A quantitative urban model for transport appraisal: Model estimation

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Short summary

Quantitative spatial models (QSM) are a specialised type of spatial computable general equilibrium (SCGE) models that enable the causal estimation of key parameters. By ensuring causality, a QSM can isolate the effects of policy interventions on land use and economic outcomes, such as the spatial distribution of wages and house prices, from confounding factors like exogenous shocks (e.g., pandemics). This paper summarises the core empirical tasks in Hörcher & Graham (2024), the first QSM framework designed to align with current transport appraisal practices. We address three key empirical challenges: the estimation of the gravity equation, which underpins location choices; the quantification of local geographical characteristics, including local amenities and firm productivity determinants, through model inversion; and the estimation of the relationship between accessibility and productivity. Through a better understunding of these empirical tasks, the study illustrates how QSMs can provide robust insights into the economic and spatial consequences of transport policy interventions.

Keywords: transport CBA; spatial equilibrium; wider economic impacts; causality

1 Introduction

Transport cost-benefit analysis (CBA), often referred to as benefit-cost analysis (BCA) in the US and transport appraisal in Europe, is one of the most impactful areas of application of transport modelling and economics. One limitation of the mainstream CBA practice is the partial equilibrium (PE) representation of the economy, i.e. it does not not capture explicit interactions between the transport market and potential welfare effects in other sectors of the spatial economy. This limitation could be resolved by a move from the partial equilibrium tradition to a suitably designed spatial general equilibrium (SGE) approach. Summarising the quest to produce spatially explicit numerical models that capture the impact of transport policies on location choice, the literature of land-use transport interaction (LUTI) and spatial computable general equilibrium (SCGE) models is review in Paulley & Webster (1991), Wegener (2011), and Robson et al. (2018). The use of these spatial models in practical CBA is hindered by the challenge of calibrating a large number of parameters to which the model outcomes may be highly sensitive.

A spatial model must reveal the *causal* impact of a transport policy on economic outcomes. Causality, a key concept in econometrics, is as critical in calibrating complex models as in single-equation analyses (Graham, 2025). For instance, transport improvements often coincide with unrelated shocks, complicating causal analysis. The Elizabeth Line, for example, opened during the pandemic, both influencing travel patterns in London. Standard spatial models can capture the impact of new railways but not overlapping public health crises. If calibration relies on *associational* relationships, it risks conflating effects, e.g., underestimating the Elizabeth Line's role in commuting due to pandemic-induced reductions. Avoiding data from such periods helps, but causal methods are essential to mitigate confounding in model calibration.

Since the mid-2010s, a new stream of research often referred to as *quantitative spatial economics* (Redding & Rossi-Hansberg, 2017) achieved a major methodological step in model estimation. The most influential examples of this literature include Allen & Arkolakis (2014), Ahlfeldt et al. (2015), Monte et al. (2018), and Heblich et al. (2020). Quantitative spatial models (QSMs) are SGE models designed specifically to facilitate the use of causal techniques in model estimation.

QSMs feature two core empirical techniques: model inversion and the theoretically coherent causal regressions.

This note is a short version of Hörcher & Graham (2024), a working paper that aims to bridge the gap between the state-of-the-art of transport analysis and quantitative spatial economics. A gap exists because mainstream QSMs, developed in isolation from the transport research community, are based on numerous simplifying assumptions about travel behaviour and transport supply which make their widespread use in contemporary transport analysis challenging. The context of the application of our model is Greater London, with nearly 1,000 spatial units. This short paper covers excerpts from Hörcher & Graham (2024), focusing on three core empirical tasks, namely the estimation of the gravity equation underlying households' residential and workplace location choices, model inversion, and the estimation of the elasticity of firm productivity with respect to transport connectivity.

2 Methodology

Model specification

Let us define the utility of a representative worker who resides in location i and commutes to location j as

$$U_{ij} = \left(\frac{L_{ij}}{1-\gamma}\right)^{1-\gamma}\left(\frac{K_{ij}}{\gamma}\right)^{\gamma} z_{ij}; \quad K_{ij} = \left(\frac{C_{ij}}{\beta}\right)^{\beta}\left(\frac{H_{ij}^R}{1-\beta}\right)^{1-\beta}. \tag{1}$$

In this specification L_{ij} is a measure of leisure time, K_{ij} is the composite subutility derived from consumption C_{ij} and residential floorspace use H_{ij}^R, γ and β are structural parameters, and z_{ij} is an idiosyncratic taste shock associated with the combination of locations i and j. To keep our notation simple, we suppress the unique identifier of households. Note, however, that z_{ij} takes a different value for each household.

