Social implications of coexistence of CAVs and human drivers in the context of route choice

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Abstract Suppose in a stable urban traffic system populated only by human driven vehicles (HDVs), a given proportion (e.g. 10%) is replaced by a fleet of Connected and Autonomous Vehicles (CAVs), which share information and pursue a collective goal. Suppose these vehicles are centrally coordinated and differ from HDVs only by their collective capacities allowing them to make more efficient routing decisions before the travel on a given day begins. Suppose there is a choice between two routes and every day each driver makes a decision which route to take. Human drivers maximize their utility. CAVs might optimize different goals, such as the total travel time of the fleet. We show that in this plausible futuristic setting, the strategy CAVs are allowed to adopt may result in human drivers either benefitting or being systematically disadvantaged and urban networks becoming more or less optimal. Consequently, some regulatory measures might become indispensable.

1 Introduction

Which route should I take? Millions of people commuting to work by car face this dilemma every day [43]. In urban settings the choice is not straightforward as there are usually multiple viable alternatives. In fact, the reasons we select a given route might be very complex [3, 5] ranging from habitual choice or everyday exploration in order to identify the best alternative to anticipating decisions of others. Moreover, people are often very different and might prefer different options in the same situation or behave seemingly irrationally [27]. Suppose now that in a future urban traffic system with stable drivers' choice strategies a proportion of human drivers (HDVs) is replaced by intelligent vehicles (CAVs) which share information and make collective route choices based on one of the predefined collective fleet strategies: Selfish (minimization of CAVs' *collective* travel time); Altruistic (minimization of HDVs' mean travel time); Social (minimization of the mean travel time of all vehicles in the system); Malicious (aiming to *maximize* HDVs' mean travel time); Disruptive (maximization of HDVs' travel time at a bounded own cost).
Will, once the system has stabilized again after such disruption, the route preferences of CAVs and HDVs be different? Will CAVs be better off than the HDVs they replaced? And, crucially, could the human drivers be significantly disadvantaged or the system-wide travel times deteriorate?
In this paper we set out to study these fundamental questions using mathematical models and simulations, see Fig. 1. Focusing on the two-route bottleneck settings, Fig. 1, which are often present in real systems [19, 33], we discover that:

- The choices of CAVs that replace a given share of HDVs differ significantly from the choices of the remaining HDVs.

