

# Dynamic Rebalancing On-Demand Service Operations for Sustainable Transportation with a Tradable Credit Scheme

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## Short summary

This study introduces a novel supply-side management strategy that leverages the Tradable Credit Scheme (TCS) to incentivize Ride-Hailing (RH) drivers to relocate from high-demand urban centers to surrounding neighborhoods. This spatial rebalancing aims to improve first/last-mile connectivity, boost public transit (PT) usage in urban centers, and enhance the integration of RH and PT for trips across districts for sustainable transportation. We employ the trip-based Macroscopic Fundamental Diagram (MFD) framework to track travelers' and RH drivers' trajectories. Our model further integrates modules for travelers' mode selection, driver operational decision-making, and network equilibrium to provide a comprehensive representation of the system dynamics. The results show that this approach effectively reduces RH selection, increases PT usage, and enhances RH-PT integration for trips between suburbs and the city center. By proposing the strategy and the dynamic evaluation framework, this study contributes to advancing supply-side management solutions for urban congestion and sustainable transportation systems.

**Keywords:** Dynamic Rebalancing, Macroscopic Fundamental Diagram, Public Transit Integration, Ride-Hailing, Sustainable Transportation, Tradable Credit Scheme.

## 1 Introduction

Within the last decade, the rapid growth of Ride-Hailing (RH) services, such as Uber and Lyft, has transformed urban traffic patterns by providing affordable, on-demand, door-to-door mobility solutions (Comini et al., 2018). However, this transformation has introduced notable consequences, such as increased urban congestion and intensified competition with public transit (PT) (Cats et al., 2022). For example, an existing study (Erhardt et al., 2019) shows that on-demand RH services negatively affect the traffic in San Francisco, raising concerns about their long-term sustainability. The reasons is that RH companies, focused on maximizing profitability, tend to deploy their vehicles in high-demand urban centers. This deployment contributes to vehicle accumulation in these areas, exacerbates congestion, draws riders away from PT, and undermines sustainable transportation systems.

To address these consequences, this study introduces a novel supply-side management strategy utilizing the Tradable Credit Scheme (TCS) to regulate RH operations. By defining the required operating licenses for RH drivers to operate in different zones and dynamically adjusting credit prices, the approach incentivizes RH drivers to shift from urban centers to surrounding neighborhoods (e.g., suburban areas). This strategy aims to limit RH operations in city centers, encourage RH-PT integration for trips among districts, and support RH drivers in offering efficient first-mile and last-mile services, ultimately fostering a more sustainable and integrated transportation system.

To evaluate the proposed strategy, we develop a dynamic traffic simulation framework comprising four interconnected modules: the TCS, the equilibrium model, the passenger-vehicle matching mechanism, and the trip-based Macroscopic Fundamental Diagram (MFD) (Mariotte et al., 2017;

Lamotte & Geroliminis, 2018). The TCS module establishes zone-based licensing requirements and dynamically adjusts credit prices to incentivize the spatial redistribution of RH drivers. The equilibrium model captures the rationality of RH drivers' behavior by evaluating decisions based on license costs, potential revenue, and whether the income aligns with their expected thresholds. The matching mechanism optimizes system performance by assigning travelers to RH, PT, RH+PT, or PT+RH modes based on travel costs and operational constraints. Finally, we use the MFD framework to capture the traffic dynamics and track RH vehicles' trajectories, including pick-up and drop-off activities, to evaluate traffic performance.

By proposing the management strategy and the dynamic evaluation framework, this study contributes to advancing supply-side management solutions for urban congestion and sustainable transportation systems. Our findings provide insights into achieving sustainable urban mobility using TCS and offer practical implications for policymakers and transport operators.

## 2 Methodology

The transportation network in this study is composed of three main groups of participants: passengers  $C$ , RH vehicles  $D$ , and PT routes, along with background traffic (i.e., private vehicle drivers). We divided the study area into  $N_r$  regions, indexed from the urban center to the outskirts. Given the coordinates of the passengers, we assign their origin and destination zones, denoted as  $r_i^{\text{origin}}$  and  $r_i^{\text{destination}}$ . The trip distance is calculated using the Manhattan distance, defined as the sum of the absolute difference between each coordinate, under the assumption of a grid-based road network. Passengers would start their trips at their given departure times ( $t^{\text{departure}}$ ).