Commuters are confronted by two constraints. First, wage w_j at workplace j and the monetary price of commuting τ_{ij} times individual labour supply x_{ij} determine the budget available for consumption, given the price of the consumption good, p_i, and the price of residential floorspace, q_i.

$$x_{ij}\left(w_j - \tau_{ij}\right) = p_i C_{ij} + q_i H_{ij}^R \tag{2}$$

Second, the sum of leisure time L_{ij} and the total time spent at work (T) and in commute (t_{ij}) cannot exceed $\bar{L}$, the daily time endowment of households.

$$\bar{L} = L_{ij} + x_{ij}\left(T + t_{ij}\right) \tag{3}$$

Again, individual labour supply $x_{ij} \geq 0$ determines how many workdays an individual is willing to provide.

The idiosyncratic utility shock is specified as an i.i.d. random draw from a Fréchet distribution:

$$F(z_{ij}) = \exp(-X_i E_j z_{ij}^{-\epsilon}), \tag{4}$$

where the average amenity (i.e. the scale parameter) is defined as the product of residence and workplace dependent local fundamentals X_i and E_j, and ϵ governs the spread of individual preferences. These assumptions lead to location choice probabilities that take the form of a commuting gravity equation.

$$\lambda_{ij} = \frac{X_i E_j \left[\frac{\upsilon_{ij}}{p_i^{\beta} q_i^{1-\beta}}\right]^{\gamma\epsilon}}{\sum_r \sum_s X_r E_s \left[\frac{\upsilon_{rs}}{p_r^{\beta} q_r^{1-\beta}}\right]^{\gamma\epsilon}} \tag{5}$$

See the derivation of the choice probability expression for a multiplicative indirect utility function and a Fréchet-distributed preference shock in the early contributions of the QSM literature, e.g. Appendix Section S.2.3 of Ahlfeldt et al. (2015), or Appendix Section C2 of Heblich et al. (2020). Workplace choice probabilities conditional on residing in i are

$$\lambda_{ij|i} = \frac{E_j\left(\upsilon_{ij}\right)^{\gamma\epsilon}}{\sum_s E_s\left(\upsilon_{is}\right)^{\gamma\epsilon}}. \tag{6}$$

To model the production side of the economy we follow the conventional approach of quantitative urban modelling more closely. Production in location j is governed by a Cobb-Douglas function of total labour input M_j^W and commercial floorspace H_j^W, with expenditure shares α and $1-\alpha$.

$$Y_j = A_j (M_j^W)^{\alpha} (H_j^W)^{1-\alpha} \tag{7}$$

We assume perfect competition in the goods market combined with zero trade cost within the urban area. Solving the firm's cost minimisation problem yields the following wage equation.

$$w_j = \alpha A_j^{1/\alpha} \left(\frac{1-\alpha}{Q_j} \right)^{\frac{1-\alpha}{\alpha}} . \tag{8}$$

Finally, our setup considers a third group of agents too: the construction sector. However, as this sector does not play a role in the rest of this short paper, we refer the reader to Hörcher & Graham (2024).

Estimating the gravity equation

Model quantification begins by estimating ϵ, the spread of the Fréchet-distributed idiosyncratic shock in household's utility function. Following Ahlfeldt et al. (2015) and Heblich et al. (2020), we express the location choice probability equation in (5) in a reduced form as

$$\log N_{ij} = \alpha_0 + \vartheta_i + \vartheta_j + \nu \cdot \log \upsilon_{ij} + \varepsilon_{ij}, \tag{9}$$

In the equation above, N_{ij} is the number of observed travellers in the commuting matrix, α_0 is the intercept capturing the denominator (multilateral resistance) in gravity equation (5), ϑ_i is a residence (origin) fixed effect, ϑ_j is a workplace (destination) fixed effect, $\nu = \gamma\epsilon$ becomes the coefficient of bilateral resistance $\ln \upsilon_{ij}$, and ε_{ij} is the error term. The main parameter of interest, ν, is transferable between this estimating equation and the location choice probability function in equation (5).