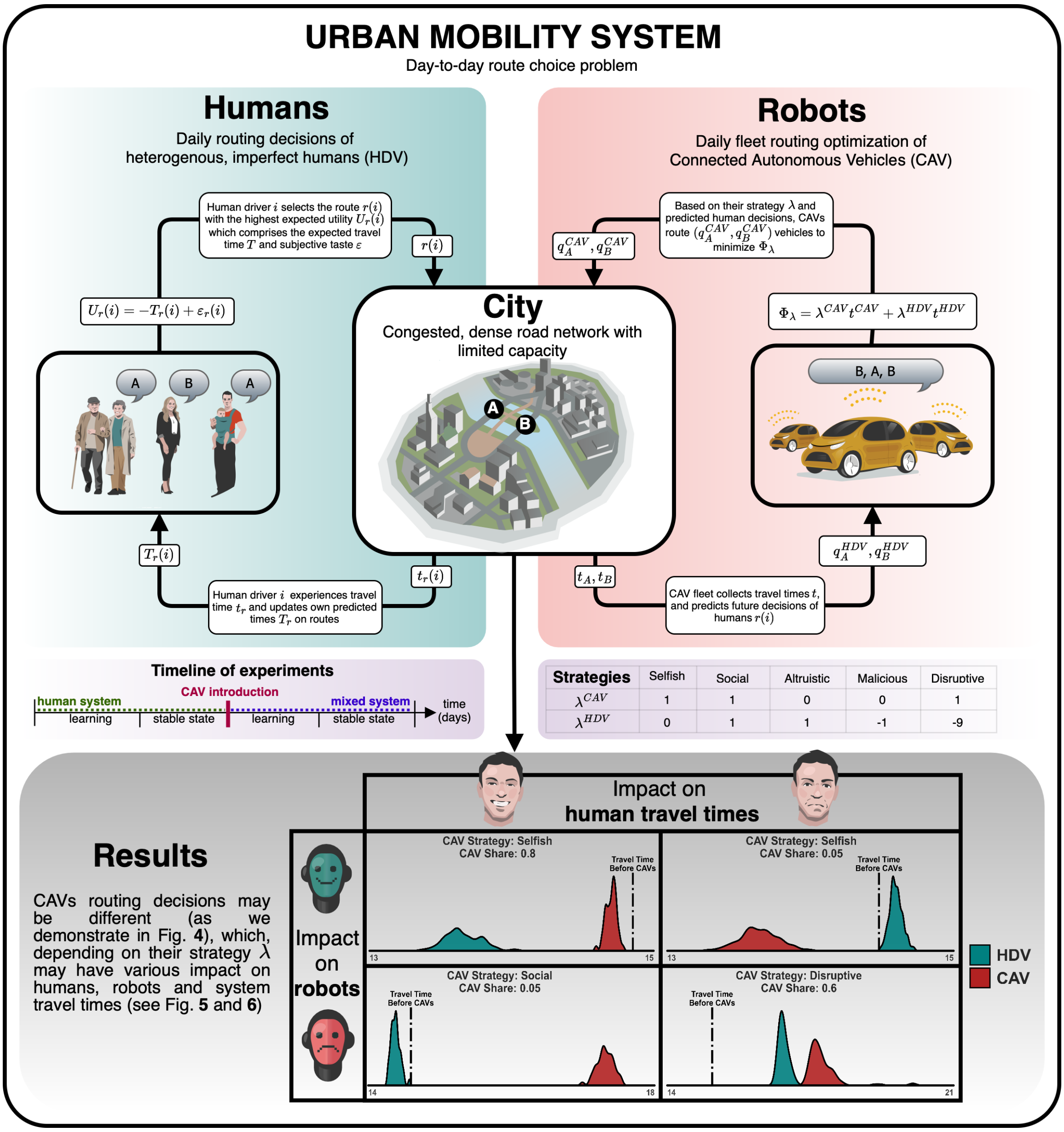

Figure 1: The learning and decision processes applied by human drivers (HDVs) and machines (CAVs). HDVs' reasoning is subjective and based on limited access to information. Contrariwise, CAVs have access to complete information on travel times and make optimal collective routing decisions. The interaction between human agents and CAVs may result in any combination of human drivers and CAVs being better off or worse off subject to the strategy applied by CAVs. In particular, the system-wide welfare may improve or deteriorate in the wake of introduction of CAVs.

- In different scenarios the average travel time of both HDVs and CAVs may increase or decrease, Fig. 1.
- If the fleet of CAVs applies the selfish strategy, it may improve its collective travel time at a cost to human drivers when the share of CAVs is small.
- For a large share of CAVs, the selfish or social strategies of CAVs may result in improvement of travel times for all the drivers. This, however, comes at a price of reduced equity.
- Human driver populations with low perception bias may be less prone to exploitation by intelligent fleets of CAVs than more diverse and less optimal populations.

- Heavily congested systems, where the choices of HDVs and CAVs tend to be similar, may be less susceptible to exploitation by CAVs. Contrariwise, uncongested networks could be easily exploited by machines.
- More elaborate, e.g. malicious, CAV strategies may result in oscillations and significant deterioration of driving conditions for all the drivers.

These conclusions seem to have been missing in the literature dealing with CAV - HDV interaction and constitute our original contribution to the subject. We obtain them from simulations by comparing the properties of the two-route bottleneck system before and after the introduction of CAVs.