The RH companies have potential RH drivers  $D_0$ , with active drivers  $D \subseteq D_0$  participating based on the reservation price. Drivers join the service only if the average RH revenue exceeds their reservation price, reflecting an elasticity in driver supply that captures real-world behavioral dynamics and trade-offs. For example, increased driver participation may raise total RH revenue but intensify congestion, reducing individual earnings and network efficiency. Besides, we consider background traffic  $C^B$ , comprising private vehicles, influences congestion and network speed and would not change mode under any circumstances. Future works could consider the relaxation of this assumption.

PT operations are defined by routes, characterized by transit speeds ( $v_{ij}^p$ ) and headways ( $h_{ij}$ ), which vary across the PT routes.

Based on the participants' settings, we have four interconnected modules that interplay with each other in the proposed traffic framework.

### Tradable credit scheme

The road network is divided into  $N_R$  regions, indexed from the city center to the outskirts. Each active RH driver receives a daily allocation of  $k$  credits from the regulator. To operate in a higher level region  $r$ , drivers must spend  $\tau_r$  credits to obtain a license. The cost decreases as the region index increases ( $\tau_r < \tau_{r-1}$ ), with the outermost region ( $NR$ ) being credit-free ( $\tau_{NR} = 0$ ). Therefore, by assuming the credit price  $p$ , drivers who wish to operate in regions other than the outermost zone must pay  $\tau_r - k$  credits, while those operating in the outermost region can sell  $k$  credits for monetary benefits.

The number of drivers licensed for region  $r$  is denoted by  $x_r$ , allowing them to operate in all regions  $r' \geq r$ . For example, an RH trip from region 3 to region 1 requires a license for region 1, enabling operation in regions 1, 2, and 3. However, a combined RH-PT trip ending in region 1 may only require a license for region 2, as the final segment is completed via PT. (See Figure 1 as an example)

The TCS operates on two distinct timescales. Drivers' activity, assignments, and credit prices are updated daily, while the regulator adjusts credit charges  $\tau_r$  over longer periods, such as weekly

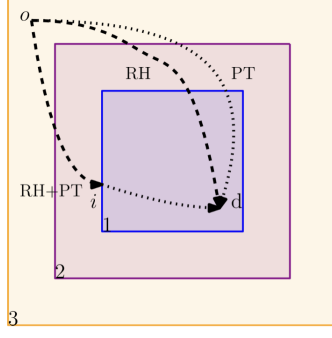


Figure 1: A trip between an origin in region 3 and a destination in region 1 has three alternatives: RH, PT, or RH till the border i and then PT.

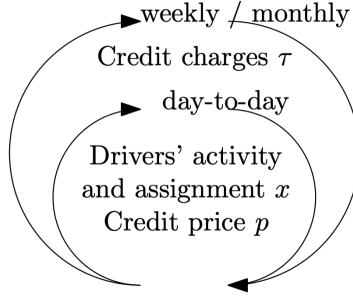


Figure 2: The two timescales of TCS: drivers' activity and assignment, and credit charge changes by the regulator.

or monthly. This two-timescale approach allows for real-time market responsiveness and long-term policy adjustments. (see Figure 2 as a reference). In the following sections, we present the traffic dynamics, mode choice models, and the supply-side equilibrium model.

