Feeder transit integration with high frequency trunk lines

Matina L.Y. Chau*[1] and Konstantinos Gkiotsalitis[2]

[1]PhD Candidate, Department of Transportation Planning & Engineering, National Technical University of Athens, Greece
[2]Assistant Professor, Department of Transportation Planning & Engineering, National Technical University of Athens, Greece

Short summary

Public transportation systems are perceived as less competitive when compared to private vehicles because of their increased travel times due to the lack of overall door-to-door availability and poor inter-modal transfer synchronisation. To address this issue, our study develops an optimisation model for the coordination of trunk lines with last-mile feeder lines. The developed linear programming model re-adjusts the dispatching times of last-mile feeder bus lines according to the timetables of trunk lines operated by metro services. Stop-specific bus holding strategies are employed to reduce the risk of missed connections and minimise prolonged waiting times. Compared against state-of-the-art regularity-based approaches for bus services, the trade-off between optimising for transfers and service regularity is analysed. A reduction of 86% in transfer waiting times is determined when compared to the original schedule, and 82.4% compared to the results of regularity-based optimisation.

Keywords: Bus Holding; Last mile; Multi-modal transport; Public Transport; Transport network modelling; Trunk-Feeder line synchronisation;

1 Introduction

Transportation, one of the three most polluting industries (UN, 2021; EC, 2022), necessitates sustainable planning to mitigate climate change. Optimizing urban transport networks enhances ecological outcomes and mobility services (Wang et al., 2018), often targeting key performance indicators of Public Transport (PT) services like reliability and passenger waiting times. These factors are key considerations in multimodal transport systems, where combining multiple modes offsets their individual weaknesses (Alessandretti et al., 2023). Multimodal networks typically include different modes and lines, with a hierarchical structure consisting of trunk and feeder lines (Gkiotsalitis, 2022b). For instance, when considering a network with a trunk and a feeder line, the first mode of the journey cannot typically change its schedule easily (i.e., metro line), and the following line usually performs the first or last mile of the passenger trips (Gkiotsalitis, 2022a). From this perspective, the trunk line may be seen as the "main" line of the multimodal system, and the feeder will need to adjust its schedule to it to reduce the passenger transfer times. First- and last-mile connectivity challenges hinder the shift from private vehicles to public transport, conversely reduced door-to-door travel times can increase ridership (Liu et al., 2023; UITP, 2015). Thus, for PT to compete with alternative modes, seamless transfer synchronizations are critical (Liu et al., 2021; Gkiotsalitis et al., 2023).

The Transfer Synchronisation Problem (TSP) addresses connectivity, reliability, and accessibility (Liu et al., 2021). Extensively studied in recent decades, advances in computation and network interconnectivity have spurred numerous literature reviews on transfer synchronisation. In the review of the TSP at the tactical level by Liu et al. (2021), most studies employed Mathematical Programming (MP) for their solution framework, using the rescheduling of dispatching times of vehicle trips as decision variables. This was observed within studies minimizing waiting times (Domschke, 1989; Saharidis et al., 2014); and maximizing successful transfers (Ibarra-Rojas & Rios-Solis, 2012)).

At the operational level, vehicle holding dominates real-time control methods, ensuring regularity and punctuality by regulating same service-line inter-trip headways (Gkiotsalitis et al., 2023). Within studies solely employing vehicle holding, Gkiotsalitis et al. (2019) developed a robust transfer synchronisation model. In the robust rescheduling and holding model for autonomous buses feeding into collector lines (Eikenbroek & Gkiotsalitis, 2020), results were resilient to travel time variation and maintained high regularity. While studies combining holding and rescheduling are notably limited (Gkiotsalitis et al., 2023), despite their respective popularity within the different planning levels of the TSP, studies by Ibarra-Rojas et al. (2019) (Mixed Integer Programming to minimise system costs) and Wu et al. (2016) (addressing delayed connection costs via timetabling and holding), highlight their potential, especially compared to the predominant combination of stop-skipping and vehicle-holding (Gkiotsalitis et al., 2023).

