A multi-objective vehicle routing and public transport rescheduling model

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Short summary

The increasing demand for Mobility-as-a-Service requires improved integration between public transport and shared mobility services. This research addresses the Vehicle Routing and Public Transport Rescheduling Problem (VRP-PTR). Initially formulated as a mixed-integer non-linear program (MINLP), the model is reformulated into a mixed-integer linear program (MILP) with the aim of reducing the operational costs of service providers and travel times of passengers. Given the multi-objective nature of the optimization problem, the ϵ-constraint method is employed, resulting in the computation of Pareto optimal solutions. A case study in Athens, Greece, is examined, which considers shuttle buses as on-demand services. These buses transport passengers from various pickup points to the most time-appropriate railway trip originating at Larissa Railway Station, with rescheduling options if necessary. The results demonstrate the model's potential as a tactical planning tool for optimizing the routes of shared modes and synchronizing them with public transport to meet dynamic passenger demand.
Keywords: ϵ-Constraint; Multi-objective Optimization; On-demand service; Railway transport service; Rescheduling; Vehicle Routing Problem.

1 Introduction

The present study focuses on the integration of on-demand and public transport services, leveraging the increasing demand for Mobility-as-a-Service (MaaS) schemes (Hensher, 2024; Research & Markets, 2024) and the establishment of Mobility Hubs across Europe (Evi De Rudder, 2024; SUM, 2024). This integration aims to reduce door-to-door travel times for passengers, making public transport a competitive alternative to private vehicles.

Shared mobility services can cover the first or last mile legs of public transport trips. These services operate without fixed routes or schedules (Machado et al., 2018). Shared mobility can be classified into three types: (i) simultaneous shared mobility, where multiple passengers travel together in the same vehicle, as in public transport, (ii) sequential shared mobility, where individual users access the same vehicle sequentially, and (iii) combined shared mobility, which merges the simultaneous and sequential shared mobility categories (Guyader et al., 2021).

Recent research in this scientific field has explored the role of Flexible On-Demand Transport Services (FDTS) in shaping urban mobility (Barreto et al., 2019). A pioneering contribution has been the development of a path-based formulation to optimize the integration of on-demand services with traditional urban bus networks, as well as improve service coverage and reduce operational costs (Steiner & Irnich, 2020). In past studies, the integration of on-demand and public transport services has been examined utilizing the MATSim agent-based mathematical framework (Kagho et al., 2021).

For the on-demand and public transport integration problem has also been modeled as a mixed-integer non-linear program (MINLP) (Kumar & Khani, 2022). Kumar & Khani (2022) explored the strategic behavior of passengers in multimodal networks using a passenger assignment model, determining the transit routes that need to be operated, their frequency, the fleet size of vehicles, and the passenger flow on both road and transit networks. The regulatory and theoretic framework in this field has explored the integration of Automated Mobility-On-Demand (AMOD) services with public transport through a tri-level modeling framework, aiming to provide sustainable urban mobility solutions and optimize policies for Mobility Service Providers (MSPs) (Dandl et al., 2021; Oeschger et al., 2020).

The integration of bike-sharing systems with public transport has been explored in past studies using trip records, demand data obtained from bike-sharing systems and public transport systems, census demographics, and points of interest (Kim, 2023). Additionally, through statistical analysis, it has been proven that factors such as trip duration, public transport frequency, and whether the rider is a subscriber or a casual customer affect the integration of public transport with bike sharing (Kong et al., 2020).

Considering the above scientific work, it is evident that there is greater emphasis on the regulatory aspects of mobility services than on the development of advanced mathematical models. The present study proposes a mixed-integer non-linear programming (MILP) model, reformulated into a mixed-integer linear (MILP) one, to unify the Vehicle Routing Problem (VRP) for on-demand shared mobility services and the Public Transport Rescheduling Problem (PTRP). The ϵ-constraint method is employed to generate a non-dominated Pareto optimal set of solutions, considering conflicting objectives such as the reducion of the fleet size and the travel times of passengers. This framework provides valuable insights for transportation authorities, offering decision support for tactical planning purposes. The remainder of the study details the problem formulation, application, and results from a real-world case study in the Athens metropolitan area.