At the same time, the functional form of (9) is common in the wider empirical literature of gravity estimation in the international trade literature (Head & Mayer, 2014). This enables us to apply one of the most robust estimators in that literature, the Poisson Pseudo-Maximum Likelihood (PPML) method of Santos Silva & Tenreyro (2006). The use of the PPML estimator is motivated by three concerns. First, N_{ij} on the left-hand-side of (9) includes many zeros in the commuting matrix, as the commuting flow is effectively zero between two-thirds of MSOA-pairs in our data. In an OLS log-log model, these observations must be removed, which implies a substantial loss of information. The second concern stems from Jensen's inequality: under heteroskedasticity, the parameters of a log-linearised model estimated by OLS lead to biased estimates, because $E[\log y] \neq \log E[y]$. Third, the identification of ν in (9) is threatened by endogeneity concerns due to the non-random placement of infrastructure as well as reverse causality due to congestion, which implies that t_{ij} is potentially endogenous. Thus, we instrument the observed travel times by the Euclidean distance between i and j, which is independent from infrastructure investment decisions as well as traffic congestion levels.

Model inversion

Based on the definitions and location choice probabilities defined above, the link between workplace and residential populations can be expressed as

$$N_j^W = \sum_i \lambda_{ij|i} \cdot N_i^R. \tag{10}$$

We can express fundamental amenity levels E_j after substituting (6) into (10):

$$E_j = N_j^W \left(\sum_i \frac{\upsilon_{ij}^{\gamma\epsilon} N_i^R}{\sum_s E_s \upsilon_{is}^{\gamma\epsilon}} \right)^{-1} \tag{11}$$

This implies an equation for each location in function of all other E_j's that we can solve for iteratively to recover the unique vector of workplace amenity levels from the observed distribution of residential and workplace populations and the observed determinants of υ_{ij}.

Residential amenities are recoverable in a similar fashion. Let us introduce

$$\tilde{X}_i = X_i \left(q_i^{1-\beta}\right)^{-\gamma\epsilon}, \tag{12}$$

which captures all location-dependent endogenous variables of location choice probability (5). The values of $\tilde{X}_i$ are the solution of

$$\tilde{X}_i = N_i^R \left(\sum_j \frac{\upsilon_{ij}^{\gamma\epsilon} N_j^W}{\sum_r \tilde{X}_r \upsilon_{rj}^{\gamma\epsilon}}\right)^{-1}. \tag{13}$$

From the solutions, X_i is quantified using data on residential floorspace prices q_i and by inverting (12) above.

The impact of connectivity on productivity

Local productivity A_j, or total factor productivity in production function (7), is measured by rearranging the wage equation in (8):

$$A_j = \left(\frac{w_j}{\alpha}\right)^{\alpha} \left(\frac{1-\alpha}{Q_j}\right)^{\alpha-1}. \tag{14}$$

As we observe both wages (w_j) and commercial floorspace prices (Q_j), the A_j vector is computed by directly substituting the observed data into (14).

Following the agglomeration literature (Combes et al., 2019; Ahlfeldt et al., 2015) we decompose local firm productivity into an exogenous component and a multiplier dependent on a measure of economic density, i.e. access to economic mass (ATEM), denoted by ρ_j.

$$A_j = a_j\, \rho_j^{\eta} \tag{15}$$

Parameter η is the agglomeration elasticity of firm productivity. We select the effective labour supply as the measure of economic activity, to consider that individual labour supply is endogenous in our model, and therefore employment (i.e., workplace population) alone is not a comprehensive measure of economic mass.

$$A_j = a_j \left[\sum_s \exp(\delta t_{sj})\, N_s^W \bar{x}_s\right]^{\eta} \tag{16}$$