## Mode choice and utility functions

While the TCS influences supply-side dynamics, the general cost of available travel options—RH, PT, or a combination of both shapes passengers' travel decisions. This general cost is determined by factors such as travel times and monetary expenses. Therefore, the mode choice costs are formulated as follows:

$$C_{o,d,PT}^t = \alpha_j T_{PT,o,d} + f_{PT} \quad (1)$$

$$C_{o,d,RH}^t = \alpha_j \left( \frac{L_{pu,o,d}}{V_{ro}(t)} + \sum_{r=r_o}^{r_d} \frac{L_r}{V_r(t)} \right) + f_{RH} L_{o,d} \quad (2)$$

$$C_{o,d,RH-PT}^t = \alpha_j \left( \frac{L_{pu,o,i}}{V_{ro}(t)} + \sum_{r=r_o}^{r_i} \frac{L_r}{V_r(t)} + T_{PT,i,d}^* \right) + f_{RH} L_{o,i} + f_{PT} \quad (3)$$

$$C_{o,d,PT-RH}^t = \alpha_j \left( T_{PT,o,i}^* + \frac{L_{pu,i,d}}{V_{ri}(t)} + \sum_{r=r_i}^{r_d} \frac{L_r}{V_r(t)} \right) + f_{RH} L_{i,d} + f_{PT} \quad (4)$$

Here,  $\alpha_j$  represents the traveler's Value of Time (VoT).  $T_{PT,o,d}$  is the PT travel time, and  $f_{PT}$  is the fixed ticket cost. The RH travel cost includes the pick-up time  $\left( \frac{L_{pu,o,d}}{V_{ro}(t)} \right)$ , travel time across regions  $\left( \frac{L_r}{V_r(t)} \right)$ , and a distance-based RH fare  $(f_{RH} \cdot L_{o,d})$ . For combined modes (RH-PT or PT-RH), the cost integrates respective portions of RH and PT travel.

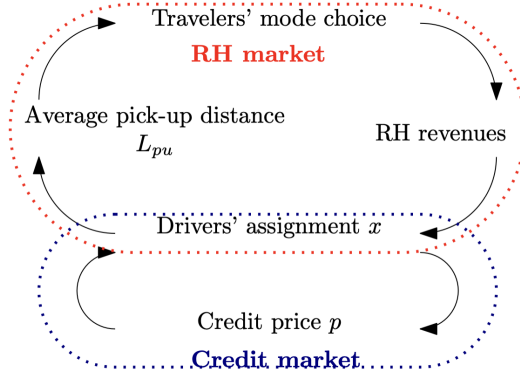


Figure 3: Inter-dependencies between drivers, travelers, and credit market.

## Matching mechanism

For passengers, at their departure time  $i^{departure}$ , travelers request a trip through a MaaS platform. The platform determines passengers' travel mode by using an optimization process to minimize total general travel costs while considering operational constraints. If passengers outnumber available drivers, those with the highest pick-up distances are reassigned to PT to maintain driver availability. The matching process is formulated as an Integer Linear Problem (ILP):

$$\min \sum_{i,j,m \in D \times C \times M} y_{i,j}^m C_{i,j}^m + \sum_{j \in C} \left( 1 - \sum_{i,m \in D \times M} y_{i,j}^m \right) C_j^{PT} \quad (5)$$

$$\text{Subject to: } \sum_{j,m \in C \times M} y_{i,j}^m \leq 1, \quad \forall i \in D \quad (6)$$

$$\sum_{i,m \in D \times M} y_{i,j}^m \leq 1, \quad \forall j \in C \quad (7)$$

$$y_{i,j}^m = 0 \text{ if } \text{lic}_i > r_j^m, \quad \forall i, j, m \in D \times C \times M \quad (8)$$

Here,  $y_{i,j}^m$ , as a binary variable, indicates the matching decision,  $C_{i,j}^m$  represents travel costs of passenger  $i$  matched with driver  $j$  in mode  $m$ , and  $C_j^{PT}$  is the transit-only cost. The first constraint, Equation 6, states that each driver is matched to at most one customer. The second constraint, Equation 7, ensures that each customer is matched to at most one driver. The third constraint, Equation 8, ensures that the driver's license  $\text{lic}_i$  allows them to serve the trip within specific zones. Specifically,  $r_j^m$  is the required license to serve customer  $j$  following the alternative  $m$ .

## Equilibrium model

The drivers' assignment  $x$  derived from the matching module balances two interconnected markets: the RH operation market, where travelers demand RH services, and the credit market, where drivers trade credits. (see Figure 3 as an reference).

