Optimal toll-pricing based on a dynamic multi-region MFD traffic model with elastic demand

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SHORT SUMMARY

Multi-region Macroscopic Fundamental Diagram (MFD) traffic models have been developed as a more easily calibratable, maintainable, and computationally efficient alternative to traditional link-based traffic assignment models with full disaggregate network representation. In this paper we use the dynamic multi-region MFD Stochastic User Equilibrium (SUE) model recently developed in Duncan et al. (2024) to explore optimal toll-pricing. We first extend the model to account for elastic demand. Then, we integrate the model within a toll-price optimisation procedure, where the tolling scheme is travel-time-based and the objective function maximises social welfare. We first test the optimisation procedure in a simple example multi-region MFD system, and then apply the procedure to optimise toll prices in a real-life large-scale and detailed case study of Zealand, Denmark. Results show that the model and optimisation procedure can effectively optimise toll prices to instigate a beneficial change in travel behaviour.

Keywords: multi-region macroscopic fundamental diagram traffic model, dynamic stochastic user equilibrium, elastic demand, toll-price optimisation, maximising social welfare

1. INTRODUCTION

Traffic congestion is a recurring problem for societies. The many hours wasted in congestion each day results in huge economic losses, and has a negative impact on the environment and public health. An effective way of alleviating congestion is to encourage a change in travel behaviour through road use tolling. The literature on toll analysis with traditional link-network traffic assignment models is vast. However, firstly, the computational burden of operating with such detailed models can limit the rigor in which analyses can be conducted. Secondly, the implementation of congestion pricing on a link-level has been found to be impractical, due to large investments being required to upgrade infrastructure and regulate the scheme. As such, most real-world tolling systems are area-based, for example in Singapore, London, Stockholm, and Milan.

Therefore, rather than analysing tolling with traditional 'microscopic' traffic models, there is a growing interest in using more aggregate 'macroscopic' traffic models, where traffic conditions are captured over entire regions through Macroscopic Fundamental Diagrams (MFDs). The appeal of such being greater computational efficiency, easier calibration, and suitability for area-based tolling scheme analysis.

Tolling with region-based MFD traffic models has been explored in several studies. Geroliminis & Levinson (2009), Amirgholy & Gao (2017), and Daganzo & Lehe (2015) use a single-region MFD model to address the morning commute problem, using dynamic tolling to alter departure times to maximise outflow / user benefits. Wang & Gayah (2021) use an MFD traffic model to explore tolling two urban regions to push traffic onto two motorway regions. Genser & Kouvelas (2022) use a multi-region MFD traffic model to identify optimal real-time dynamic tolls for maintaining system optimum. Zheng & Geroliminis (2013) use a multimodal-MFD model to optimise dedicated bus lane allocation and congestion pricing, and Zheng & Geroliminis (2020) adapt the work to maximise equity through different value of time groups. Many studies have used a traffic simulation software package to simulate traffic given tolling in a single zone, but we do not class these as using region-based MFD traffic models.

There are numerous gaps in the research into tolling with region-based MFD traffic models. In particular, few studies:

- Explore tolling with a multi-region MFD traffic model.
- Account for stochasticity within the regional path choice.
- Account for the impacts of tolling on travel demand.
- Aim to maximise social welfare.
- Apply methodologies to real-life case studies.

This study addresses each these gaps concurrently. In Duncan et al. (2024) we developed a new dynamic multi-region MFD SUE traffic model, and calibrated and applied it in a real-life large-scale and detailed case study of Zealand, Denmark with 135 regions and motorways considered separately. The current study first extends this model to account for elastic demand, and then integrates it within a time-based toll-price optimisation process for maximising social welfare. The model and toll-price optimisation process are tested in an illustrative example and then applied to the real-life case study.

The paper is structured as follows. In Section 2 we introduce the dynamic multi-region MFD SUE traffic model developed in Duncan et al (2024). In Section 3 we describe how a time-based tolling scheme can be captured by the model. In Section 4 we extend the model to account for elastic demand. In Section 5 we integrate the model within a toll-price optimisation process. In Section 7 we optimise the tolls in a real-life case study and examine the results. In Section 8 we summarise the work and provide thoughts on future research.