2 Methodology

The present Vehicle Routing and Public Transport Rescheduling Problem (VRP-PTR) strives to synchronize two primary modes of transportation: on-demand shuttle buses and a railway public transport service. A set of passengers, located at distinct pickup points, forms the target of this synchronization, aiming to reduce the passenger travel and waiting times from their origin location until their boarding to the public transport service.

In formulating this problem, the following key assumptions have been made:

1. All on-demand shuttle buses are stationed at the railway station.
2. Each shuttle bus route begins and ends at the railway station.
3. Each passenger demand point is served exactly once.
4. All shuttle bus trips follow the shortest route to each assigned pickup point.
5. Travel times between different vertices of the transport network are known in advance.

Consider a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ is divided into two subsets: $\mathcal{O} \cup \mathcal{P}$. The set $\mathcal{P}$ represents the pickup locations, numbered $1, ..., p$, with each passenger demand point having the railway station as its destination. The set $\mathcal{O}$ contains two copies of the railway station, denoted as 0 and $p + 1$. Vertex 0 represents the starting point of the shuttle, and vertex $p + 1$ represents the endpoint. The arc set A defines the feasible shuttle paths, structured as follows: $\mathcal{A} = \{\{(0, j) : j \in \mathcal{P}\} \cup \{(i, j) : i \in \mathcal{P}, j \in \mathcal{P}, i \neq j\} \cup \{(j, p + 1) : j \in \mathcal{P}\}$. Let $\mathcal{K}$ be the set of the available on-demand shuttle buses. The model's primary objective is to align the shuttle bus services with the railway schedule, where each railway trip $r \in \mathcal{R}$ is associated with a specific departure time $y_r \in \mathbb{R}_{\geq 0}$.

Each vertex $i \in \mathcal{V}$ is associated with a pickup passenger demand ($q_i \in \mathbb{R}_{\geq 0}$). This demand needs to be picked up at a specific time instance ($e_i \in \mathbb{R}_{\geq 0}$). Note that on-demand shuttle buses depart from the railway station empty $q_0 = 0$. For the boarding and alighting process of each passenger, a fixed time $\beta \in \mathbb{R}_{\geq 0}$ is required, and reaching the railway station after alighting takes an additional fixed time $\alpha \in \mathbb{R}_{\geq 0}$.

Each shuttle bus $k \in \mathcal{K}$ has a predefined capacity $Q_k \in \mathbb{R}_{\geq 0}$ and a maximum allowed running duration $T_k \in \mathbb{R}_{\geq 0}$. The cost of the on-demand service fleet in this model is determined by its size. The travel time from vertex $i \in \mathcal{P}$ to vertex $j \in \mathcal{P}$ is denoted as $t_{ij} \in \mathbb{R}_{\geq 0}$, with $t_{ii} = +\infty$ to prevent loop arcs. Additionally, a maximum ride time $\mathcal{L} \in \mathbb{R}_{\geq 0}$ for passengers ensures service quality. A parameter $s \in \mathbb{R}_{\geq 0}$ is also defined, representing the allowable timetable deviation for railway trips. The problem's notation is presented in Table 1.

