Solution methods for an integrated airline schedule planning and revenue management model

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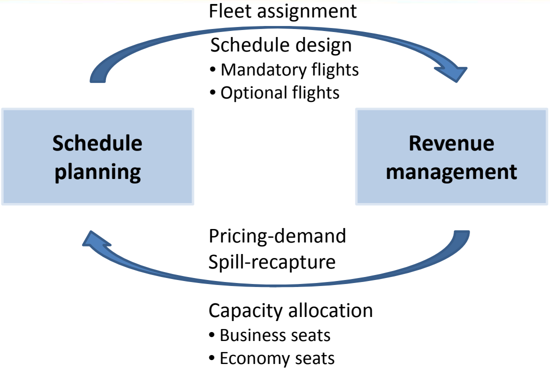
Motivation

- Demand responsive transportation systems
  - Better representation of demand $\Rightarrow$ Appropriate demand models
  - Integration of supply-demand interactions in transportation models
- Today's talk:
  - A brief description of the integrated model
  - A heuristic method
  - Transformation of the problem & Generalized Benders Decomposition

Integrated schedule planning and revenue management



- *Aim: to take better fleeting decisions with the information provided by the demand model*

Integrated model - Schedule planning

$$Max \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s^{'})} (d_i - \sum_{j \in I_s} t_{i,j} + \sum_{j \in (I_s \setminus I_s^{'})} t_{j,i} b_{j,i}) p_i - \sum_{\substack{k \in K \\ f \in F}} C_{k,f} x_{k,f} : \text{ revenue - cost} \quad (1)$$

$$s.t. \sum_{k \in K} x_{k,f} = 1 : \text{ mandatory flights} \qquad \forall f \in F^M \quad (2)$$

$$\sum_{k \in K} x_{k,f} \leq 1 : \text{ optional flights} \qquad \forall f \in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f \in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in Out(k,a,t)} x_{k,f} : \text{ flow conservation} \qquad \forall [k,a,t] \in N \quad (4)$$

$$\sum_{a \in A} y_{k,a,minE_a^-} + \sum_{f \in CT} x_{k,f} \leq R_k : \text{ fleet availability} \qquad \forall k \in K \quad (5)$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+} : \text{ cyclic schedule} \qquad \forall k \in K, a \in A \quad (6)$$

$$\sum_{h \in H} \pi_{k,f}^h \leq Q_k x_{k,f} : \text{ seat capacity} \qquad \forall f \in F, k \in K \quad (7)$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \quad (8)$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \quad (9)$$

- *Itinerary-based fleet assignment*
- *Spill and recapture*

Integrated model - Revenue management

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})}\delta_{i,f}d_i - \sum_{j\in I_s}\delta_{i,f}t_{i,j} + \sum_{\substack{j\in(I_s\setminus I_s^{'})\\ i\neq j}}\delta_{i,f}t_{j,i}b_{j,i} \leq \sum_{k\in K}\pi_{k,f}^h : \textit{demand-capacity} \qquad \forall h\in H, f\in F \quad (10)$$

$$\sum_{\substack{j\in I_s\\ i\neq j}} t_{i,j} \leq d_i : \textit{total spill} \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \quad (11)$$

$$\tilde{d}_i = D_s \frac{\exp(V_i(p_i,z_i,\beta))}{\sum_{j\in I_s}\exp(V_j(p_j,z_j,\beta))} : \textit{logit demand} \qquad \forall h\in H, s\in S^h, i\in I_s \quad (12)$$

$$b_{i,j} = \frac{\exp(V_j(p_j,z_j,\beta))}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k(p_k,z_k,\beta))} : \textit{recapture ratio} \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}), j\in I_s \quad (13)$$

$$d_i \leq \tilde{d}_i : \textit{realized demand} \qquad \forall h\in H, s\in S^h, i\in I_s \quad (14)$$

$$LB_i \leq p_i \leq UB_i : \textit{bounds on price} \qquad \forall h\in H, s\in S^h, i\in I_s \quad (15)$$

$$t_{i,j}, b_{i,j} \geq 0 \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}), j\in I_s \quad (16)$$

$$\pi_{k,f}^h \geq 0 \qquad \forall h\in H, k\in K, f\in F \quad (17)$$

Heuristic method

Available solvers[1] are able to converge on instances with about 35 flights. We devised a heuristic procedure based on two simplified versions of the model:

- FAM^{LS}: price-inelastic schedule planning model $\Rightarrow$ MILP
  - Explores new fleet assignment solutions based on a local search
  - Price sampling
  - Variable neighborhood search (VNS)
- REV^{LS}: Revenue management with fixed capacity $\Rightarrow$ NLP
  - Optimizes the revenue for the explored fleet assignment solution

[1] BONMIN: Bonami et at. (2008), An algorithmic framework for convex mixed integer nonlinear programs. Discrete Optimization, 5(2):186-204

Heuristic method

Require: x^0, y^0, d^0, p^0, t^0, b^0, π^0, time_{max}, n_{min}, n_{max}, $\mathrm{notImpr}$, $\mathrm{tabuListSize}$

  $g := 0$, $\mathrm{time} := 0$, $n_{\mathrm{fixed}} := n_{\mathrm{min}}$, $\mathrm{notImpr} := 0$, $z^* := -\mathrm{INF}$, $\mathrm{tabuList} := \emptyset$

  repeat

    $p^g :=$ Price sampling$(t^{g-1}, p^{g-1}, d^{g-1})$

    $\{d^g, b^g\} :=$ Logit model(p^g)

    $L :=$ Fixing$(x^{g-1}, t^{g-1}, n_{\mathrm{fixed}})$

    $\{x^g, y^g, \pi^g, t^g\} :=$ solve $z_{\mathrm{FAM^{LS}}(p^g, d^g, b^g, L)}$

    if $(\bar{x}^g \notin \mathrm{tabuList})$ **then**

      $\mathrm{tabuList} := \mathrm{tabuList} \bigcup x^g$

      $\{p^g, d^g, b^g, \pi^g, t^g\} :=$ solve $z_{\mathrm{REV^{LS}}(x^g, y^g)}$

      if $(z_{\mathrm{REV^{LS}} \geq z^*})$ **then**

        Update z^*

        Intensification: $n_{\mathrm{fixed}} := n_{\mathrm{fixed}} + 1$ when $n_{\mathrm{fixed}} < n_{\mathrm{max}}$

        $\mathrm{notImpr} := 0$

      else if $(\mathrm{notImpr} == 3)$ **then**

        Diversification: $n_{\mathrm{fixed}} := n_{\mathrm{fixed}} - 1$ when $n_{\mathrm{fixed}} > n_{\mathrm{min}}$

        $\mathrm{notImpr} := \mathrm{notImpr} - 1$

      end if

    end if

    $g := g + 1$

  until $\mathrm{time} \geq \mathrm{time}_{max}$

Local search

- Neighborhood solutions are visited based on the spill rather than a fully random search
- Price sampling:
  - A random price is drawn for each itinerary
  - If the spilled passengers are higher than the average ⇒ decrease the price
  - Otherwise ⇒ increase the price
- Fixing FAM solutions - VNS:
  - The itineraries are sorted according to their spilled number of passengers
  - Low spill value ⇒ associated flights have a higher probability to be fixed to their current aircraft
  - If the solution is improved more assignments are fixed and vice versa.

Performance of the heuristic

The omitted instances are the ones where the sequential approach has the same solution as the integrated model.