2. THE DYNAMIC MULTI-REGION MFD SUE MODEL

In this section, we discuss the overall framework of the dynamic multi-region MFD SUE model introduced in Duncan et al. (2024).

We begin by detailing general multi-region MFD traffic modelling concepts. The study area is partitioned into a set of regions. The traffic conditions in each region are described by a speed-MFD function that maps accumulation (number of vehicles in the region) to average speed of the vehicles in the region. As accumulation increases, average MFD speed decreases. There is a set M of both internal and external Origin-Destination (OD) movements, i.e. trips originating and destinating in the same region and trips originating in one region and destinating in another, respectively. A regional path is defined as a sequence of regions traversed when travelling an OD movement. P_m is the choice set of regional paths for OD movement $m \in M$. The total runtime period of the system is split into an indexed set Ψ of discrete time-slices, each with duration ε. The travel demands d_m^τ for each regional OD movement $m \in M$ departing at a given time-slice

$\tau \in \Psi$, are obtained by aggregating travel demands from the underlying network ODs over the time-slice between the OD regions. The travel demand d_m^τ for OD movement m departing at time-slice τ is split among the available regional paths $p \in P_m$ according to a regional path choice model, to give the regional path flows $f_{m,p}^\tau$. For a given accumulation level and thereby average MFD speed in a region at a given moment in time, the travel time of a region when travelling a particular regional path at that moment in time, is obtained by dividing the regional-path-specific region length by MFD speed.

The traffic dynamics of the dynamic multi-region MFD SUE model are described by a traffic propagation model utilising features of a Space-Time Graph (STG). Due to the word constraints of this extended abstract, we briefly describe the model. The travel demand for each time-slice is assumed to depart uniformly and continuously, and, throughout each time-slice, all drivers are assumed to be experiencing the same speed in a region. Vehicles departing at the beginning and end of each time-slice travelling each regional path are tracked from origin to destination on the STG, based on region travel times, see Fig. 1. Occupied STG areas of regional path flows are then used to calculate accumulation levels, which feed back to determine average vehicle speeds in a region during a time-slice (through the speed-MFD function), and thereby region travel times. The traffic propagation model is thus naturally expressed as a fixed-point problem in terms of region travel times.

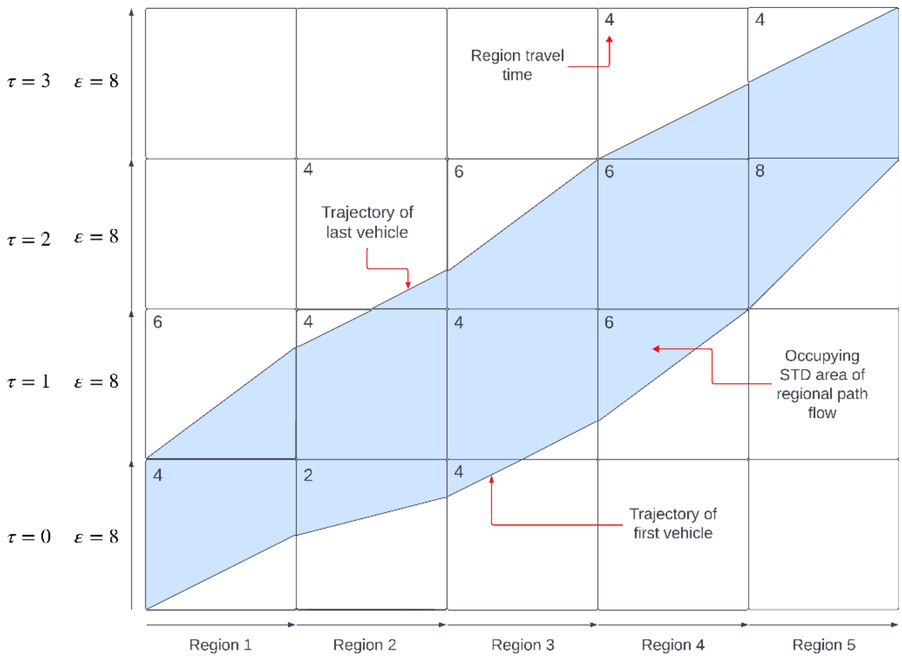

Fig. 1. Demonstration of traffic propagation on space-time graph.