Within the scope of the TSP, feeder lines often transfer passengers by feeding them into urban rail trunk lines (Chowdhury & I-Jy Chien, 2002; Yang et al., 2020). Addressing multimodal synchronisation with hierarchy, Gkiotsalitis (2022a) outlined the Feeder-Trunk Line Synchronisation Problem. The non-convex MP was reformulated into a convex quadratic program with continuous decision variables of dispatching, arrival and headway times. Solved to global optimality using exact methods, the rescheduling variables minimised the excessive waiting times of passengers throughout the feeder line stops, as well as the transferring waiting times between the feeder and trunk line modes.

Considering the literature on the passenger synchronisation of multimodal urban transport networks, our study formulates the Trunk-Feeder Line Synchronisation Problem with Holding Times (TFLSPHT). Introduced at the tactical level, considering the dispatching- and the vehicle holding times at stops as decision variables, the formulation consists of a linear program that can be efficiently solved and potentially applied for real-time control in future studies. Among the reviewed studies, the extent to which rescheduling and vehicle-holding can complement or impede one another within the context of transfer synchronisation has not been explored. Our study aims to fill this research gap by developing a mathematical model, testing and analysing the interrelations between transfer synchronisation and the service regularity objectives. Our paper departs from the Feeder-Trunk Line Synchronisation Problem presented in Gkiotsalitis (2022a) by contributing the following:

(1) develop a mathematical model to minimise transfer times between a trunk- and a collecting feeder line that performs the last-mile leg of passenger journeys, resulting in a formulation for the Trunk-Feeder Line Synchronisation Problem.

(2) incorporate holding times into the formulation of transfer synchronisation problems.

(3) analyse the side effects and extent to which vehicle holding and rescheduling can affect service regularity and transfer synchronisation.

For this to be achieved, our TFLSPHT model is validated on real-world data of the multimodal public transit network of Athens, Greece. The problem scenario is suited in supporting the planning of PT services during off-peak time periods where missed connections can exacerbate passenger waiting times at transfer points. By optimising the TFLSPHT model, the rescheduling of the feeder line and the implementation of vehicle holding control measures result in reducing prolonged door-to-door travel times of passengers.

2 Mathematical Modelling

Trunk-Feeder Line Synchronisation Model with Holding Times (TFLSMHT)

To realise our TFLSP with Holding Times model, our study has made the following assumptions:

(i) The service of the trunk line (i.e., metro) is frequent. Thus, passengers transferring from the feeder line (i.e., bus) to the trunk line will not have to wait long.

(ii) The service of the feeder line (i.e., bus) is infrequent. Thus passengers transferring from the trunk- to the feeder line will have to wait long if the departure times of the feeder line from the transfer station are not synchronised to the arrival times of the trunk line.

(iii) The number of scheduled feeder line trips cannot be increased, as this will result in additional requests for drivers and vehicles that may not be available.

(iv) Feeder line trips can change their dispatching times, but are not allowed to overtake each other.

The TFLSPHT formulation is presented as follows. Consider a multimodal public transit network consisting of feeder and trunk line stops $s \in S_f$ and $s \in S_m$, respectively. Every day, each line operates a certain number of trips ($n \in N_f$ for feeder modes and $n \in N_m$ for trunk modes) according to its assigned schedule. Transfer points at which passengers egress from the trunk line to then board onto the feeder are captured in the set $B = S_f \bigcap S_m$. At each transfer station in set $b \in B$, it takes w_b seconds of walking time for the passengers to alight from the trunk until they board the feeder line.

Bus trips are dispatched at a pre-determined dispatch time δ_n, according to their daily schedule. To align with the availability of staff and vehicles, the difference between the decision variable x_n of the re-scheduled dispatch time of the feeder trip $n \in N_f$, and its pre-determined dispatch time δ_n, is bounded by a time window of $[U_{lb}, U_{ub}]$, as defined in Eq. (1).

$$U_{lb} \leq x_n - \delta_n \leq U_{ub} \qquad \forall n \in N_f \tag{1}$$

To prevent daily operations from commencing before crew members are prepared to start their daily work schedule, the re-scheduled dispatch time of the first trip of the day (x_1) cannot start earlier than time δ_{min}. Meanwhile, the time point δ_{max} constraints the last feeder trip $x_{|N_f|}$ of the schedule so that it cannot start its operations later than that. This is added as a constraint to avoid schedule sliding, prohibiting trips from being operated after the end of the work day. The two aforementioned constraints related to the start time of the first and last trip of the day are presented below.