	Best solution reported by BONMIN		**Sequential approach (SA)**		**Heuristic results** *Average over 5 replications*				
	Profit	Time (sec) *max 43,200*	Profit	% dev.	Profit	% dev.	%imp. over SA	Time (sec) *max 3,600*	%time reduction
2	37,335	27	35,372	-5.26%	37,335	0.00%	5.55%	13	53.33%
4	46,037	2,686	43,990	-4.45%	46,037	0.00%	4.66%	3	99.90%
5	70,904	2,479	69,901	-1.42%	70,679	-0.32%	1.11%	6	99.75%
7	87,212	42,628	84,186	-3.47%	87,212	0.00%	3.59%	60	99.86%
8	906,791	12,964	904,054	-0.30%	906,791	0.00%	0.30%	2	99.98%
11	94,203	1,724	93,920	-0.30%	94,203	0.00%	0.30%	10	99.42%
12	858,544	7,343	854,902	-0.42%	858,545	0.00%	0.43%	1	99.99%
13	138,575	37,177	137,428	-0.83%	138,575	0.00%	0.83%	173	99.54%
14	96,486	17,142	93,347	-3.25%	96,486	0.00%	3.36%	89	99.48%
16	38,205	240	37,100	-2.89%	38,205	0.00%	2.98%	1	99.50%
18	53,128	141	52,369	-1.43%	53,128	0.00%	1.45%	1	99.53%
20	146,467	31,945	146,464	-0.00%	147,506	0.71%	0.71%	380	98.81%
21	207,434	4,848	217,169	4.69%	219,136	5.64%	0.91%	1,395	71.22%
22	153,789	4,387	163,114	6.06%	163,393	6.24%	0.17%	126	97.12%
23	227,364	22,174	226,615	-0.33%	227,284	-0.04%	0.30%	1,283	94.21%
24	194,598	42,360	208,561	7.18%	210,395	8.12%	0.88%	791	98.13%
25	463,731	31,535	469,136	1.17%	470,494	1.46%	0.29%	1,117	96.46%

Log transformation of the problem

$$\tilde{d}_i = D_s \cdot \frac{\exp(\beta \ln(p_i) + c_i)}{\sum_{j \in I_s} \exp(\beta \ln(p_j) + c_j)} \qquad \forall h \in H, s \in S^h, i \in I_s$$

A new variable υ_s [2] is defined:

$$\upsilon_s = \frac{1}{\sum_{j \in I_s} \exp(\beta \ln(p_j) + c_j)}$$

$$\mathrm{Prob}_i^s = \upsilon_s \exp(\beta \ln(p_i) + c_i)$$

$$\sum_{i \in I_s} \mathrm{Prob}_i^s = 1$$

$$\tilde{d}_i = D_s \mathrm{Prob}_i^s$$

[2] As proposed by Cornelia Schön (2008)

Log transformation

The log transformation for the choice probability:

$$\ln(\mathrm{Prob}_i^s) = \ln(\upsilon_s) + \beta \ln(p_i) + c_i$$
$$\mathrm{Prob}_i^{s'} = \upsilon_s^{'} + \beta p_i^{'} + c_i,$$

where $\mathrm{Prob}_i^s > 0, \upsilon_s > 0, p_i > 0$

- We get rid off the non-convexity of the demand model...

Transformed revenue model - no spill effects

The fleet assignment decision variables are fixed ($x_{k,f}$, $y_{k,a,t}$), we have an NLP.

$$\max \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s^{'})} \exp(p_i^{'} + d_i^{'}) \Rightarrow p_i^{'} + d_i^{'} \quad (18)$$

$$\text{s.t.} \boxed{\sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s^{'})} \delta_{i,f} \exp(d_i^{'}) \leq \sum_{k \in K} \pi_{k,f}^h} \quad \forall f \in F^*, h \in H \quad (19)$$

$$\boxed{\sum_{h \in H} \pi_{k,f}^h = Q_k \boldsymbol{X}_{k,f}} \quad \forall f \in F, k \in K \quad (20)$$

$$d_i^{'} \leq \upsilon_s^{'} + \beta p_i^{'} + c_i + \ln D_s \quad \forall h \in H, s \in S^h, i \in I_s \quad (21)$$

$$\sum_{i \in I_s} \exp(\upsilon_s^{'} + \beta p_i^{'} + c_i) = 1 \quad \forall h \in H, s \in S^h \quad (22)$$

$$\ln(LB_i) \leq p_i^{'} \quad \forall h \in H, s \in S^h, i \in I_s \quad (23)$$

$$p_i^{'} \leq \ln(UB_i) \quad \forall h \in H, s \in S^h, i \in I_s \quad (24)$$

$$d_i^{'}, p_i^{'}, \upsilon_s^{'} \in \Re \quad \forall h \in H, s \in S^h, i \in I_s \quad (25)$$