Table 1: Nomenclature

Sets	
$\mathcal{P}$	set of all pickup vertices
$\mathcal{O}$	set of railway station copies
$\mathcal{V}$	set of all pickup vertices and the railway station, where $\mathcal{V} = \mathcal{O} \cup \mathcal{P}$
$\mathcal{A}$	arc set of all feasible routes where the on-demand vehicle can travel
$\mathcal{K}$	set of the available on-demand shared vehicles (shuttle buses)
$\mathcal{R}$	set of all scheduled railway trips during an operational day
Parameters	
y_r	departure time of the scheduled railway trip $r \in \mathcal{R}$
q_i	pickup demand at each vertex $i \in \mathcal{V}$ for on-demand service
e_i	demand time of each vertex $i \in \mathcal{V}$ for on-demand service
β	fixed time for the boarding/alighting of each passenger to the shared vehicle
α	fixed time for reaching the railway station dock after the alighting process
Q_k	capacity of each shuttle bus $k \in \mathcal{K}$
T_k	maximum allowed running time of a shuttle bus $k \in \mathcal{K}$
d_{ij}	minimum travel distance of a feasible arc $(i, j) \in \mathcal{A}$
t_{ij}	travel time of traversing a feasible arc $(i, j) \in \mathcal{A}$
L	maximum allowed ride time of passengers from any pickup vertex $i \in \mathcal{P}$
s	the allowable timetable deviation for railway trips $r \in \mathcal{R}$
M	a very large positive number
Variables	
x_{ij}^k	$x_{ij}^k \in \{0, 1\}$, where $x_{ij}^k = 1$ if the on-demand shuttle bus $k \in \mathcal{K}$ serves vertices $(i, j) \in \mathcal{A}$ sequentially, and $x_{ij}^k = 0$ if not
θ_i^k	$\theta_i^k \in \{0, 1\}$, where $\theta_i^k = 1$ if the on-demand shuttle bus $k \in \mathcal{K}$ serves pickup point $i \in \mathcal{V}$, and $\theta_i^k = 0$ if not
l_{ir}	$l_{ir} \in \{0, 1\}$, where $l_{ir} = 1$ if the passengers from pickup point $i \in \mathcal{P}$ are assigned to the railway trip $r \in \mathcal{R}$, and $l_{ir} = 0$ if not
z_k	$z_k \in \{0, 1\}$, where $z_k = 1$ if the on-demand shuttle bus $k \in \mathcal{K}$ serves at least one pickup vertex $i \in \mathcal{P}$, and $z_k = 0$ if not
u_i^k	$u_i^k \in \mathbb{R}_{\geq 0}$, indicating the final time at which shuttle bus $k \in \mathcal{K}$ starts servicing vertex $i \in \mathcal{V}$
g_j^k	$g_j^k \in \mathbb{R}_{\geq 0}$, indicating the time at which shuttle bus $k \in \mathcal{K}$ starts servicing vertex $j \in \mathcal{V}$, considering the time it served the previous vertex $i \in \mathcal{V} : i \neq j$
τ_k	$\tau_k \in \mathbb{R}_{\geq 0}$, indicating the return time of the on-demand shuttle bus $k \in \mathcal{K}$ at the railway station after unloading all serviced passengers from the pickup vertices
b_i	$b_i \in \mathbb{R}_{\geq 0}$, indicating the arrival time of passengers from pickup vertex $i \in \mathcal{P}$ at the railway station
h_r	$h_r \in \mathbb{R}$, indicating the allowable timetable modifications for a railway trip $r \in \mathcal{R}$
m	$m = [m_1, ..., m_i, ..., m_{\lvert P \rvert}]^\intercal \in \mathbb{R}_{\geq 0}$, waiting time for the passenger demand $i \in \mathcal{P}$ from when it calls the on-demand service until its pickup time
n	$n = [n_1, ..., n_i, ..., n_{\lvert P \rvert}]^\intercal \in \mathbb{R}_{\geq 0}$, in-vehicle travel time for the passenger demand $i \in \mathcal{P}$ corresponding to the $(i, p+1)$ request
w	$w = [w_1, ..., w_i, ..., w_{\lvert P \rvert}]^\intercal \in \mathbb{R}_{\geq 0}$, waiting time for the passenger demand $i \in \mathcal{P}$ after its arrival at the railway station until the departure time of the next closest railway trip $r \in \mathcal{R}$

Considering the presented nomenclature, the mathematical model of the VRP-PTR of the on-demand and public transport services is presented below.

$$(\tilde{Q}):$$

$$\min\ f_1(m, n, w) \tag{1}$$

$$\min\ f_2(z) \tag{2}$$

$$\text{s.t.:}\quad f_1(m, n, w) = \sum_{i \in \mathcal{P}} (m_i + n_i + w_i) \tag{3}$$

$$f_2(z) = \sum_{k \in \mathcal{K}} z_k \tag{4}$$

$$z_k \geq x_{ij}^k \qquad \forall (i, j) \in \mathcal{A}, \forall k \in \mathcal{K} \tag{5}$$

$$\sum_{k\in\mathcal{K}}\sum_{j:(i,j)\in\mathcal{A}} x_{ij}^k = 1 \qquad \forall i\in\mathcal{P} \tag{6}$$

$$\sum_{j:(0,j)\in\mathcal{A}} x_{0j}^k = \sum_{j:(j,p+1)\in\mathcal{A}} x_{jp+1}^k \qquad \forall k\in\mathcal{K} \tag{7}$$