The general form of our estimating equation based on (15) is

$$\log \hat{A}_j = \eta \log \rho_j(\delta) + \vartheta_{j\in z} + \varepsilon_j, \tag{17}$$

in which $\hat{A}_j$ are the productivity residuals recovered via model inversion in the previous step, $\vartheta_{j\in z}$ are borough fixed effects, and ε_j is the error term. The core endogeneity concerns well-known in the literature are that (i) ATEM may be correlated with unobserved local characteristics, e.g. natural advantages/endowments, that also affect productivity, and (ii) through the endogenous location choice of firms and competition forces, productivity may also affect the magnitude of agglomeration, fueling reverse causality (Combes et al., 2019; Graham & Gibbons, 2019). To identify the causal effect of agglomeration on productivity, we deploy instrumental variables and control function techniques in three alternative specifications.

3 Results

Commuting gravity

The results for six model specifications are summarised in Table 1. Models (1) and (2) are OLS estimates of the model, with and without origin and destination fixed effects. These models rely on a restricted sample because OLS cannot handle zero flows after the log transformation. It is remarkable though that the elasticity estimate in model (2) is close to our most preferred one in

(4). Models (3) and (4) are Poisson models with fixed effects. A common endogeneity concern is that impedence between i and j may not be independent from the flows themselves. One potential reason may be the non-random placement of intrastructure. To address this concern, we instrument υ_{ij} by the Euclidian distance in model (4), which gives our preferred estimate.

Table 1: Estimating commuting gravity

	Dependent variable: log commuting flow					
	(1)	(2)	(3)	(4)	(5)	(6)
Impedance	real wage	real wage	real wage	real wage	travel time	gen.cost
Notation	υ_{ij}	υ_{ij}	υ_{ij}	υ_{ij}	t_{ij}	$\bar{v}t_{ij} + \tau_{ij}$
Method	OLS†	OLS†	PPML	PPML+IV	PPML	PPML
Instrument				Eucl.dist		
Estimate	2.959***	12.303***	19.035***	10.193***	-0.893***	-0.855***
	(0.011)	(0.021)	(0.060)	(0.028)	(0.01)	(0.009)
Fixed effects	no	yes	yes	yes	yes	yes
RMSE	1.078	0.657	8.815	8.684	13.948	13.686
AIC	1,052,942	708,040	2,767,492	2,836,697	5,290,953	5,524,191
BIC	1,052,974	729,218	2,790,654	2,859,856	5,314,115	5,547,353
# of obs.	352,300	352,300	966,289	965,306	966,122	966,122

†: Only origin-destination pairs with positive flows are included.
Standard errors in parentheses, ***: 99%, **: 95%, *: 90%

Assuming that commuters have a fixed travel time budget of 1 hour on average (Kung et al., 2014), such that $\gamma = (T+1)/24$, $\epsilon = \nu/\gamma$ is computed from our preferred empirical estimate of $\hat{\nu} = 10.193$. Note that all four elasticities and the resulting $\hat{\epsilon}$ values are higher in models (2) to (4) than the single-digit estimates in other QSM studies in the literature. In models (5) and (6) we estimate the gravity equation using travel times and generalised travel costs as impedance measures. This exercise confirms that we get lower values when the regular impedance measure is used.