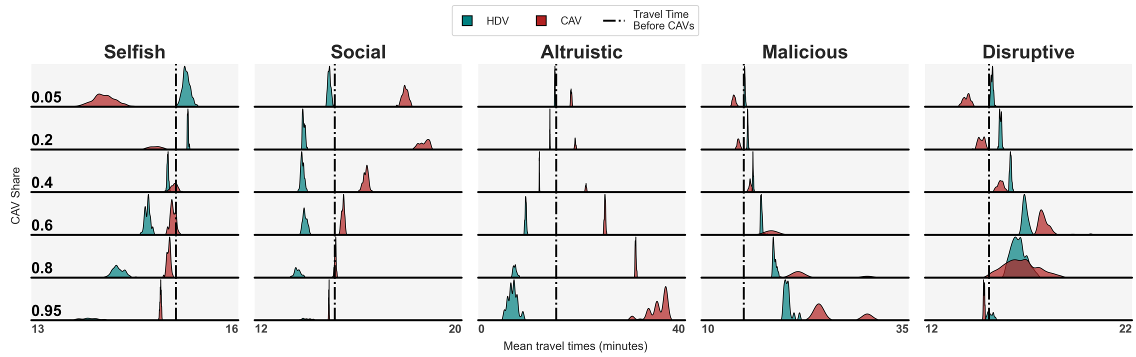

Figure 2: Kernel density estimations of HDV (teal) and CAV (red) travel times over the final 100 simulation days for various CAV shares and strategies. In selfish strategies, CAV times improve while HDV times worsen at low shares; both improve at high shares, favoring HDVs. Altruistic and social strategies raise CAV times and lower HDV times, except at very high CAV shares, where both improve. Malicious and disruptive strategies resemble selfish at low shares but increase travel times for all at high shares.

The standard econometric framework used to quantify choice is the expected utility theory [40], which posits that people choose the alternative with the highest expected utility. In the route choice setting with no access to external sources of information, the main component of utility is the predicted travel time [7]:

$$U_r = -T_r + \text{other factors}, \tag{1}$$

where U_r is the utility of route r and T_r is the expected (by a given agent) travel time on route r. If other factors are negligible, the rational HDV choice is to select the route with the highest utility, which corresponds to the shortest expected travel time. In the case of bottlenecks with two alternatives A and B, Fig. 1, this amounts to choosing

$$\arg\min_{r \in \{A,B\}} T_r. \tag{2}$$

Transport systems analysts typically assume that the system is in or close to equilibrium [42]. This means that the numbers of drivers traveling along alternative routes within a given time interval, e.g. the morning peak hour, are stable across consecutive days. This also implies stability of travel times (which may be assumed to depend monotonically, via the BPR [10] function, on the number of drivers) on different routes.
The most classical and widely-accepted traffic equilibrium, postulated by Wardrop [51], occurs when no single driver, who is assumed to have infinitesimal influence on the system as a whole, has an incentive to swap routes provided other drivers do not modify their choices the following day. Quantitatively, the drivers are assumed to make choices according to formula (2), where T_r are equilibrium travel times [51]. This so-called User Equilibrium (UE), is reminiscent of Nash equilibrium [39] in game theory and in simple

settings can be explicitly computed. When the number of agents is finite, however, the setting becomes an atomic congestion game which is inherently unstable [1, 29], see also Appendix, and often admits multiple Nash equilibria, [54].

A more realistic setting, adopted in our study, assumes that there exist other components of utility in equation (1), such as tastes or fluctuations in driving conditions, which are incorporated via formula

$$U_r = -T_r + \varepsilon_r, \tag{3}$$

where ε_r are random variables. This setting, the subject of random utility theory [38, 48] implies that, for ε_r independent identically distributed Gumbel variables commonly used in the field of transportation, the expected proportion of drivers choosing alternative A is given by the logit formula:

$$P_A = \frac{\exp(-T_A/\beta)}{\exp(-T_A/\beta) + \exp(-T_B/\beta)}, \tag{4}$$

which is pervasive in transport modelling [13]. In (4), β is the spread of subjective HDV tastes (perception bias). Low spread corresponds to HDVs preferring routes close to optimal in terms of travel time. High spread makes the choices more random. Assuming that the number of vehicles is very large, Daganzo and Sheffi [16] postulated the so-called stochastic user equilibrium (SUE), in which no agent *believes* they can improve their travel time by unilaterally changing routes.