The earnings of RH drivers are derived from their assigned passengers, considering trip distances and license costs. This feedback loop creates a dynamic interplay: passenger mode choices influence driver earnings, which in turn determine driver participation in the network. The platform iteratively updates these decisions to achieve equilibrium.

To compute the equilibrium, we jointly model the number of active drivers, their assignments  $x$ , and the credit price  $p$ . For simplicity, we assume the monetary expense of passengers using RH directly goes to RH drivers, so the RH revenue  $R_r$  is the sum of fees paid by travelers using RH

for trips requiring access to region  $r$  but not  $r - 1$ . The average RH gain  $G_r^{\text{avg}}$  for operating with a license for region  $r$  is given as:

$$G_r^{\text{avg}} = \sum_{r' \leq r} \frac{R_{r'}}{\sum_{r'' \leq r'} x_{r''}} - p \cdot \tau_r \quad (9)$$

where  $\tau_r$  is the credit requirement for operating in region  $r$ . The average RH revenue for all regions combined is:

$$R^{\text{avg}} = \frac{\sum_{r \in [1, NR]} R_r}{|D|} \quad (10)$$

where  $|D|$  is the number of active drivers. The equilibrium is achieved when the selected licenses correspond to the maximum gain value,  $G_{\text{avg}}^{\text{max}} = \max_r (G_r)^{\text{avg}}$ , over the licenses. The equilibrium is formulated as:

The equilibrium is reached when the chosen licenses correspond to the maximum average gain, formulated as:

$$(G_{\text{max}}^{\text{avg}} - G_r^{\text{avg}}) \cdot x_r = 0, \quad \forall r \in [1, NR - 1] \quad (11)$$

$$x_r \geq 0, \quad \forall r \in [1, NR] \quad (12)$$

$$\sum_{r=1}^{NR} x_r = |D| \quad (13)$$

$$\text{if } R^{\text{avg}} \geq P_i^{\text{res}}, \text{ then } i \in D, \text{ otherwise } i \notin D, \quad \forall i \in D_0 \quad (14)$$

$$\sum_{r=1}^{NR} x_r \cdot (\tau_r - \kappa) \leq 0 \quad (15)$$

$$\sum_{r=1}^{NR} x_r \cdot (\tau_r - \kappa) = 0 \quad (16)$$

$$p \geq 0 \quad (17)$$

Equation 11 means that any licenses chosen by at least one driver must yield the maximum gain. Equation 12 and Equation 13 ensure non-negativity and conservation of the number of drivers, respectively. Equation 14 states that a driver is active if and only if its reservation price is below the average revenue. Equation 15 is the credit cap and Equation 16 is the market clearing condition (MCC)(Balzer & Leclercq, 2022): all credits are used, or their price is zero. while Equation 17 states the credit price is non-negative. The last three constraints demonstrate the MCC condition specific to the TCS.

The equilibrium equations are theoretical and challenging to solve for most scenarios due to the integer nature of  $x_r$  and the nonlinearities in average gains and revenues. Therefore, we reformulate the equilibrium as a minimization problem. The cost function  $J$  consists of two parts:

1. The deviation of average gains from the maximum gain:

$$J_1 = \frac{1}{|D_0| \cdot NR} \sum_{r=1}^{NR} (G_{\text{max}}^{\text{avg}} - G_r^{\text{avg}}) \cdot x_r \quad (18)$$

2. The misclassification of active/inactive drivers:

$$J_2 = \sum_{i \in D_0} \xi(i, R^{\text{avg}}) \quad (19)$$

where  $\xi(i, R^{\text{avg}}) = 1$  if  $i \in D$  but  $R^{\text{avg}} < P_i^{\text{res}}$ , or if  $i \notin D$  but  $R^{\text{avg}} > P_i^{\text{res}}$ ; otherwise,  $\xi(i, R^{\text{avg}}) = 0$ .