$$\sum_{i:(0,i)\in\mathcal{A}} x_{0i}^k \leq 1 \qquad \forall k\in\mathcal{K} \tag{8}$$

$$\sum_{i:(0,i)\in\mathcal{A}} x_{0i}^k \geq \sum_{i:(i,j)\in\mathcal{A}}\sum_{j\in\mathcal{V}} x_{ij}^k \qquad \forall k\in\mathcal{K} \tag{9}$$

$$\sum_{i:(i,g)\in\mathcal{A}} x_{ig}^k - \sum_{j:(g,j)\in\mathcal{A}} x_{gj}^k = 0 \qquad \forall g\in\mathcal{P}, \forall k\in\mathcal{K} \tag{10}$$

$$\sum_{i\in\mathcal{P}}\sum_{j:(i,j)\in\mathcal{A}} q_i x_{ij}^k \leq Q_k \qquad \forall k\in\mathcal{K} \tag{11}$$

$$\frac{1}{M}\sum_{j:(i,j)\in\mathcal{A}} x_{ij}^k \leq \theta_i^k \leq \sum_{j:(i,j)\in\mathcal{A}} x_{ij}^k \qquad \forall i\in\mathcal{V}, \forall k\in\mathcal{K} \tag{12}$$

$$u_i^k \geq e_i \qquad \forall i\in\mathcal{V}, \forall k\in\mathcal{K} \tag{13}$$

$$u_0^k = e_i - t_{0i} \mid x_{0i}^k = 1 \qquad \forall i\in\mathcal{P}, \forall k\in\mathcal{K} \tag{14}$$

$$g_j^k = u_i^k + \beta q_i \theta_i^k + t_{ij} \mid x_{ij}^k = 1 \qquad \forall (i,j)\in\mathcal{A} : i\in\mathcal{V}, \forall k\in\mathcal{K} \tag{15}$$

$$u_j^k \geq g_j^k \qquad \forall j\in\mathcal{V}, \forall k\in\mathcal{K} \tag{16}$$

$$m_i = u_i^k - e_i \qquad \forall i\in\mathcal{P}, \forall k\in\mathcal{K} \tag{17}$$

$$\tau_k = g_{p+1}^k + \beta\sum_{i\in\mathcal{P}} q_i\theta_i^k \qquad \forall k\in\mathcal{K} \tag{18}$$

$$g_{p+1}^k - u_0^k \leq T_k \qquad \forall k\in\mathcal{K} \tag{19}$$

$$b_i = \tau_k + \alpha \mid \theta_i^k = 1 \qquad \forall i\in\mathcal{P}, \forall k\in\mathcal{K} \tag{20}$$

$$n_i = b_i - u_i^k \qquad \forall i\in\mathcal{P}, \forall k\in\mathcal{K} \tag{21}$$

$$n_i \leq L \qquad \forall i\in\mathcal{P} \tag{22}$$

$$\sum_{r\in\mathcal{R}} l_{ir} = 1 \qquad \forall i\in\mathcal{P} \tag{23}$$

$$-s \leq h_r \leq s \mid l_{ir} = 1 \qquad \forall i\in\mathcal{P}, \forall r\in\mathcal{R} \tag{24}$$

$$(y_r + h_r) l_{ir} \geq b_i \qquad \forall i\in\mathcal{P}, \forall r\in\mathcal{R} \tag{25}$$

$$w_i = (y_r + h_r) - b_i \mid l_{ir} = 1 \qquad \forall i\in\mathcal{P}, \forall r\in\mathcal{R} \tag{26}$$

The present VRP-PTR problem of on-demand and railway transport services has two objectives. The first objective (1), expressed in Eq. (3), minimizes the total door-to-rail passenger travel time, which consists of the cumulative waiting time of passengers in three stages: (1) waiting for the shuttle pickup (m_i), (2) in-vehicle travel time to the railway station (n_i), and (3) waiting time at the railway station for the next railway train (w_i). The second objective (2), expressed in Eq. (4), minimizes the shuttle bus fleet size using the binary variable z_k, where $z_k = 1$ if on-demand shuttle $k \in \mathcal{K}$ serves at least one pickup point and 0 otherwise (5). These two objectives form a multi-objective optimization problem (MOOP).