Fast-forward to 2024, the logit choice, based on Gumbel-distributed random terms in (3), and its variants [6] is still the most popular family of human route choice models, see also [9, 22, 35, 53] for other approaches. Accordingly, we adopt a plausible logit-type model, called ϵ-Gumbel, in this paper, noting that for normally distributed error terms the results are similar, see Appendix. Importantly, the logit choice formula can be derived not only based on the error in perceived utility as per Daganzo and Sheffi [16] but also within the more recent framework of rational inattention [21, 37]. The equilibrium notions of SUE as well as UE, see also BRUE [18, 36], however, seem to be poorly suited to more realistic state-of-the-art models of multi-agent simulations, initiated 40 years ago by Horowitz [30] and employed in this paper. Therefore, instead of assuming that the system is strictly in equilibrium, such as SUE, we study experimentally systems which stabilize, see Appendix, bearing in mind that the stable states can be very complex [14, 52] or nonunique [46].

As the system is not exactly in equilibrium [11], the drivers do not know precisely the travel times they will experience selecting different alternatives. Therefore, T_r in formula (3) can only be approximate and we assume that every driver adjusts (in their minds) these estimates every day. There are various mechanism by which the human agents may adjust their day-to-day route choices [1]. In this paper we only consider the most popular mechanism called, depending on the source, exponential filter or Gawron/Horowitz/Erev-Roth learning [12, 20, 25, 30], omitting explicit modeling of habitual choice or bounded rationality [12, 36, 53] or direct anticipation of decisions of others based on game theory [1, 44]. We assume, namely, that every driver maintains implicitly/subconsciously estimates of travel times on alternative routes and these estimates are updated daily by combining previous knowledge and most recent travel times. There exist two basic mechanisms, *experience only* and *full information* as well as a whole spectrum of models, where only partial information is available [23, 37]. In this paper, we focus on the *experience only* mechanism, in which human drivers' knowledge is updated based on the experienced travel times only and there is no access to past or real-time travel times on alternative routes.

In our simulations, the human-only system stabilizes as a result of human learning and adjustment. Once this has happened, a given share of HDVs is replaced by a fleet of CAVs which is centrally controlled and pursues a pre-defined collective goal.

Once the goal is set, we assume that, every day, the fleet operator decides how many CAVs will be routed via each alternative. Once this decision has been made the CAVs set off onto the prescribed routes and, during the process of driving, behave similarly to

HDVs. In particular, we assume that CAVs do not utilize more efficient driving techniques such as platooning [34, 50]. The only aspect differentienting CAVs from HDVs that we consider in this paper is collective route choice based on superior access to information about the system and prediction of human drivers' behaviour. Once the modified system has stabilized (in most cases) again, we compare the statistics of the system *before* and *after* the introduction of CAVs and reach our conclusions.

Let us note that similar frameworks under the name of guidance systems, Advanced Travel Information Systems (ATIS) or Stackelberg congestion games [28, 44, 49, 55, 56] have been considered in the literature. However, in contrast to them our goal is to demonstrate a range of outcomes with emphasis on the ordinary human driver as well as system-wide welfare when confronted with a centrally-guided fleet of CAVs rather than to show how the traffic system could be made more efficient or brought closer to system optimum, compare [32, 57]. Furthermore, we explicitly consider the decision process and gradual adaptation of human drivers as opposed to a typical Stackelberg game setting [56] of a Cournot-Nash company with market power and individual rational price-takers. We also treat human drivers as separate entities with different tastes who take time to adapt without aggregating them into a single User Equilibrium player which can instantaneously arrive at an optimal equilibrium assignment [49]. Moreover, in contrast to the repeated game setting typical in reinforcement learning [47], we assume that human agents only take myopic decisions to minimize the current perceived travel time without optimizing their long-term pay-offs. Finally, our point of view is distinct regarding the CAVs. Namely, the fleet of CAVs, even if it represents a robo-taxi company carrying people who switched from their own cars or caters for people who have subscribed to a collective route guidance system (like online routing services), is a separate entity with its own target which might diverge from the goal of city authorities or even be a reflection of hidden hostile motives. In this vein, we do not assume that the system necessarily stabilizes after introduction of CAVs. Indeed, for some fleet strategies, as we demonstrate, a stable state is an undesirable feature, and keeping the system away from it allows the fleet of CAVs to maximize its specific collective target, compare [2] for more general multi-agent targets. Finally, the fleet has full information regarding the system travel times and can predict how many HDVs will choose every alternative before making their own routing decision, see also [2] for reinforcement learning-based city-scale scenarios.