$$\delta_{min} \leq x_1 \tag{2}$$
$$x_{|N_f|} \leq \delta_{max} \tag{3}$$

The second set of variables of the TFLSPHT model are $\{a_{ns}\}$, representing the arrival time of the bus trip n at stop $s \in S_f$. The change in dispatch time x_n has a trickle down effect onto the overall change in arrival times of the feeder line by redefining the relative shift in expected inter-station travel times t_{ns} between stops $s-1 \in S_f$ and $s \in S_f$ as follows:

$$a_{n2} = x_n + t_{n2} \qquad \forall n \in N_f \tag{4}$$
$$a_{ns} = a_{n,s-1} + t_{ns} + l_{n,s-1} \qquad \forall n \in N_f \ \forall s \in S_f \setminus \{1\} \tag{5}$$
$$l_{ns} \leq l_{max} \qquad \forall n \in N_f \ \forall s \in S_f \setminus \{1\} \tag{6}$$

As presented in constraints (4), re-scheduling the dispatch time of a bus trip alters its arrival time at the second stop, resulting in a shift of re-scheduled arrival times throughout the remaining stops of the trip's journey. This can be seen in constraints (5), where the arrival time a_{ns} at stop $s \in S_f$ depends on that of $a_{n,s-1}$ at stop $s-1 \in S_f$. Additionally, the arrival time also factors in the third set of decision variables of this problem, namely $\{l_{n,s}\}$. We note that $l_{n,s-1}$ is the holding time of the bus trip n at the previous stop. Throughout the journey, the holding time of vehicle trip n at stop s, denoted as l_{ns}, cannot exceed the maximum time limit l_{max}, as shown in constraints (6). This ensures that vehicle trips will not be held at a stop for an extended amount of time, resulting in significantly increased travel times for the on-board passengers (Gkiotsalitis, 2020).

On account of the feeder line's schedule starting no earlier than its corresponding dispatch time, the following constraints are applied:

$$a_{n1} = x_n \qquad \forall n \in N_f \tag{7}$$

As overtaking among vehicles of the same feeder line is prohibited, the arrival time of every trip $n \in N_f \setminus \{1\}$ of the feeder at any stop s is scheduled to always take place at a later point in time compared to its prior trip:

$$a_{n-1,s} \leq a_{ns} \qquad \forall n \in N_f \setminus \{1\} \; \forall s \in S_f \tag{8}$$

Within a day's schedule of feeder trips, vehicles may operate two successive trips of the same line. Given that we already know the successive trips operated by the same vehicle (defined at the vehicle scheduling phase) we can set $\Phi_{nn'} = 1$ for trips n and n' operated by the same vehicle, and $\Phi_{nn'} = 0$ for trips that are not. We note that $\Phi_{nn'}$ are binary parameters. If two trips n and n' are performed by the same vehicle, one should ensure that trip n' starts after trip n has ended, considering also the required layover time Ψ between the trips. This is modelled below:

$$\Phi_{nn'}\Psi \leq \Phi_{nn'}(x_{n'} - a_{n|S_f|}) \qquad \forall n, n' \in N_f \tag{9}$$

Constraints (9) ensure that the re-scheduled dispatch time of the later trip $x_{n'}$ is at least equal to the end time of the previous trip operated by the same vehicle, $a_{n|S_f|}$, plus the required layover Ψ.

Despite the central objective of the TFLSPHT being the optimisation of waiting times at the transfer point $b \in B$ between trunk and feeder lines, not all trips of the feeder and trunk lines need to be synchronised at stop b. Thus, we use binary parameters Y_{bnm}, where $Y_{bnm} = 1$ if vehicle trips $n \in N_f$ and $m \in N_m$ need to synchronise their transfer at the transfer point b, and 0 otherwise.