Constraints (6) ensure that each pickup point with an on-demand request is visited exactly once by an on-demand shuttle, ensuring the complete coverage of passenger demand. Constraints (7) enforce a round-trip structure for each shuttle bus, maintaining the railway station as a fixed start and end location. Constraints (8) and (9) ensure that each shuttle bus can only be assigned to one route and that any route it serves must start from the railway station if it is assigned to serve an $(i,j) \in \mathcal{A}$ pair. Constraints (10) enforce the flow conservation and constraints (11) impose capacity limits on the shuttle bus to prevent overloading during pickups.

Constraints (12) map shuttle buses to passenger demand, using the two-index binary variable θ_i^k and applying a big-M approach. Constraints (13) ensure that the service time u_i^k of a pickup point by a shuttle bus is at least equal to the time instance e_i, which represents the pickup time demand for point $i \in \mathcal{V}$. Constraints (14) calculate the exact time at which a shuttle bus should leave the railway station to reach a pickup point (u_0^k), provided it is scheduled to service it. To maintain the proper sequence between consecutive pickup points, constraints (15) calculate the start time for servicing vertex $j \in \mathcal{V}$ (g_j^k) if the same shuttle bus serves the arc $(i,j) \in \mathcal{A}$ ($x_{ij}^k = 1$). This setup also determines the time g_{p+1}^k when vehicle $k \in \mathcal{K}$ returns to the railway station after servicing all the assigned arcs $(i,j) \in \mathcal{A}$. Together, constraints (13) and (16) ensure that the final service time u_j^k of a vertex j by the on-demand shuttle bus equals to the maximum of g_j^k and its pickup time demand e_j.

Constraints (17) calculate the first component of door-to-rail passenger travel time (m_i), representing the waiting time from the moment passengers request the on-demand shuttle service until the shuttle bus picks them up (u_i^k). Constraints (18) determine the return time τ_k of shuttle vehicle $k \in \mathcal{K}$ at the railway station, and constraints (19) maintain their maximum allowable operating time T_k. Constraints (20) define the arrival time b_i of passengers from each pickup point at the railway station as the sum of the arrival time of vehicle k at the railway station (τ_k), if it has serviced this pickup point, and the estimated time for passengers to alight from the vehicle and reach the railway dock (α).

Constraints (21) calculate the second component of door-to-rail passenger travel time (n_i), representing the in-vehicle travel time for passengers. Constraints (22) ensure that the passenger ride time remains below the maximum allowable limit L, which also restricts shuttle bus detours. This approach promotes more efficient route selection for the on-demand service, falling under the general category of the Vehicle Routing Problem (VRP). Constraints (23) assign each pickup point to exactly one railway trip, which can be rescheduled (h_r) within an allowable range, as defined by constraints (24) and a predefined timetable deviation parameter s. Constraints (25) adjust the departure time of the railway trip by applying the time modification, if it exists ($y_r + h_r$), to closely match it with the arrival time of pickup point (b_i) when assigned to that railway trip. Finally, constraints (26) calculate the third component of door-to-rail passenger travel time (w_i), the waiting time for passengers in the railway station. This waiting time is the difference between their arrival at the railway station and the departure time of the next closest railway trip, which may have been rescheduled.

The multi-objective nature of this problem creates conflicts, as improving one objective (e.g., reducing the number of available shuttle buses) may deteriorate the other objective (e.g., increasing the door-to-rail passenger travel time), and vice versa. To solve this MOOP, the ϵ-constraint solution method will be used, resulting in the computation of a representative subset of the Pareto front. The Pareto front refers to the set of all Pareto-optimal outcomes, which define the best trade-offs between competing objectives (Gkiotsalitis, 2022). Using the ϵ-constraint method, we will treat objective function (2) as a constraint by not allowing it to exceed a constant value ϵ, while keeping objective function (1) as the primary objective. With this consideration, the objective function (2) is reformulated into the following constraint:

$$\sum_{k \in \mathcal{K}} z_k \leq \epsilon \tag{27}$$

The constant ϵ is allowed to take various values, and in this way, the problem is solved repeatedly, making the ϵ-constraint an *a posteriori* (generator) solution approach (Gkiotsalitis, 2022). The aforementioned mathematical program ($\tilde{Q}$) is non-convex, due to the nonlinearities in constraints (14), (15), (20), (24) - (26). The model is reformulated into a mixed-integer linear problem (MILP) ($\hat{Q}$), which can be solved to global optimality using the exact Branch-and-Cut methodology.