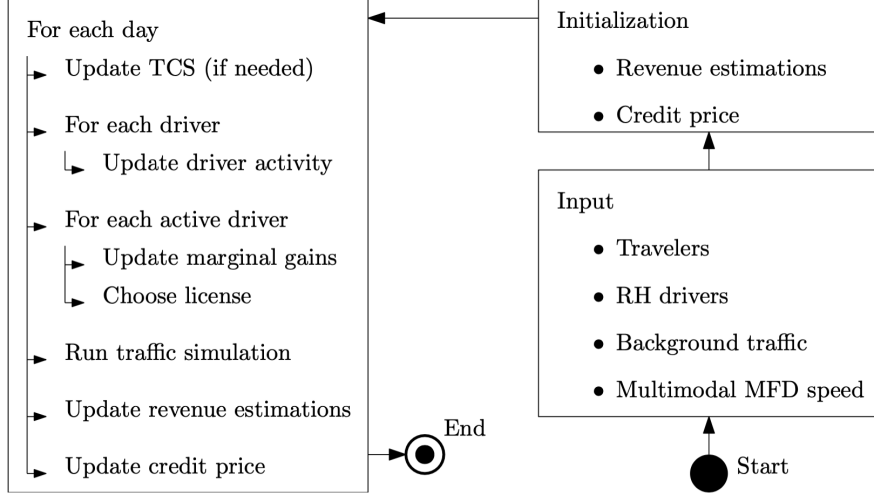


Figure 4: Simulation of the day-to-day RH operations.

The combined cost function is:

$$J = J_1 + J_2 \quad (20)$$

which is minimized to approximate the equilibrium.

To reduce problem size, the driver conservation equation Equation 13 and MCC Equation 16 are used to eliminate variables  $x_{NR-1}$  and  $x_{NR}$ . Assuming  $p > 0$ , the credit cap equation Equation 15 simplifies, leading to the following reformulations:

$$x_{NR-1} = \frac{|D|(\kappa - \tau_{NR}) - \sum_{k=1}^{NR-2} (\tau_k - \tau_{NR})x_k}{\tau_{NR-1} - \tau_{NR}} \quad (21)$$

$$x_{NR} = |D| - \sum_{r=1}^{NR-1} x_r \quad (22)$$

These simplifications reduce the problem to solving for  $N_R$  number of variables:  $NR-2$  regional assignments  $x_r$ , the credit price  $p$ , and the active drivers  $|D|$ , allowing for efficient computation.

### 3 Day to day simulation

To evaluate equilibrium prediction quality, convergence speed, and transition smoothness, we simulate the day-to-day evolution of traffic states under TCS constraints. Given the relatively small size of the credit market and the potential difficulty for drivers in finding buyers or sellers, RH drivers interact with a centralized credit bank. The bank regulates credit prices based on supply-demand imbalances and maintains a neutral budget by balancing purchases and sales. The simulation begins each day with an update to the TCS settings if needed, influencing driver participation and operational choices. Traffic dynamics are then modeled to capture the resulting shifts in driver behavior, credit market activity, and network performance. The iterative process continues until the system reaches equilibrium, where credit prices stabilize, and revenue patterns reflect consistent driver and passenger decisions. The overall process is illustrated in Figure 4.

To model the dynamic transition to equilibrium under specified TCS constraints, active drivers select their operating licenses based on marginal gains ( $MG_r$ ) for switching between regions. The marginal gain of accessing region  $r$  is defined as:

$$MG_r = \tilde{G}_r^{\text{avg}} - \tilde{G}_{r+1}^{\text{avg}} = \frac{\tilde{R}_r}{\sum_{r' \leq r} x_{r'}} - p(\tau_r - \tau_{r+1}),$$

where  $\tilde{R}_r$  represents the estimated RH revenue for region  $r$ ,  $\sum_{r' \leq r} x_{r'}$  is the total number of drivers accessing region  $r$ , and  $p$  is the credit price. A positive  $MG_r$  indicates additional revenue exceeding the extra credit cost for the required license, while a negative  $MG_r$  suggests the cost outweighs the benefit. Drivers evaluate  $MG_r$  for all potential licenses and update their choices accordingly. As the license distribution  $x$  evolves, marginal gains are recalculated to reflect changes in revenue distribution.