The dynamic multi-region MFD SUE model embeds the fixed-point traffic propagation model within an overall traffic user equilibrium model for equilibrating regional path flows. Regional path choice is based on regional path travel times actually experienced (rather than instantaneous travel times). The travel cost function for r-path $p \in P_m$ is assumed in this study to depend on experienced travel time and distance (and later toll), so that the travel cost for regional path $p \in P_m$ is as follows:

$$c_{m,p}^\tau = \alpha_{tt} \cdot \bar{T}_{m,p}^\tau + \alpha_l \cdot L_{m,p},$$

(1)

where $\bar{T}_{m,p}^{\tau}$ is the experienced travel time in [min] of regional path $p \in P_m$ when departing at time-slice τ, $L_{m,p}$ is the distance in [km] travelled along regional path $p \in P_m$, α_{tt} is the Value of Time (VOT) in [DKK/min], and α_l is the Value of Distance (VOD) in [DKK/km]. $c_{m,p}^{\tau}$ is thus in DKK.

Dynamic multi-region MFD SUE conditions are established as follows: A demand-feasible universal regional path flow vector $\boldsymbol{f}^*$ of all regional path flows departing at all time-slices, is a dynamic multi-region MFD SUE solution iff:

$$f_{m,p}^{\tau} = d_m^{\tau} Q_{m,p}^{\tau}\left(\boldsymbol{c}\left(\bar{\boldsymbol{T}}\left(\boldsymbol{t}^*(\boldsymbol{f})\right)\right)\right), \qquad \forall p \in P_m, \forall m \in M, \forall \tau \in \Psi,$$

(2)

where $Q_{m,p}^{\tau}$ is the choice probability of regional path $p \in P_m$ at time-slice $\tau \in \Psi$, given the vector of regional path travel costs $\boldsymbol{c}$, given the experienced regional path travel times $\bar{\boldsymbol{T}}$, given the vector of equilibrated region travel times $\boldsymbol{t}^*$ (from the traffic propagation model), given the regional path flows $\boldsymbol{f}$.

To demonstrate, consider the example multi-region system in Fig. 2, where there are four regions, one OD movement from region 1 to 4, and two regional paths. Fig. 3 plots the demand profile over the course of the day, where as can be seen there is a morning and evening peak. Upon solution of the dynamic multi-region MFD SUE model with a time-slice grain of 30-minutes and a Multinomial Logit (MNL) regional path choice model, Fig. 4A displays how the regional path travel costs vary across the day and Fig. 4B displays how the regional path choice probabilities vary. As can be seen, the travel cost of r-path 1 is much cheaper than r-path 2 off-peak, and so the majority of travellers take r-path 1. During peak hours, accumulation levels in region 2 increase and therefore so does the travel time and thus travel cost of r-path 1. The cost difference is less between r-paths 1 & 2 during congested periods and thus drivers distribute themselves more evenly between the two r-paths.

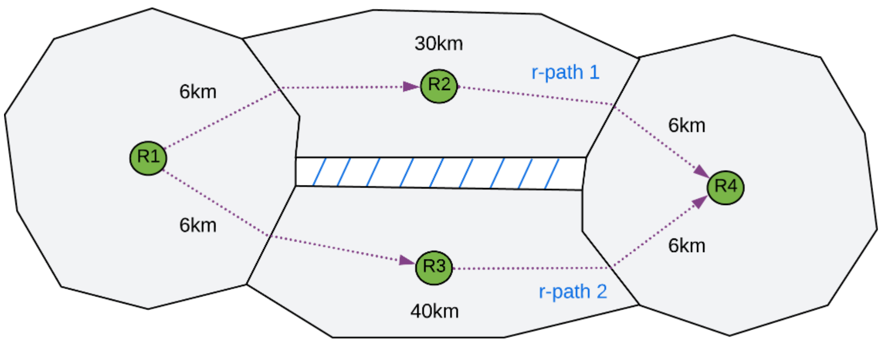

Fig. 2. Example multi-region system.