3 Results and discussion

The application of the proposed model was tested in a real-world case study with actual data from the Athens, Greece region. Larissa railway station was selected as the central railway hub. This station, one of Athens' primary railway stations, serves as a key connection point for national and international rail services, supporting both intercity and suburban railway networks across Greece. In the current experiments, the main north-south railway line service connecting Athens and Thessaloniki was examined, focusing on the trip originating in Athens and terminating in Thessaloniki.

The passenger demand pickup points were situated within Athens Municipality and its neighboring regions, emphasizing areas where accessing Larissa railway station requires the use of multiple public transport modes. In such locations, alternatives like private vehicles, taxi, or on-demand services provide a more convenient way of reaching the railway station. The study area includes $|\mathcal{P}| = 24$ pickup points with a total number of 117 passengers. Pickup problems with $|\mathcal{P}| > 24$ could not be solved within a reasonable time frame on a conventional computer machine, as the computational complexity increases exponentially with the addition of $|\mathcal{P}| \times |\mathcal{K}|$ and $|\mathcal{P}| \times |\mathcal{R}|$ binary variables. The time at which each pickup vertex requested on-demand service is known in advance. The earliest pickup demand is at 7:10 a.m., and the latest at 11:40 p.m. The aforementioned pickup points were serviced by an on-demand shuttle bus fleet, with sizes ranging from $|\mathcal{K}| = 9$ to $|\mathcal{K}| = 30$ vehicles. These fleet sizes also represent the values assigned to the ϵ-constant in constraints (27) during repeated model runs. Fleet sizes below the $|\mathcal{K}| < 9$ vehicles were insufficient to service the external pickup demand resulting in infeasibilities. For the railway service, we have considered $|\mathcal{R}| = 7$ trips that are daily scheduled for the Athens-Thessaloniki line during an operational day, with the first scheduled trip departing at 7:18 a.m., and the last one at 11:55 p.m. The examined network is illustrated in Figure (1).