At the end of each day, the traffic simulation updates the estimated RH revenue for each region ( $\tilde{R}_r$ ) using the equation:

$$\tilde{R}_r(\text{day}+1) = \tilde{R}_r(\text{day}) - \frac{1}{p_{\text{day}} - T_\tau} (\tilde{R}_r(\text{day}) - R_r),$$

where  $T_\tau$  is the time interval between TCS updates, and  $R_r$  is the observed revenue for the day. Similarly, the average RH revenue across all regions is updated as:

$$\tilde{R}_{\text{avg}}(\text{day}+1) = \tilde{R}_{\text{avg}}(\text{day}) - \frac{1}{p_{\text{day}} - T_\tau} \left( \tilde{R}_{\text{avg}}(\text{day}) - \frac{\sum_r R_r}{|D|} \right),$$

where  $|D|$  represents the total number of active drivers. These updates allow the framework to capture real-time adjustments in revenue distribution and driver behavior.

The credit price ( $p$ ) is adjusted daily to maintain a budget-neutral state for the credit bank, ensuring all credits sold equal credits purchased. The price is updated using the equation:

$$p(\text{day}+1) = \max \left( 0, p(\text{day}) + \Delta p \frac{1}{p_{\text{day}} - T_\tau} \sum_{i \in D} (\tau_{\text{lic}_i} - \kappa) \right),$$

where  $\Delta p$  is a sensitivity parameter set by the regulator,  $\tau_{\text{lic}_i}$  is the credit requirement for the license chosen by driver  $i$ , and  $\kappa$  is the credit cap. The adjustment process incorporates a decay factor to smooth price trajectories over time and facilitate convergence.

By iteratively updating license choices, revenue estimations, and credit prices, the framework captures the dynamic interactions between drivers and the TCS, modeling the transition toward equilibrium. These steps, incorporating marginal gain evaluations and daily adjustments, ensure the system adapts to constraints while promoting stable and efficient outcomes.

## 4 Case Study

This section presents a case study to evaluate the impact of TCS settings. The fictive city is a 12 km square divided into  $N_R = 3$  regions (Figure 5), with distances computed using the Manhattan distance. The case study includes 1,000 MaaS users and 3,000 background vehicles within an hour, with Value of Time (VoT) drawn from a uniform distribution between 20 and 100 EUR/h. The departure times follow a normal distribution over an hour as shown in Figure 6. The origins coordinates are uniformly generated within the boundary of the study area, but 90%, 8%, and 2% of the destinations are located in the region 1, region 2, and region 3, respectively, representing a morning peak hour scenario.

The city operates as a single Macroscopic Fundamental Diagram (MFD) region to compute trips. Congestion dynamics are modeled using the affine MFD speed function:

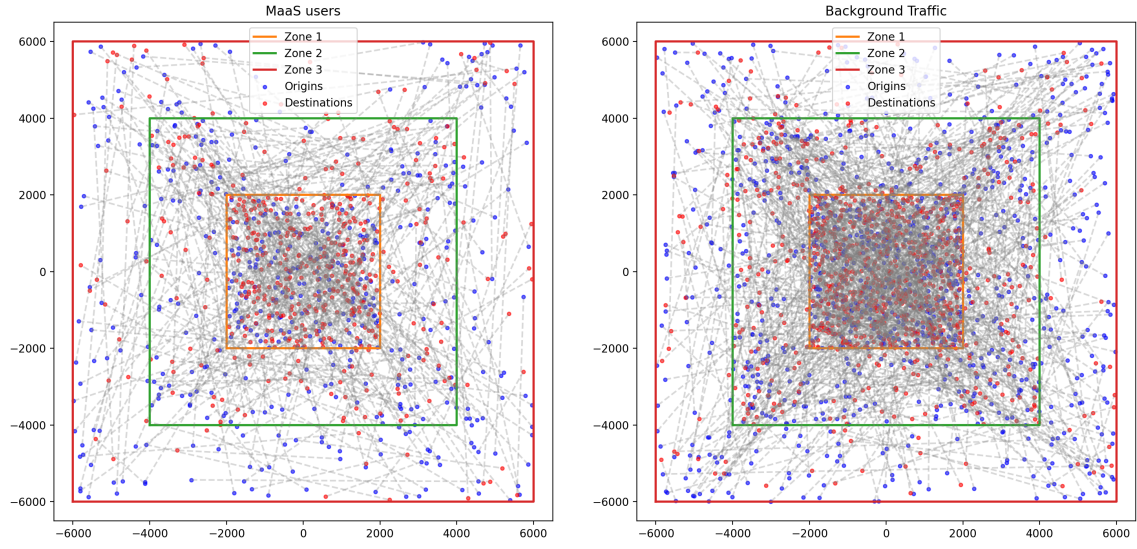


Figure 5: The example of the fictive city zones and the origin and destination spots of study riders and background traffic

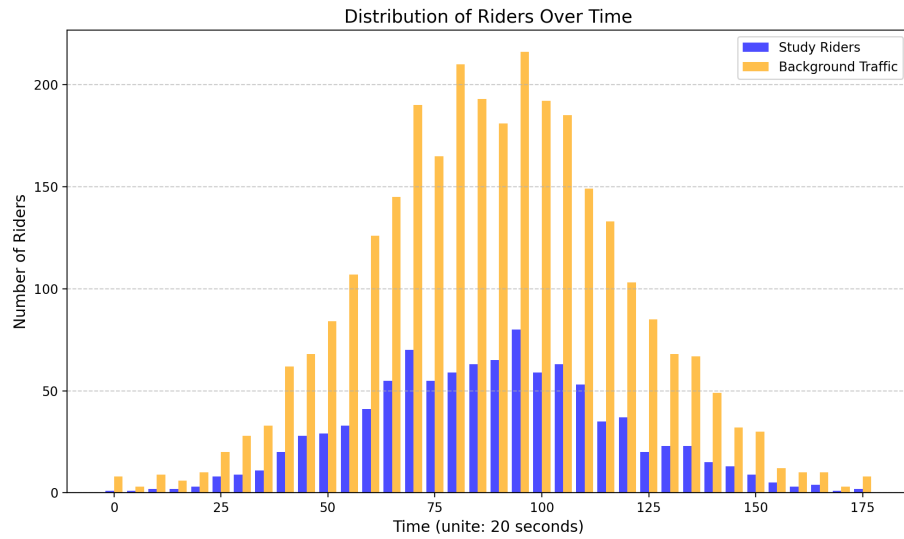


Figure 6: Departure time distribution of the background traffic and the MaaS customers.



$$V(n) = V_{\max} \left( 1 - \frac{n}{n_{\max}} \right),$$

where  $V_{\max} = 10$  m/s and  $n_{\max} = 5000$  vehicles represent the maximum speed and maximum accumulation, respectively.

The RH system includes 150 potential drivers ( $D_0 = 150$ ) with reservation prices uniformly distributed between 10 and 50 EUR. RH operations charge a distance-based fare of 2 EUR/km. The system buffers RH requests for 20-second matching periods. If a passenger is not matched after three attempts, they default to PT.

Each driver was assigned  $\kappa = 10$  free credits per day. For the first five days, the credit price is set to zero. Subsequently, the required credit amounts are adjusted to  $\tau_1 = 15$ ,  $\tau_2 = 10$ , and  $\tau_3 = 0$ . This adjustment allows us to observe the resulting changes in the number of travelers for each mode and the number of active drivers, along with the changing in credit price.

Mean PT speeds and access times vary by origin-destination pairs, as detailed in Table 1. Transit is faster and more frequent in the city center, while access times adjust based on the highest trip region. For combined RH-PT trips, access time is halved as RH vehicles reduce the distance to transit stations.

Table 1: PT mean speeds (m/s) and headtimes (in parentheses, min) for OD pairs.

Origin/Destination	1	2	3
1	7 (5)	6 (10)	6 (15)
2	6 (10)	6 (10)	5 (15)
3	6 (15)	5 (15)	5 (15)

The PT fare ( $f_{PT}$ ) is fixed at 1 EUR per trip, regardless of distance or the number of regions crossed, reflecting pricing for small to medium-sized cities.