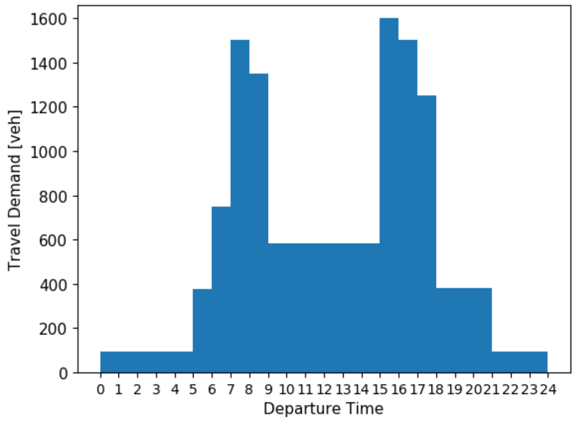

Fig. 3. Travel demand departing over the course of the day.

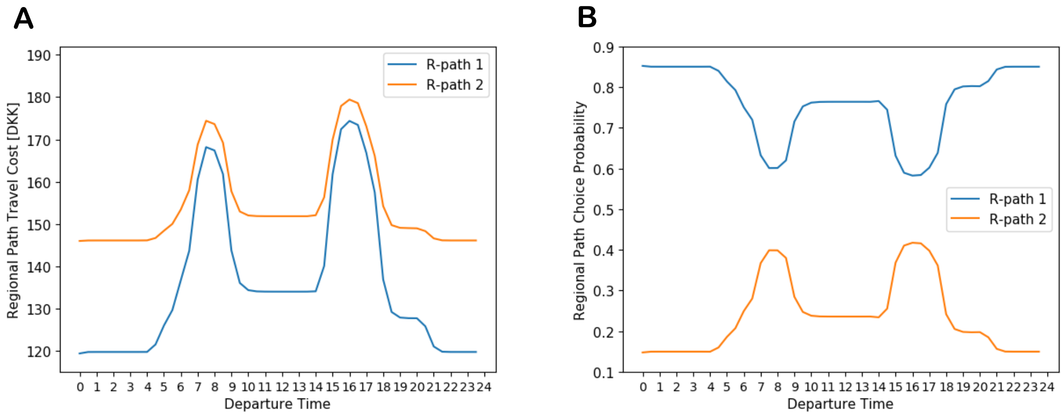

Fig. 4. No Tolling Scenario. A: Regional path travel costs across the day. B: Regional path choice probabilities.

3. TIME-BASED TOLLING

The tolling scheme we explore is travel-time-based. A time-based toll is imposed when travelling within specified regions during specified times. The price of the toll depends on the time spent travelling in the region, which could be traveller's actual time spent, or the travel time predicted from the model they would have spent given their departure time and regional path. Let ω_r^τ be the toll-price in [DKK/min] for travelling in region r during time-slice τ, where $\boldsymbol{\omega}$ is the vector of all tolls. By combining the travel time experienced in each region of a regional path with the toll-price in the region at the time-slice of travel, one can calculate the experienced toll $\kappa_{m,p}^\tau$ in [DKK] of regional path $p \in P_m$ when departing during time-slice τ. This is then added to the travel cost function in (1).

To demonstrate, consider the example multi-region system in Fig. 2. Suppose that a time-based toll is imposed upon travelling in region 2 between 7-9am and 3-6pm, i.e. the morning and evening peaks, with price $\omega_2 = 0.5$ [DKK/min]. Fig. 5A displays how the regional path travel costs vary across the day and Fig. 5B displays how the regional path choice probabilities vary. As shown, with the toll for travelling r-path 1 during the peak hours, the travel costs are almost equal during the peak hours, resulting in almost even split r-path choice probabilities.