Contrary to the flexibly adjustable a_{ns} arrival times of feeder transit, the arrival times of trunk lines are typically fixed and cannot be changed (Liang et al., 2016) because they operate on the same infrastructure (train and/or metro tracks) with other trains, and a potential change can result in domino effects (Şahin, 1999). Hence, the arrival times of the trips of the trunk lines are considered to be fixed and represented as parameters γ_{mb}, where γ_{mb} is the arrival time of vehicle trip m of the trunk line at transfer point b. Let us now consider two vehicle trips m and n of the trunk and feeder line, respectively. To ensure that the trip of the feeder line n will arrive at the transfer point after the trip of the trunk line m, and that there will be sufficient time available for walking between the two lines, one should enforce the following:

$$Y_{bnm}(\gamma_{mb} + w_b) \leq a_{nb} \qquad \forall b \in B \; \forall n \in N_f \; \forall m \in N_m \tag{10}$$

That is, for every occurring synchronisation, the travel time of passengers arriving at the trunk's transferring station $b \in B$ and then walking to the bus stop, is to be no greater than the arrival time of the feeder line at the same transferring point. As the objective of this paper is a minimisation problem, the combined trunk arrival (γ_{mb}) and passenger walking time (w_b) serves as a boundary to the optimised arrival times of the feeder trips, as presented in constraints (10). This ensures the minimisation of a_{nb} in such a manner, that its scheduling will be as close as feasibly possible to the passenger's arrival time at the feeder stop, reducing the waiting times during transfers.

In addition to the synchronisation of transfers, as this study seeks to not significantly deteriorate the regularity of the feeder line service, we add the following constraints to ensure that the headway between two successive trips of the same feeder line does not deviate considerably from the target headway of the line h_n^*:

$$h_{ns} := a_{ns} - a_{n-1s} \qquad \forall n \in N_f \setminus \{1\} \; \forall s \in S_f \setminus \{1\} \tag{11}$$

$$|h_{ns} - h_n^*| \leq h_{max} \qquad \forall n \in N_f \setminus \{1\} \; \forall s \in S_f \setminus \{1\} \tag{12}$$

Constraints (11) set the value of the decision variable of feeder headways h_{ns} to equal the value of the difference between the arrival times of successive trips at the same stop. This provides the feeder line with a relative measure of service regularity, as longer headways represent longer passenger waiting times at the associated stop of the feeder trip. To promote service regularity, constraints (12) prevent the absolute difference between the optimised values of h_{ns} from significantly differing from the ideal target headway h_n^*, capping it by a maximum deviation of $h_{\max}$. By including these constraints, this study accounts for the service regularity of the feeder line.

Inequality constraints (12) introduce non-linearity into the feasible region of the problem, which becomes non-convex and thus, optimality cannot be guaranteed. As a result, the absolute terms of inequality constraints (12) are linearised as follows.

$$h_{ns} - h_n^* \leq h_{max} \qquad \forall n \in N_f \setminus \{1\}\ \forall s \in S_f \setminus \{1\} \tag{13}$$

$$-h_{max} \leq h_{ns} - h_n^* \qquad \forall n \in N_f \setminus \{1\}\ \forall s \in S_f \setminus \{1\} \tag{14}$$

Combining constraints (13) and (14), the absolute value condition of (12) is replaced by its equivalent linearisations. This, along with the similarly affine constraints of (1) - (11), results in a feasible region which is a convex set.

The complete model formulation of the Trunk-Feeder Line Synchronisation Problem with Holding Times is summarised below, with the objective function (15) optimising the rescheduled arrival times of the feeder line according to the requirements of the Trunk-Feeder transfer. Transfers are accounted for trunk-feeder line trips m and n only if they are supposed to exchange passengers at transfer point b, denoted by parameter $Y_{bnm} = 1$. If this is the case, the difference between the feeder line's arrival a_{nb} and the sum of the arrival of the trunk line γ_{mb} plus the required walking time from the trunk to the feeder line w_b is minimised. Subject to the constraints of the problem, this objective function requests the dispatching schedule of the feeder mode to be adjusted to the arrival times of the trunk line trips. The feeder trips can also be held at different stops of the line to reduce the transfer times between the trunk and feeder line. This will result in reduced door-to-door travel times for passengers when performing the last-mile of their journey.

Trunk-Feeder Line Synchronisation Model with Holding Times (TFLSMHT)

$$\min \sum_{n \in N_f} \sum_{m \in N_m} \sum_{b \in B} Y_{bnm}(a_{nb} - \gamma_{mb} - w_b) \tag{15}$$

$$\text{s.t. } (x, a, h, l) \in \mathcal{F}(x, a, h, l) = \{(x, a, h, l) \in R \mid (x, a, h, l) \text{ satisfy Eqs. (1)-(11), (13)-(14))}\} \tag{16}$$

The feasible region $\mathcal{F}$ of the mathematical problem is a convex set and the objective function is an affine function. Thus, our TFLSPHT is formulated as a linear program that can be solved to global optimality in polynomial time using interior point solution methods (Karmarkar & Ramakrishnan, 1991).