## 5 Results and discussion

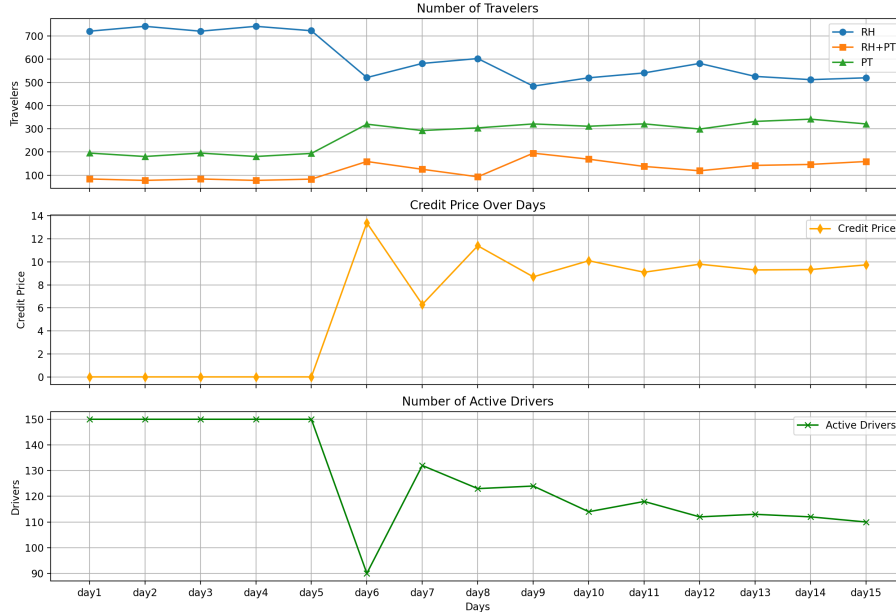


Figure 7: Trends in mode choice, credit price, and active drivers over 15 days under the TCS.

Figure 7 illustrates the impact of the TCS on both the demand side (i.e., MaaS users' behavior)

and the supply side (i.e., RH drivers’ participation) over a 15-day period in the numerical study. As shown in the plots, as the required credit increased (from  $\tau_1 = 0$ ,  $\tau_2 = 0$ , and  $\tau_3 = 0$  to  $\tau_1 = 15$ ,  $\tau_2 = 10$ , and  $\tau_3 = 0$ ), leading to a rise in credit prices (as shown in the middle plot), the number of active drivers decreased, prompting a partial shift of MaaS travelers toward PT and RH+PT modes.

The result provides several key insights. First, the proposed TCS strategy proves to be an effective lever in affecting mode choice, encouraging a shift from RH to PT and RH+PT. Second, RH remains the most popular mode among study travelers, highlighting its still appealing, which suggests the need to explore additional TCS configurations or coupling effective policies to further reduce RH usage and promote more sustainable alternatives. Finally, the increased adoption of PT and RH+PT indicates that supply-side management strategies effectively shape demand-side behavior, motivating more travelers to incorporate PT into their trips.

## 6 Conclusions and future work

In this study, we proposed a novel management strategy using TCS to incentivize RH drivers to relocate from high-demand urban centers to surrounding neighborhoods, thereby promoting sustainable transportation traveling options, such as PT. To evaluate the effectiveness of the proposed strategy, we develop an integrated dynamic traffic simulation framework. Furthermore, the results highlight the applicability of our evaluation framework, showing its ability to capture the dynamics of traveler behavior and system performance under different TCS configurations. They also highlight the effectiveness of the proposed strategy in reducing RH usage, promoting PT, and fostering a balanced and sustainable urban mobility system.

For the full version of the paper, we would consider incorporating background traffic as responsive demand and further consider elastic demand to better capture variations in traveler behavior under different TCS configurations and policy scenarios and conduct more numerical studies (e.g., variants of TCS settings) to reduce the appealing of RH and evaluate the long-term impacts of TCS strategies on mode shifts, system efficiency, and urban mobility sustainability.

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