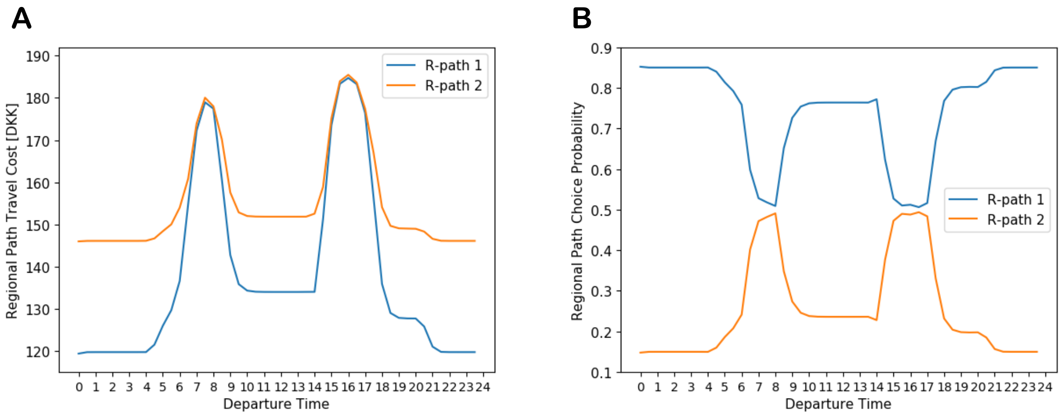

Fig. 5. Tolling Scenario. A: Regional path travel costs across the day. B: Regional path choice probabilities.

4. ELASTIC DEMAND

To account for how travel demand may vary depending on the Level of Service (LoS), we incorporate elastic demand within the model. For a given setting of the tolls $\boldsymbol{\omega}$, the LoS for OD movement m and departing time-slice τ is calculated as follows:

$$\tilde{C}_m^\tau(\boldsymbol{\omega}) = \sum\nolimits_{r \in R_{m,p}} Q_{m,p}^\tau(\boldsymbol{\omega}) \cdot c_{m,p}^\tau(\boldsymbol{\omega}), \tag{3}$$

where $Q_{m,p}^\tau$ and $c_{m,p}^\tau$ are the choice probability and cost of regional path $p \in P_m$ when departing during τ. $\tilde{C}_m^\tau$ is thus the average predicted cost of travelling OD movement m when departing during τ.

The elastic demand function we assume is a power law function:

$$\tilde{d}_m^\tau(\boldsymbol{\omega}) = d_m^\tau \left(\frac{\tilde{C}_m^\tau(\boldsymbol{\omega})}{\tilde{C}_m^{\tau,NTS}} \right)^{-\gamma}, \tag{4}$$

where $\tilde{C}_m^{\tau,NTS}$ is the LoS under the No Tolling Scenario (NTS) and $\gamma \geq 0$ is the demand elasticity parameter for car. Dynamic multi-region MFD SUE with elastic demand replaces d_m^τ in (2) with $\tilde{d}_m^\tau$.

To demonstrate, consider again the example multi-region system in Fig. 2. Under the same tolling scenario as described in Section 3, Fig. 6A displays the percentage difference in travel demand across the day upon solution of the elastic demand model with $\gamma = 0.75$. As can be seen, the demand decreases for time-slices in which travellers travel in region 2 during the tolled peak hours, decreasing by up to around 5.3%. Demand increases slightly for time-slices around these decreased demand time-slices. Fig. 6B shows how the demand during the peak hours decreases as γ increases.

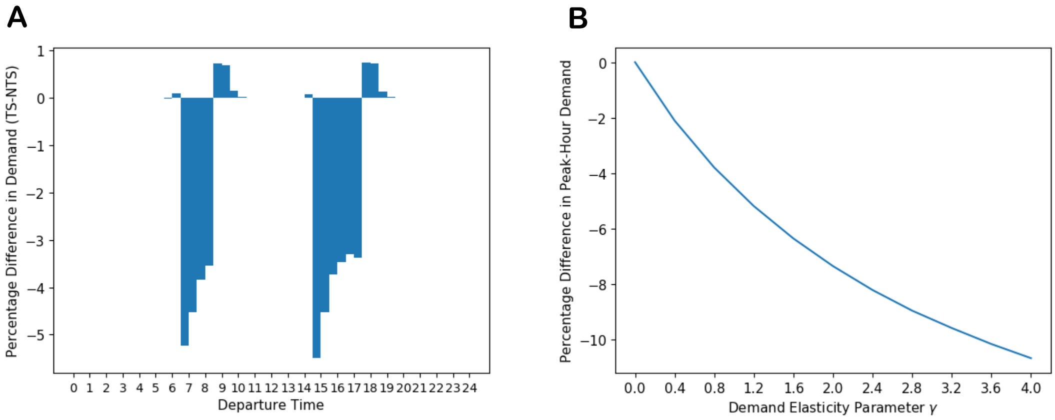

Fig. 6. Tolling Scenario. A: Percentage differences in demand across the day between tolling and no tolling scenarios. B: How peak hour demand decreases with the elasticity parameter.