Regularity-based Model with Holding Times (RMHT)

We developed the TFLSMHT to minimise the transfer times of passengers transferring from a trunk line to a feeder line at a transfer point. Traditionally, operators of feeder line services aim to improve service regularity without the consideration of passenger waiting times at transfer points. Past studies have explored several regularity-based optimization models with holding times, minimising the waiting times of feeder line passengers at all stops, without considering the waiting times experienced at the transfer points. In such studies, the objective minimises the squared difference between actual and target headways at all stops of the feeder line. This objective is used to increase the regularity of PT service lines as much as possible (see Gkiotsalitis (2022a)). The formulation of a Regularity-based Model with Holding Times (RMHT) is presented below and results in a quadratic programming model.

Regularity-based Model with Holding Times (RMHT)

$$\min \sum_{n \in N_f \setminus \{1\}} \sum_{s \in S_f \setminus \{1\}} \left(\frac{h_{n,s}}{2} - \frac{h_n^*}{2} \right)^2 \tag{17}$$

$$\text{s.t.: Eqs. } (1) - (11) \tag{18}$$

This quadratic program maximising feeder line service regularity without considering the transfer synchronisation between the trunk and the feeder modes will be referred to as the Regularity-based Model with Holding Times (RMHT). This model is widely used when optimising the regularity of PT operations, and it is presented here to allow us to perform a comparison with our TFLSMHT, which minimises the waiting times at transfer points but does not optimise the regularity of feeder line services. In the following section, a comparison between our TFLSMHT and the RMHT is performed, demonstrating the potential benefit of using each model in terms of passenger waiting times at transfer points and the service regularity of feeder lines.

3 Real world case study: Results and discussion

Our case study is based on the controlled corridor of Alexandras Avenue, Athens, with multiple trunk and feeder lines interconnecting throughout this area. This includes the M3 line of the metro, following the route of Doukissis Plakentias to Dimotiko Theatro. Among the feeder transit that intersects with the trunk line's network, the B5 bus line following the Ag. Paraskevi - L. Alexandras - St. Larisis route is considered. To evaluate this study's framework in the context of real-world applications, data provided by the public transport authority of Athens, known as Athens Urban Transport Organisation (OASA), was processed to define the the input parameters of the case study. This includes the publicly available schedule of the M3, and the real-time telematics data supplied by OASA for the trips of bus line B5 on the 1st of March, 2024.

Metro line M3 serves $S_m = \{1, 2, ..., 22\}$ metro stations. Depending on the line performing the trip as well as the scheduled period, the M3's frequency varies between every 2-7 minutes throughout the peak hours, and 6-15 minutes during off-peak. On an average workday, the M3 performs around $N_m = \{1, 2, ..., 42\}$ trips during the hours of 19:00 - 24:00. As the model reschedules the feeder line based on the trips of the trunk line, we are only interested in the arrival times γ_{ms} of all metro trains to the point of transfer, denoted using an asterisk '*'. That is, all arrival times of the 42 trips of the M3 at metro station 9*, as depicted below. Figure 1 visualises the synchronisation between the M3 metro- and B5 bus line. The first and last stations, as well as the transfer stops of the routes are additionally provided. Passenger walking time to the location of the transfer point (w_{36}) is calculated based on the distance between the trunk line station and the feeder line stop.

