, \forall w \in W_i \qquad (11)$$

$$h^w_{ik} = \alpha^w_{ik} + \beta^w_{ik} \qquad \forall w \in W_i, \forall k \in M \qquad (12)$$

$$\beta^w_{ik} = V_w r^i_k \qquad \forall i \in N, \forall w \in W_i, \forall k \in M \qquad (13)$$

$$r^i_k = \sum_{p \in P} (r^p_k \lambda_{ip}) / Q_i \qquad \forall i \in N, \forall k \in M \qquad (14)$$

$$\sum_{p \in p} \phi_{ip} \leq F \qquad \forall i \in N \qquad (15)$$

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Model Go back III

$$\pi_w^p + \pi_u^q \leq 1 \qquad \forall w \in W, \forall u \in \bar{W}(w), \forall p \in P, \forall q \in \bar{P}(p) \qquad (16)$$

$$\sum_{i \in N} \omega_t^{ip} \leq 1 \qquad \forall p \in P, \forall t \in H \qquad (17)$$

$$\sum_{w \in W} \pi_w^p \leq 1 \qquad \forall p \in P \qquad (18)$$

$$\phi_{ip} \leq \pi_w^p \qquad \forall i \in N, \forall w \in W_i, \forall p \in P \qquad (19)$$

$$\omega_t^{ip} \geq \phi_{ip} + \theta_{it} - 1 \qquad \forall i \in N, \forall p \in P, \forall t \in H \qquad (20)$$

$$\omega_t^{ip} \leq \phi_{ip} \qquad \forall i \in N, \forall p \in P, \forall t \in H \qquad (21)$$

$$\omega_t^{ip} \leq \theta_{it} \qquad \forall i \in N, \forall p \in P, \forall t \in H \qquad (22)$$

$$\sum_{t \in H} \theta_{it} = c_i \qquad \forall i \in N \qquad (23)$$

Model Go back IV

$$t + B(1 - \theta_{it}) \geq m_i + 1 \qquad \forall i \in N, \forall t \in H \qquad (24)$$

$$t \leq m_i + c_i + +B(1 - \theta_{it}) \qquad \forall i \in N, \forall t \in H \qquad (25)$$

$$Q_i = \sum_{p \epsilon P} \lambda_{ip} \qquad \forall i \in N \qquad (26)$$

$$\lambda_{ip} \leq \phi_{ip} Q_i \qquad \forall i \in N, \forall p \in P \qquad (27)$$

$$\phi_{ip} \leq \lambda_{ip} \qquad \forall p \in P \qquad (28)$$

$$\lambda_{ip} \leq \sum_{w \in W_i} \sum_{t \in H} (R_w \omega_t^{ip} + B(1 - \pi_w^p)) \qquad \forall i \in N, \forall p \in P \qquad (29)$$

$$s_k^i \in \{0, 1\} \qquad \forall i \in N, \forall k \in M \qquad (30)$$

$$x_{ik} \in \{0, 1\} \qquad \forall i \in N, \forall k \in M \qquad (31)$$

$$y_{ij} \in \{0, 1\} \qquad \forall i, j \in N \qquad (32)$$

Model ◂ Go back V

$$
\begin{aligned}
z_{ij} &\in \{0,1\} & \forall i,j \in N && (33)\\
\pi_w^p &\in \{0,1\} & \forall p \in P, \forall w \in W && (34)\\
\omega_t^{ip} &\in \{0,1\} & \forall i \in N, \forall p \in P, \forall t \in H && (35)\\
\phi_{ip} &\in \{0,1\} & \forall i \in N, \forall p \in P && (36)\\
\theta_{it} &\in \{0,1\} & \forall i \in N, \forall t \in H && (37)
\end{aligned}
$$