5. MAXIMISING SOCIAL WELFARE

The objective function to maximise social welfare is as follows:

$$Z_{SW}(\boldsymbol{\omega}) = Z_{CB}(\boldsymbol{\omega}) + Z_{PB}(\boldsymbol{\omega}),$$

where Z_{CB} and Z_{PB} are the consumer benefit and producer benefit, respectively:

$$Z_{CB}(\boldsymbol{\omega}) = \sum_{\tau\in\Psi}\sum_{m\in M}\int_{d_m^\tau}^{\tilde{d}_m^\tau(\boldsymbol{\omega})} \tilde{C}_m^{\tau,NTS}\left(\frac{x}{d_m^\tau}\right)^{\gamma}.dx + \sum_{\tau\in\Psi}\sum_{m\in M}\left\{d_m^\tau \tilde{C}_m^{\tau,NTS} - \tilde{d}_m^\tau(\boldsymbol{\omega})\tilde{C}_m^\tau(\boldsymbol{\omega})\right\},$$

and,

$$Z_{PB}(\boldsymbol{\omega}) = \sum_{\tau\in\Psi}\sum_{m\in M}\sum_{p\in P_m} f_{m,p}^{\tau,*}(\boldsymbol{\omega})\cdot\kappa_{m,p}^\tau(\boldsymbol{\omega}).$$

The first term in Z_{CB} is the inverse demand component of consumer benefit and the second term is the improvement in LoS from the NTS. Z_{PB} is equal to the toll revenue. This social welfare objective function has been used in a variety of studies such as Watling et al. (2015), where a discussion of the derivation of this measure can be found.

To demonstrate, consider again the example multi-region system in Fig. 2. Fig. 7A displays, for fixed demand, how social welfare varies as the time-based toll price in region 2, ω_2, is varied. Fig. 7B displays results with elastic demand. As shown, the social welfare objective functions for both fixed and elastic demand are concave around optima, which are around 0.5DKK/min and 0.7DKK/min, respectively.

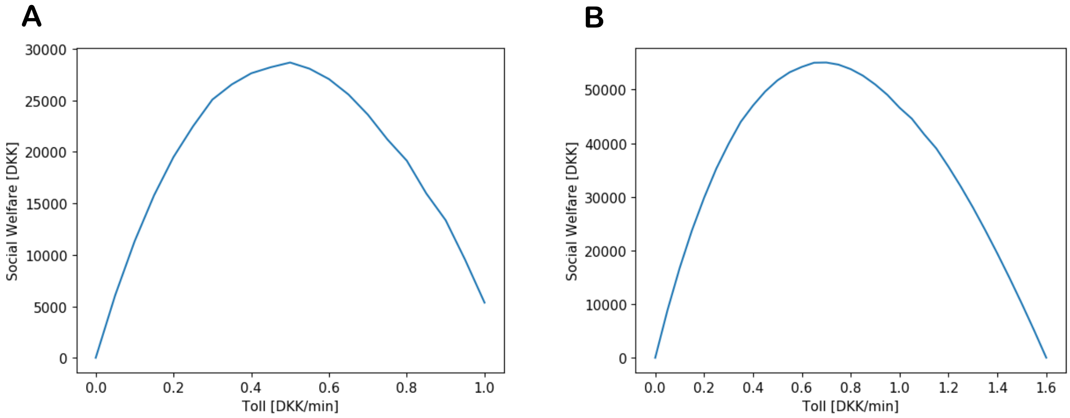

Fig. 6. Tolling Scenario. How social welfare varies with time-based toll-price. A: Fixed demand. B: Elastic demand.