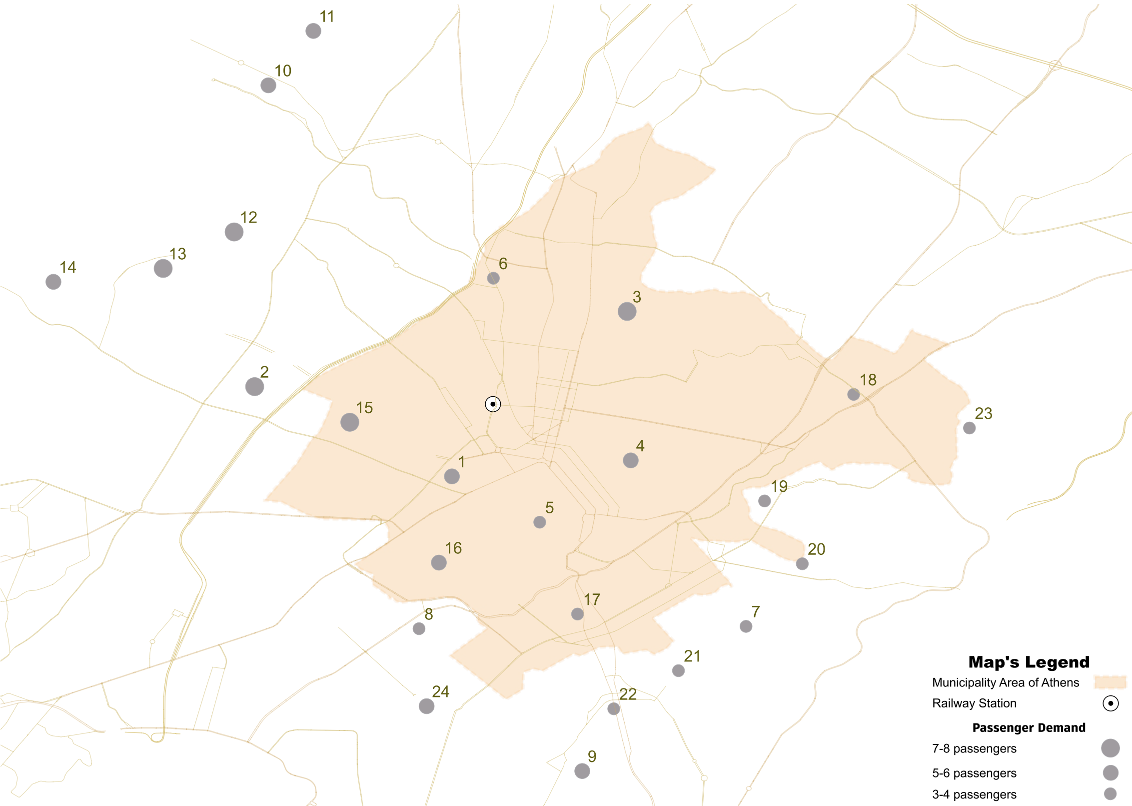

Figure 1: Athens case study - Location of the railway station and the pickup vertices, along with the corresponding passenger demand.

Several key considerations remain consistent across all model applications. The passenger demand at each pickup location is lower than the capacity of each shuttle bus, which is $Q_k = 13$ passengers. The boarding and alighting time per passenger on the on-demand vehicle is fixed at $\beta = 7$ seconds, while the time to reach the railway platform is set at $\alpha = 5$ minutes. The maximum allowable ride time for any passenger on the on-demand service is set to $L = 45$ minutes. In addition to the passenger demand-related parameters, the on-demand service operates from 6:00 a.m. until the last passenger request and the final railway trip, with a maximum operating duration of $T_k = 60$ minutes per vehicle. Shuttle buses operate at an average speed of 30 km/h, based on Athens' Urban Transportation Organization (OASA) velocity statistics. Finally, railway timetable modifications are limited to $s = 2$ minutes due to the inability to significantly alter the pre-determined schedules of railway lines.

For the experiments, Python 3.12 was utilized along with many of its standard libraries. To solve the mixed-integer linear program formulated in our study, the Branch-and-Cut solution method was implemented by the commercial solver Gurobi. The model runs were conducted on an external server with 3.20 GHz processor and 15.6 GB of RAM.

Table (2) outlines the main outcomes of each application of the model, with the primary differentiating factor being the value of the ϵ-constant. This constant, representing the upper limit on the total number of shuttle buses, increases incrementally by one, starting from the minimum feasible number needed to service the network (9 vehicles) and reaching up to 30 vehicles. A total of 22 cases were examined. This table also presents the best objective function value occurred at each case, corresponding to the total door-to-rail passenger travel time for all 24 pickup points, along with the progress rate of this value, which exhibits a declining tendency as ϵ increases, confirming the conflicting nature of the two objectives. Specifically, solutions up to case #6 ($\epsilon = 14$ vehicles) are Pareto optimal (non-dominated), as there is no other combination which improves the total door-to-rail passenger travel time without deteriorating the number of shuttle buses. Beyond this point, the total door-to-rail passenger travel time stabilizes at 682.971 minutes, indicating that adding more than 14 on-demand shuttle buses does not affect the total passenger travel time. It is also evident that as the number of on-demand vehicles increases from the minimum of nine, the decline rate of the objective value progressively decreases. The largest decline in the total door-to-rail passenger travel time, which equals 0.91%, occurs in the case with $\epsilon = 10$ vehicles compared to the previous case with $\epsilon = 9$ vehicles.

Table 2: Summary of examined cases - Maximum on-demand fleet size (ϵ), best objective value (in minutes), and its decline rate (%) between consecutive cases.

Case	ϵ-Constant value	Best Objective Value	Decline Rate
1	9	698.440	0.91%
2	10	692.112	
3	11	687.312	0.69%
4	12	684.912	0.35%
5	13	683.232	0.25%
6	14	682.971	0.04%
7	15	682.971	0.04%
8	16	682.971	0.04%
9	17	682.971	0.04%
...	...	682.971	0.04%
22	30	682.971	0.04%

The trade-off between the two objectives, f_1 and f_2, is illustrated in Fig.2, presenting the Pareto front generated by the ϵ-constraint method, along with the optimal values of the objective functions f_1 and f_2. For values of $\epsilon > 15$, no points are shown in the figure because these solutions are dominated and therefore not part of the Pareto Front.