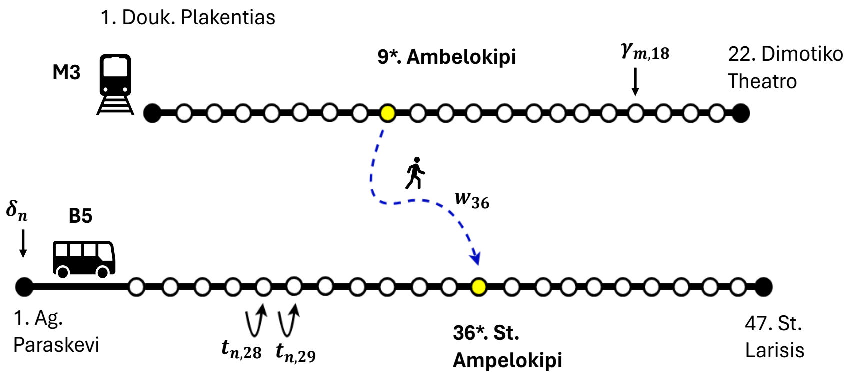

Figure 1: Transfer synchronisation between the main transit line M3 and the collecting feeder line B5

The high-capacity articulated buses of the B5 feeder line serves $S_f = \{1, 2, ..., 47\}$ stops. Its headways vary throughout the feeder line's off-peak hours of 19:00 - 24:00, on a typical weekday. On average, the lowest headway is around 15 minutes, while the highest is around 30 minutes. This results in the B5 completing $N_f = \{1, 2, ..., 12\}$ trips during this time period.

The Continuous Linear Program of our TFLSPHT model is solved using the provided data by OASA, and the interior point method on an Intel(R) Core(TM) i3-1005G1 CPU @ 1.20GHz PC. Solved in 0.03s of 108 iterations (0.00 work units), an objective cost of **992** was obtained. The optimised $\{l_{ns}\}$ of the feeder trips $n \in N_f$ at all stops $s \in S_f$ showed that vehicle holding was frequently employed throughout almost all feeder trips, right until the point of the transfer $s \in S_f =$ **36***, where from then on, all variables equated 0 seconds. This demonstrates the targeted efficacy of vehicle holding measures for improving transfer synchronisation.

To further demonstrate our model's capability in optimising the waiting time at transfer points, Tables 1 and 2 make a direct comparison of the original transfer schedule of the B5 bus line, with that of its TFLSMHT optimised equivalent, as well as the competing regularity-based optimisation model of Eq. (17)'s RMHT. This analysis allows us to determine the trade-offs between specifically optimising for service-regularity and transfer synchronisation, as well as the extent to which vehicle holding and rescheduling can affect the objectives. The original schedule was created based on OASA's data, while TFLSMHT and RMHT's schedules were calculated using their respective optimised decision variables, as depicted in the tables below.

Table 1: Difference in transfer waiting times between the original, transfer synchronisation and regularity- based schedules

Trips	**Original**	**TFLSMHT**	**RMHT**
	$(\delta_n + \sum_{s=1}^{b=36} t_{ns})$ - w_b - γ_{mb}	a_{nb} - w_b - γ_{mb}	a_{nb} - w_b - γ_{mb}
1	83s	0s	332.6s
2	801s	0s	348.9s
3	81s	0s	594.9s
4	826s	0s	0s
5	46s	0s	358.5s
6	99s	0s	639.1s
7	251s	0s	625s
8	82s	0s	579.3s
9	1542s	0s	558.3s
10	2737s	197s	739s
11	37s	74s	74s
12	284s	721s	777s
Sum:	6869s	992s	5626.6s

The original transfer waiting times of trips 2, 4, 9 and 10 of the feeder line are significantly high because passengers missed their transfer connection and had to wait for a prolonged period of time until the arrival of the next bus trip. TFLSMHT mitigates this, as shown in Table 1's column of optimised transfer waiting times, where 75% of journeys are seamlessly synchronised (trips with 0s transfer waiting times). By comparing the sums of the waiting times of the respective schedules, a major decrease in 85.56% of the originally occurring transfer waiting times is obtained. The missed transfers of trips 2, 4, 9 and 10 were rectified, and an excessive waiting time of 721s was only reported in trip 12. This demonstrates the improvement potential of the TFLSPHT model in terms of the reduction of passenger transfer waiting times. In contrast, the optimisation of the headway-objective of the regularity-based RMHT is found to decrease the B5's transfer waiting times by 18.09%, calculated above from Table 1. This demonstrates an 82.4% improvement potential of transfers from optimising the TFLSMHT when compared to those provided by the RMHT. Furthermore, the application of the regularity-based model resulted in only 1 trip out of the 12 (8.33%) to be seamlessly coordinated, as opposed to the TFLSMHT's 75%.