6. CASE STUDY

For details on the setup of the real-life large-scale case study and how it was set-up, we direct the reader to Section 5 of Duncan et al. (2024). The large-scale study area of Zealand, Denmark (see Fig. 8) was partitioned into 39 underlying urban and rural areas (see Fig. 9) and 96 motorway regions superimposed upon these underlying regions and treated separately (see Fig. 10). The tolled regions are all those with underlying regions 3, 4, 7, 8, 9, or 10, which are the regions constituting the city of Copenhagen. Tolls are enforced between 7-9am and 3-6pm.

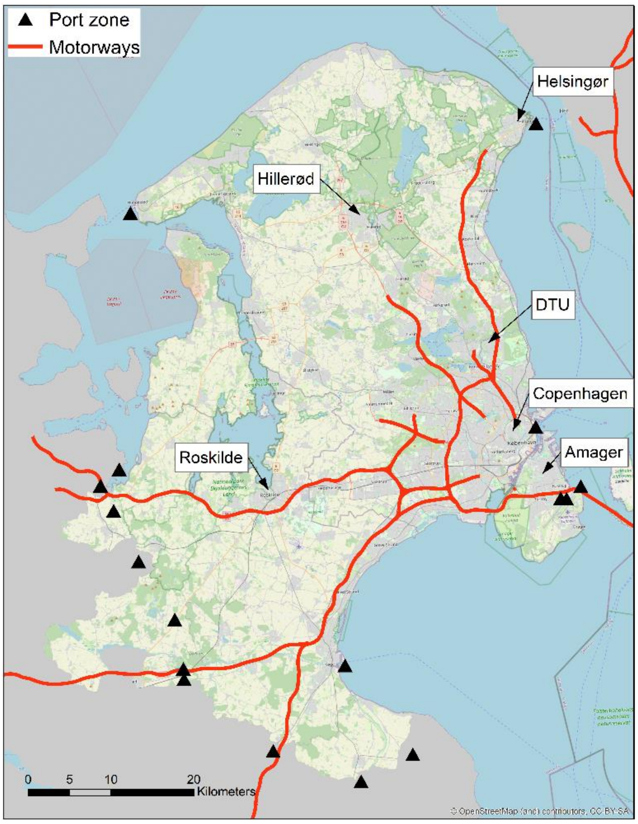

Fig. 8. Large-scale case study area of Zealand, Denmark.