Set Partitioning

Go back

$$\min \sum_{a\in\Omega} c_a \cdot \lambda_a \tag{38}$$

$$\text{s.t.} \sum_{a\in\Omega} A_a^i \cdot \lambda_a = 1, \quad \forall i \in N, \tag{39}$$

$$\sum_{a\in\Omega} B_a^{kt} \cdot \lambda_a \leq 1, \quad \forall k \in K, \forall t \in T, \tag{40}$$

$$\sum_{a\in\Omega} C_a^{lw} \cdot \lambda_a - ct_w \cdot \mu_w^l \leq 0, \quad \forall l \in L, \forall w \in W, \tag{41}$$

$$\sum_{w\in W} \mu_w^l \leq 1, \quad \forall l \in L, \tag{42}$$

$$\mu_w^l + \mu_{\overline{w}}^{\overline{l}} \leq 1, \quad \forall l \in L, \forall \overline{l} \in \overline{L}, \forall w \in W, \forall \overline{w} \in \overline{W}, \tag{43}$$

$$\sum_{a\in\Omega} D_a^{lt} \cdot \lambda_a \leq 1, \quad \forall l \in L, \forall t \in T, \tag{44}$$

$$\lambda_a \in \{0,1\}, \quad \forall a \in \Omega, \tag{45}$$

$$\mu_w^l \in \{0,1\}, \quad \forall l \in L, \forall w \in W. \tag{46}$$

Sub-Problem

Go back

$$\textit{minimize}\ (c+s-a)-(\alpha+\sum_{k\in K}\sum_{t\in T}\beta_{kt}\cdot \textit{beta}_{kt}+\sum_{l\in L}\sum_{t\in T}\gamma_{lt}\cdot \textit{gamma}_{lt}+\sum_{l\in L}\sum_{w\in W}\delta_{lw}\cdot \textit{delta}_{lw}) \quad (47)$$

$$
\begin{aligned}
s-a &\geq 0, & & (48)\\
c &\geq ht_k\cdot \textit{fraction}_{jk}-M\cdot(1-ss_j), & \forall j,k\in K, & \quad (49)\\
\sum_{j\in K} ss_j &= 1, & & (50)\\
\sum_{j\in K} ss_j\cdot sc_j+\textit{length} &\leq ql, & & (51)\\
\sum_{k\in K} o_{jk}\cdot ss_j &= x_j, & \forall j\in K, & \quad (52)\\
\sum_{l\in L} \textit{split}_l &\leq Z, & & (53)\\
\textit{split}_l &\leq \textit{delta}_{lw}, & \forall l\in L, & \quad (54)\\
\sum_{l\in L} cs_l &= \textit{quantity}, & & (55)\\
cs_l &\leq \textit{split}_l\cdot \textit{quantity}, & \forall l\in L, & \quad (56)\\
\textit{split}_l &\leq cs_l, & \forall l\in L, & \quad (57)\\
td_k &= \left(\sum_{l\in L} d_{kl}\cdot cs_l\right)/\textit{quantity}, & \forall k\in K, & \quad (58)\\
ht_k &= F/\textit{cranes}_k+V_w\cdot td_k, & \forall k\in K, & \quad (59)\\
\sum_{t\in T} \textit{time}_t &= c, & & (60)\\
t+M\cdot(1-\textit{time}_t) &\geq s+1, & \forall t\in T, & \quad (61)\\
t &\leq s+c+M\cdot(1-\textit{time}_t), & \forall t\in T, & \quad (62)\\
\textit{beta}_{kt} &\geq x_k+\textit{time}_t-1, & \forall k\in K,\forall t\in T, & \quad (63)\\
\textit{beta}_{kt} &\leq x_k, & \forall k\in K,\forall t\in T, & \quad (64)\\
\textit{beta}_{kt} &\leq \textit{time}_t, & \forall k\in K,\forall t\in T, & \quad (65)\\
\textit{gamma}_{lt} &\geq \textit{split}_l+\textit{time}_t-1, & \forall l\in L,\forall t\in T, & \quad (66)\\
\textit{gamma}_{lt} &\leq \textit{split}_l, & \forall l\in L,\forall t\in T, & \quad (67)\\
\textit{gamma}_{lt} &\leq \textit{time}_t, & \forall l\in L,\forall t\in T. & \quad (68)
\end{aligned}
$$