Table 2: Difference in feeder line service regularity between the original, transfer synchronisation and regularity-based schedules

Trips	**Original**	**TFLSMHT**	**RMHT**
$n \in N_f$	$\sum_{s=2}^{47}$ original $(h_{ns} - h_n^*)^2$	$\sum_{s=2}^{47} (h_{ns} - h_n^*)^2$	$\sum_{s=2}^{47} (h_{ns} - h_n^*)^2$
2	704,071	72,861	12,329
3	290,089	1,894,049	19,238
4	4,579,625	9,679,857	68,784
5	1,660,113	4,238,111	82,718
6	1,958,513	2,621,342	198,127
7	813,597	135,537	21,865
8	492,510	79,455	21,911
9	1,495,641	113,764	32,019
10	589,423	234,418	165,274
11	35,291,273	25,896,369	4,328,962
12	45,584,906	23,472,517	19,676,883
Total squared deviation (s^2):	93,459,761	68,438,280	24,628,109

To analyse how service regularity is impacted by our model, the deviation of the schedules' headways from the target headways h_n^* needs to be determined. This was done for the headways of all trips $n \in N_f$ at all stops $s \in S_f$ of the B5. By comparing the sum of the squared deviations of the headways of each trip to the original schedule as presented in Table 2, the TFLSMHT achieves a

26.77% reduction in service headways. In contrast, the RMHT objective minimises the headway deviation by 73.7%.

Table 3: Service improvements categorised by the model's objective

Key performance indicator	**TFLSMHT optimisation**	**RMHT optimisation**
Transfer waiting time improvement	85.56%	18.09%
Squared headway deviation improvement	26.77%	73.70%
Overall vehicle holding time of solution	3186s	4553s

As summarised in Table 3, the objective that aligns itself with the service being optimised possesses the highest proportion of performance improvement by a significant margin. In totalling the overall use of vehicle holding in the respective scenarios, arguably, the TFLSMHT model's strategic vehicle holding to the point of the transfer station allows a more efficient application of less vehicle holding. The TFLSMHT's overall comparatively high and efficient metrics validates the efficacy of the TFLSPHT's use of rescheduling and vehicle holding in targeting transfer synchronisation and service regularity. The control measures show favourable performance in reducing transfer waiting times. As the computation time of the linear TFLSMHT is very low, it can be applied for real-time control by adjusting the dispatch and holding times of bus trips to the traffic conditions of the network, reducing transfer waiting times and maintaining even headways between successive trips.

4 Conclusions

This study presents and validates a mathematical model, optimising transfer times between a main transit line and a collecting feeder mode. Collecting feeder modes can extend beyond public transport and include other sustainable modes, such as e-scooter or bike-sharing services. This model changes the dispatching times of trips and applies vehicle holding to feeder line services to reduce the waiting time of passengers at transfer points. Tested using data from the public transit network of Athens, the model's efficacy was analysed and trade-offs between the objectives of reducing transfer waiting times and headway irregularities were determined. The Trunk Feeder Line Synchronisation Problem with Holding Times was modelled as a linear program and solved to global optimality using exact approaches. Passenger transfer waiting times were successfully reduced by 86% and the feeder line's service regularity was enhanced by 27%. As a result, the TFLSMHT demonstrated an improvement potential of 82.4% when compared to the transfer times of the RMHT. In future research, the TFLSPHT can be applied to real time problems, dynamically reducing transfer waiting times. Its application at the real-time control phase poses computational challenges, as the simultaneous rescheduling and holding of several vehicles operating the same line would need to account for travel time fluctuations, as well as for any dynamic unaccountabilities of the trunk line (i.e., delayed arrival times at the transfer point). Furthermore the performance of the control strategies of TFLSPHT depends on the accurate estimation of the future system states (i.e., future changes in passenger demand and travel times). This is a direction worth exploring when addressing the problem in terms of real-time applications.

Acknowledgements

This work is funded by the European Union's Horizon Europe research and innovation programme CONDUCTOR (Grant Agreement no 101077049).

References

Alessandretti, L., Natera Orozco, L. G., Saberi, M., Szell, M., & Battiston, F. (2023). Multimodal urban mobility and multilayer transport networks. *Environment and Planning B: Urban Analytics and City Science*, *50*(8), 2038–2070.