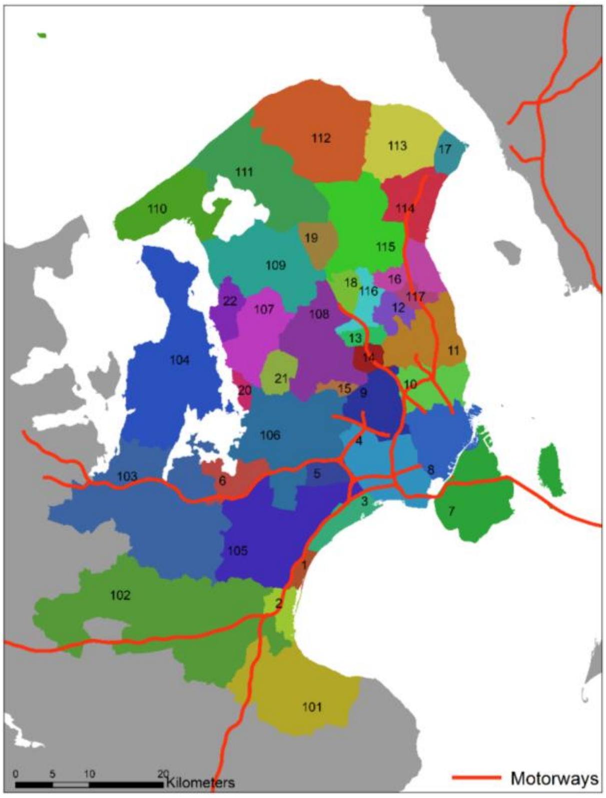

Fig. 9. Underlying urban/rural region partitioning

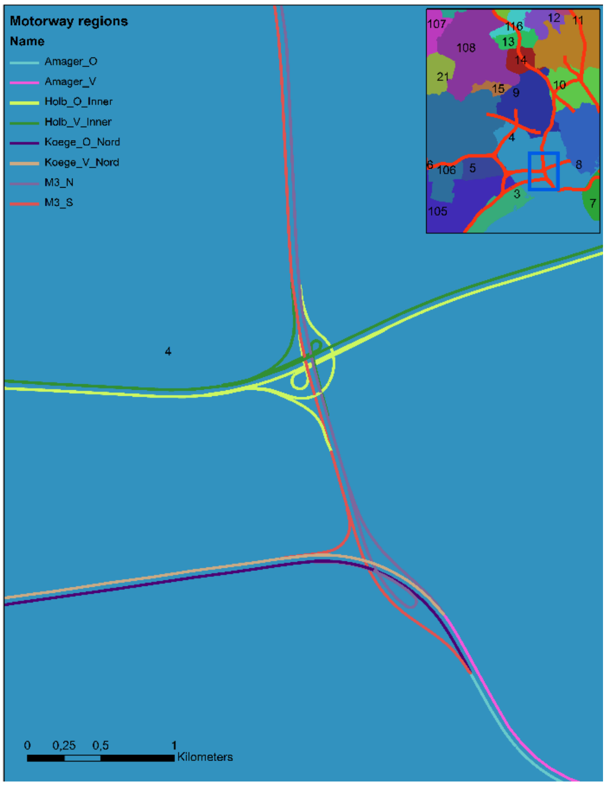

Fig. 10. Demonstration of superimposed motorway region partitioning.

Fig. 11A displays, for fixed demand, how social welfare varies as the time-based toll price in the tolled regions, ω, is varied. Fig. 11B displays results with elastic demand. As shown, the optimal toll-price is 12 DKK/min for fixed demand and 15 DKK/min for elastic demand. These values are rather high, indicating a strong reluctance for drivers to travel around the tolled regions.

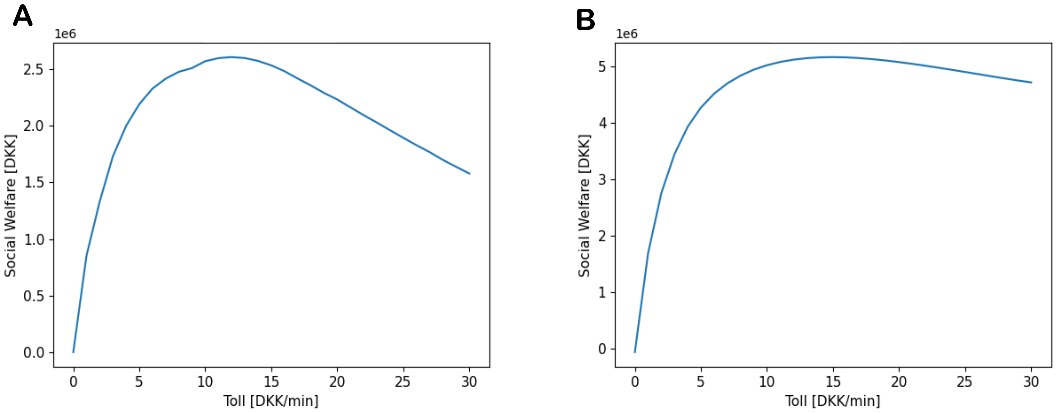

Fig. 11. Real-life case study. How social welfare varies with time-based toll-price. A: Fixed demand. B: Elastic demand.

7. SUMMARY AND FUTURE RESEARCH

This study has extended the dynamic multi-region MFD SUE model introduced in Duncan et al. (2024) to account for elastic demand, and then integrated it within a time-based toll-price optimisation process for maximising social welfare. In both a simple example multi-region system and a real-life case study we have shown that the model and optimisation procedure can effectively optimise toll prices to instigate a beneficial change in travel behaviour.

Although not reported in this extended abstract due to the word constraints, we have also extended the model to account for the effects of tolling on departure time choice. These will be presented at the conference. In future research we aim to consider heterogeneity in VOTs and equity within the proposed toll-price optimisation procedure, as well as explore a multi-objective objective function balancing maximising social welfare with minimising emissions. Beyond toll-price optimisation, we also aim to use the model to explore behaviour changes / emissions under different low emission zone policy specifications in Copenhagen.

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