$$\pi_{k,f}^h \geq 0 \quad \forall h \in H, k \in K, f \in F \quad (26)$$

Master problem - GBD

$$\max \quad \alpha$$

$$\text{s.t.} \quad \alpha \leq \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s')} \exp(\boldsymbol{P}_i^{c'} + \boldsymbol{D}_i^{c'}) - \sum_{k\in K}\sum_{f\in F} C_{k,f}\boldsymbol{X}_{k,f}^c$$

$$+ \sum_{k\in K}\sum_{f\in F}(Q_k \ \mathbf{MR}_{k,f}^c - C_{k,f})[x_{k,f} - \boldsymbol{X}_{k,f}^c] \qquad \forall c \in \mathrm{CUTS} \qquad (27)$$

$$\sum_{k\in K} x_{k,f} = 1 \qquad \forall f \in F^M \qquad (28)$$

$$\sum_{k\in K} x_{k,f} \leq 1 \qquad \forall f \in F^O \qquad (29)$$

$$y_{k,a,t^-} + \sum_{f\in \mathrm{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in \mathrm{Out}(k,a,t)} x_{k,f} \qquad \forall [k,a,t] \in N \qquad (30)$$

$$\sum_{a\in A} y_{k,a,\mathrm{minE}_a^-} + \sum_{f\in CT} x_{k,f} \leq R_k \qquad \forall k \in K \qquad (31)$$

$$y_{k,a,\mathrm{minE}_a^-} = y_{k,a,\mathrm{maxE}_a^+} \qquad \forall k \in K, a \in A \qquad (32)$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \qquad (33)$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \qquad (34)$$

- *Li and Sun 2006 - Generalized Benders Decomposition*

Lagrangian multipliers - Marginal revenue of each seat

$$\mathbf{MR}^{c}_{k,f} = \frac{\sum_{h \in H} \boldsymbol{\Pi}^{h}_{k,f} \boldsymbol{\lambda}_{\boldsymbol{k,f,h}}}{Q_k} \quad \forall f \in F, k \in K, \boldsymbol{X}^{c}_{k,f} = 1$$

- $\boldsymbol{\lambda}_{\boldsymbol{k,f,h}}$ are the Lagrangian multipliers related to the demand-capacity constraints
- price of one seat at flight f on class h and plane type k.
- obtained with the application of the optimality conditions

GBD framework

- Initial FAM solution ($x_{k,f}$)
- Repeat until UB $\leq$ LB
  - Solve REV subproblem which is an NLP and obtain...
    - price, demand, allocated seats ($p_i^{'}$, $d_i^{'}$, $\pi_{k,f}^{h}$)
    - Lagrangian multipliers $\Rightarrow$ Benders cuts
    - A lower bound (LB) for the problem
  - Solve the FAM master problem which is a MILP and obtain...
    - an updated FAM solution ($x_{k,f}$)
    - An upper bound (UB) for the problem

A small example

- 2 airports CDG-MRS
- 4 flights - all are mandatory
- 2 aircraft types: 37-50 seats

We start with an initial FAM solution:

	AC1	AC2
F1	X	
F2	X	
F3	X	
F4	X	

A small example - GBD iterations

Iteration 1		
	Sub	Master
	12522.8	16923.4
	LB	**UB**
	12522.8	16923.4
	AC1	AC2
F1		X
F2		X
F3		X
F4		X

⟹

Iteration 2		
	Sub	Master
	10734.4	14822.8
	LB	**UB**
	12522.8	14822.8
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	

Iteration 3		
	Sub	Master
	12696.8	14822.8
	LB	**UB**
	12696.8	14822.8
	AC1	AC2
F1	X	
F2		X
F3		X
F4	X	

⟹

Iteration 4		
	Sub	Master
	12474.4	12696.8
	LB	**UB**
	12696.8	**12696.8**
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	

Conclusions and on-going work

- The integrated model has promising results
- ... which motivates the effort in devising solution methodologies

- GBD will be tested
- The spill effects will be added back to the model

Thank you for your attention!