Chowdhury, S. M., & I-Jy Chien, S. (2002). Intermodal transit system coordination. *Transportation Planning and Technology*, *25*(4), 257–287.

Domschke, W. (1989). Schedule synchronization for public transit networks. *Operations-Research-Spektrum*, *11*(1), 17–24.

EC. (2022). *European commission, climate action, transport.* Retrieved from https://climate.ec.europa.eu/eu-action/transport_en

Eikenbroek, O. A., & Gkiotsalitis, K. (2020). Robust rescheduling and holding of autonomous buses intertwined with collector transit lines. In *2020 ieee 23rd international conference on intelligent transportation systems (itsc)* (pp. 1–7).

Gkiotsalitis, K. (2020). Bus holding of electric buses with scheduled charging times. *IEEE transactions on intelligent transportation systems*, *22*(11), 6760–6771.

Gkiotsalitis, K. (2022a). Coordinating feeder and collector public transit lines for efficient maas services. *EURO Journal on Transportation and Logistics*, *11*, 100057.

Gkiotsalitis, K. (2022b). *Public transport optimization.* Springer.

Gkiotsalitis, K., Cats, O., & Liu, T. (2023). A review of public transport transfer synchronisation at the real-time control phase. *Transport reviews*, *43*(1), 88–107.

Gkiotsalitis, K., Eikenbroek, O. A., & Cats, O. (2019). Robust network-wide bus scheduling with transfer synchronizations. *IEEE transactions on intelligent transportation systems*, *21*(11), 4582–4592.

Ibarra-Rojas, O. J., Muñoz, J. C., Giesen, R., & Knapp, P. (2019). Integrating frequency setting, timetabling, and route assignment to synchronize transit lines. *Journal of Advanced Transportation*, *2019*(1), 9408595.

Ibarra-Rojas, O. J., & Rios-Solis, Y. A. (2012). Synchronization of bus timetabling. *Transportation Research Part B: Methodological*, *46*(5), 599–614.

Karmarkar, N., & Ramakrishnan, K. (1991). Computational results of an interior point algorithm for large scale linear programming. *Mathematical Programming*, *52*, 555–586.

Liang, X., de Almeida Correia, G. H., & Van Arem, B. (2016). Optimizing the service area and trip selection of an electric automated taxi system used for the last mile of train trips. *Transportation Research Part E: Logistics and Transportation Review*, *93*, 115–129.

Liu, T., Cats, O., & Gkiotsalitis, K. (2021). A review of public transport transfer coordination at the tactical planning phase. *Transportation Research Part C: Emerging Technologies*, *133*, 103450.

Liu, T., Ji, W., Gkiotsalitis, K., & Cats, O. (2023). Optimizing public transport transfers by integrating timetable coordination and vehicle scheduling. *Computers & Industrial Engineering*, *184*, 109577.

Saharidis, G. K., Dimitropoulos, C., & Skordilis, E. (2014). Minimizing waiting times at transitional nodes for public bus transportation in greece. *Operational Research*, *14*, 341–359.

Şahin, İ. (1999). Railway traffic control and train scheduling based oninter-train conflict management. *Transportation Research Part B: Methodological*, *33*(7), 511–534.

UITP, A. P. T. (2015). Mobility in cities database. *Synthesis report*.

UN. (2021). *United nations, sustainable transport conference, fact sheet climate change.* Retrieved from https://www.un.org/sites/un2.un.org/files/media_gstc/FACT_SHEET_Climate_Change.pdf

Wang, L., Xue, X., Zhao, Z., & Wang, Z. (2018). The impacts of transportation infrastructure on sustainable development: Emerging trends and challenges. *International Journal of Environmental Research and Public Health*, *15*(6). Retrieved from https://www.mdpi.com/1660-4601/15/6/1172 doi: 10.3390/ijerph15061172

Wu, W., Liu, R., & Jin, W. (2016). Designing robust schedule coordination scheme for transit networks with safety control margins. *Transportation Research Part B: Methodological*, *93*, 495–519.

Yang, Y., Jiang, X., Fan, W., Yan, Y., & Xia, L. (2020). Schedule coordination design in a trunk-feeder transit corridor with spatially heterogeneous demand. *IEEE Access*, *8*, 96391–96403.