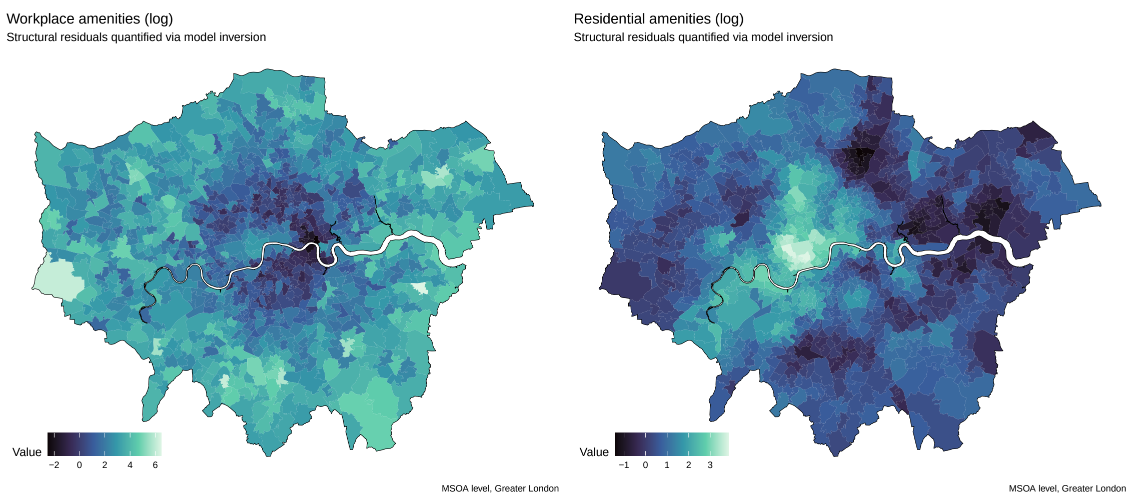

Figure 1: Workplace and residential amenities recovered as structural residuals of the model, assuming that the observed data captures a spatial equilibrium

Fundamental local amenities

Next, let us plot the $\{E_j\}$ residential amenity and $\{X_j\}$ workplace amenity vectors after model inversion. The residuals in Figure 1 reveal interesting patterns. The results show that the locations in Central London – with the exception of Westminster and the nearby MSOAs – are not particularly pleasant places to work, controlling for the very high wages offered by these places. An inner ring surrounding the City of London and Canary Wharf has particularly low amenities for working. By contrast, some of the locations in the suburbs are more attractive for working than what their wage levels would justify. Lower density and congestion externalities (i.e. noise, pollution) may explain some of these results. By contrast, the most appealing residential locations are clustered around the central areas of Finchley Road and Swiss Cottage, Kensington and Chelsea, and the low-density residential neighbourhoods of Richmond.

Access and firm productivity

The models reported in Table 2 differ in the specification of ρ_j, the inclusion of fixed effects, and the estimation method determined by the identification strategy. In models (1) and (2), the general functional form in (15) remains unchanged but we ignore agglomeration spillovers between locations, so ρ_j does not depend on the spatial impedance between MSOAs. More specifically, ρ_j is the density of employment. Model (1) is a crude OLS estimate. In model (2) we instrument the log of the employment density by a set of historic and geographical variables.

Models (3) and (4) have been estimated in a two-step process which captures the decay in spillover effects between nearby MSOAs, following Koster (2024). In the first step we create 2.5-minute-wide concentric doughnuts around location j denoted by $\mathcal{R}_{jr}$, aggregate the effective labour supply in each ring into and estimate the contribution of each distance ring to the measure of ATEM in j via the parameter vector $\{d_r\}$. As $\log \hat{A}_j$ depends on η and the ring-specific parameters non-linearly, we estimate the model with nonlinear least squares (NLS). Then, in the second step we fit a curve on the coefficient estimates to quantify δ as the parameter of τ_r, the mean travel time between locations in ring r and location j. This non-linear specification is no longer suitable to apply instrumental variables, as one would need to instrument a function of a set of unknown parameters. Thus, we follow Koster (2024) again and apply a control function approach.

The standard errors reported in Table 2 are computed by bootstrapping the entire model inversion and parameter estimation process 200 times.

Table 2: Agglomeration economies and distance decay

	Dependent variable: log productivity residual			
	(1)	(2)	(3)	(4)
ATEM measure	Emp.density	Emp.density	Total emp.†	Total emp.†
Method	OLS	2SLS	NLS+CF	NLS+CF
Productivity elasticity, η	0.125***	0.170***	0.155***	0.149***
	(0.005)	(0.016)	(0.009)	(0.011)
Distance decay, δ			-0.082***	-0.079***
			(0.012)	(0.012)
Borough fixed effects	no	yes	yes	yes
RMSE	0.08	0.06	0.06	0.06
AIC	-2122.05	-2573.61	-2665.81	-2610.97
BIC	-1950.88	-2402.44	-2372.37	-2317.53
# of obs.	983	983	983	983

†: Total employment is aggregated in 2.5-min travel time bands.
Standard errors in parentheses, ***: 99%, **: 95%, *: 90%

The naïve model in column (1) yields an agglomeration elasticity of 12.5%. The 2SLS regression with employment density as the ATEM measure produces a higher result. The two control function specifications lead to estimates in between the OLS and 2SLS results. Our preferred model is (4), with an elasticity of $\hat{\eta} = 14.9\%$ and a distance decay of $\hat{\delta} = -0.079$, which implies that spillovers fade quickly after 15 to 20 minutes of travel time.