In all the solutions generated for each pickup vertex across the different cases, the constraints outlined in Section 2 are fully satisfied. Specifically, all pickup points are served sequentially based on their demand times, with passenger ride times remaining below the maximum limit of $L = 45$ minutes. At the railway hub, passengers board the next available railway trip with the closest timing. Additionally, all on-demand shuttle bus trips in each case select the most efficient route paths to optimize the passenger service. To ensure high-quality standards for passengers, all railway trips assigned passenger demand were rescheduled within the predefined time limits of [-2, 2] minutes, with most adjustments falling at the lower bound.

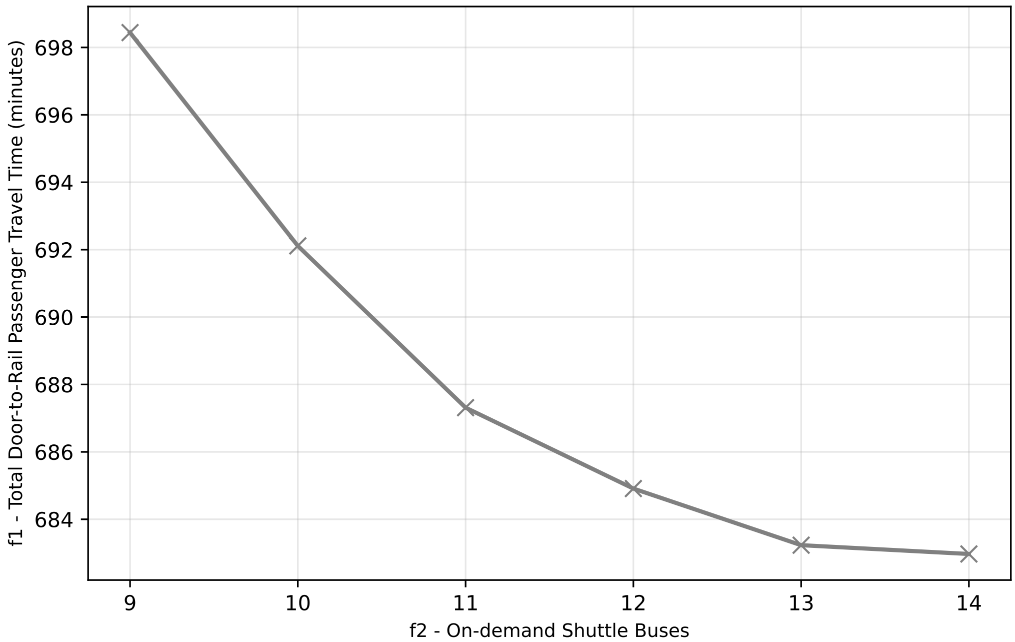

Figure 2: Approximated Pareto Front when solving VRP-PTR with ϵ-Constraint Method - f_1 (minutes), f_2 (number of On-demand Shuttle Buses).

4 Conclusions

This study proposes a multi-objective mathematical optimization model for the Vehicle Routing and Public Transport Rescheduling Problem (VRP-PTR). The model integrates shuttle buses (on-demand service) and the railway service (fixed-line service). A real-world case study is examined in Athens, Greece, considering various pickup points where passengers request on-demand services to transport them to the railway station, targeting the most time-appropriate railway trip on the Athens-Thessaloniki line. The goal is to minimize two key factors: (a) the shuttle bus fleet size, and (b) the total door-to-rail passenger travel time, creating a multi-objective optimization problem. This work advances the field by: (a) developing a MILP that addresses the VRP-PTR, ensuring global optimality for large network instances, and (b) allowing the integration of on-demand service with any public transport mode and its trips. This model serves as a valuable tool for operational planning regarding the synchronization of two distinct transport modes, one operating on-demand services and another operating fixed-line services. The model includes constraints for the efficient travel routes of the on-demand services and time service continuity for the external passenger demand. Employing the ϵ-constraint method reveals the trade-offs between the conflicting objectives and presents Pareto optimal outcomes, demonstrating that a reduction of approximately 15 minutes in total door-to-rail passenger travel times requires the addition of five shuttle buses.

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