The estimated agglomeration elasticity $\hat{\eta} = 14.9\%$ is at the higher end of the values found in the previous literature (see Graham & Gibbons, 2019). However, our result does not stand out from previous studies focusing on Greater London, specifically. Dericks & Koster (2021) developed a QSM using the same case study context. Exploiting the exogenous variation in density caused by

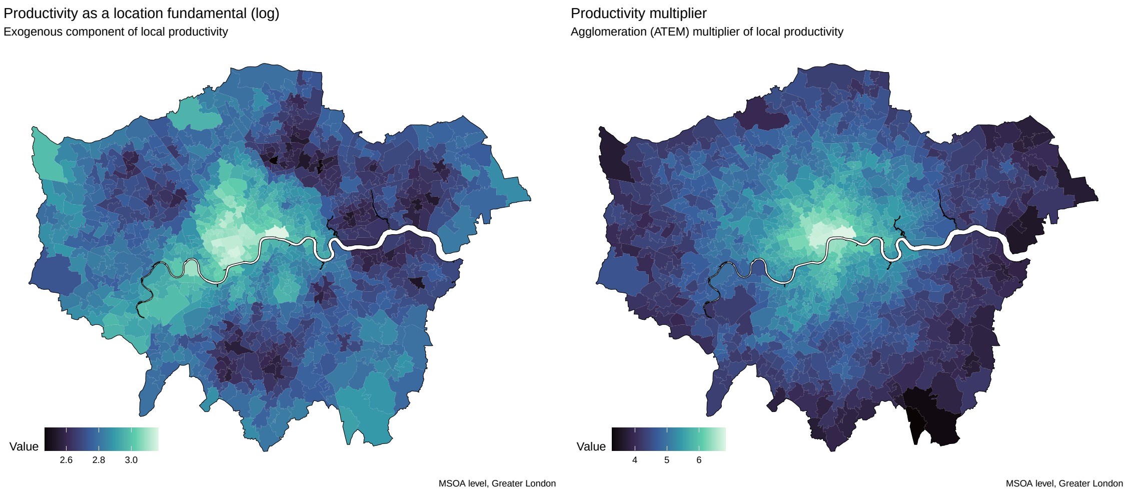

Figure 2: Productivity as a location fundamental and the multiplier generated by agglomeration

bombings during WW2, the agglomeration elasticity they estimate is somewhat higher than ours, 19.6%. As a sensitivity check, we re-estimate model (3) using MSOAs under and above the median distance from the CBD. We find that the elasticity goes up to 19.1% for the central subsample while it reduces to 6.3% for the more perpheral half of London. As we move away from the centre of the city, this clearly non-linear pattern seems to converge close to the UK-wide average of 4.3% recommended by the UK Transport Appraisal Guidance and the 4.7% mean of the global literature reported by Graham & Gibbons (2019).

Given the above estimated $\hat{\eta}$ and $\hat{\delta}$ parameters, the observed travel time matrix, and the quantified values of A_j, we recover the $\{\hat{a}_j\}$ vector from (16).

$$\hat{a}_j = \hat{A}_j \left[\sum_s \exp(\hat{\delta} t_{sj})\, N_s^W \bar{x}_s \right]^{1/\hat{\eta}} \tag{18}$$

The $\{\hat{a}_j\}$ vector captures fundamental geographical properties that make firms more productive in certain locations, controlling for the level of access to economic mass. Figure 2 provides a visual illustration of the decomposition outlined in equation (15).

4 Conclusions

This short paper delivers insights into the core steps of estimating the quantitative spatial model of Hörcher & Graham (2024). The process of model estimation yields empirical results with immediate policy relevance and contributions to the related literature. The fully quantified model can be used to predict and evaluate various transport policy interventions at a transformative scale. For a detailed impact assessment of the Elizabeth Line in London, using the model presented in this paper, the interested reader is referred to Hörcher & Graham (2024).

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