2 Results

In the main experiment we study the long-term consequences of different proportions of HDVs becoming a centrally-coordinated fleet of CAVs in our two-route scenario. We compare the choices and travel times of HDVs and CAVs and summarize the results in Figs. 2, 3, 4, 5. In the second experiment, Fig. 6, we examine the dependence of the results on perception bias of human agents. Finally, in the third experiment, Fig. 7, we study how the results depend on congestion.

Experimental setting

In the experiments, run in a custom simulation software, we let the system composed of only HDVs stabilize and, after 200 days (on M-day) we replace a given share of HDVs by CAVs. We study the system purely experimentally in the stable regime of parameters. For human drivers we assume the ϵ-Gumbel model. For CAVs, we consider five possible strategies. After M-day, we run the simulation for another 100 days, see Fig. 1, after which we record HDV and CAV travel times and flows (vehicle counts) on both routes and compare them to the respective values before M-day. We distinguish five phases:

- Days 1-100: Stabilization of HDV-only system composed of, by default, 1000 drivers.
- Days 101-200: Stable state in which we capture various statistics for HDVs.

- Day 200 (M-day): a given share of HDVs is replaced by a centrally-coordinated fleet of CAVs.
- Days 201 - 300: Stabilization of the system in the new reality.
- Days 301 - 400: Stabilized (for most cases) state in which we compute the same statistics, this time for both HDVs and CAVs. We compare them to each other as well as to the statistics from days 101-200.

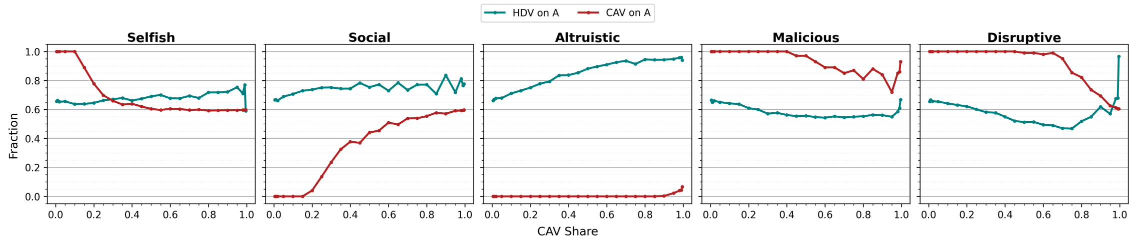

Figure 3: Comparison of CAV and HDV fractions on route A for varying CAV shares shows selfish CAVs dominate A initially, declining with larger shares, while HDVs remain stable. Social CAVs shift to A, opposing HDV preferences. Malicious and disruptive strategies align closely, routing more vehicles via A as shares increase.

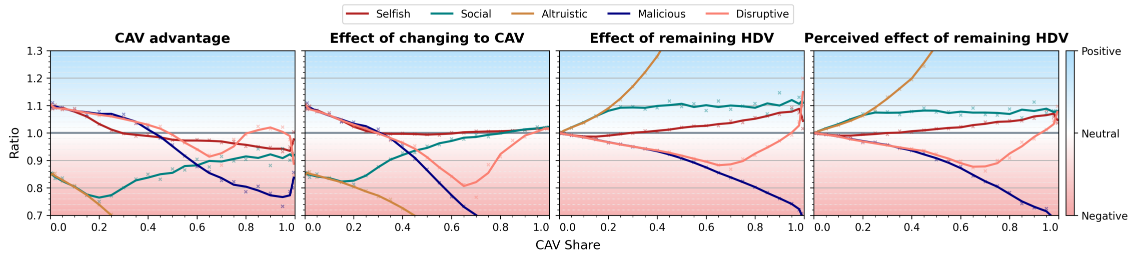

Figure 4: Outcomes of replacing HDVs with CAVs, creating varying CAV shares under baseline HDV perception bias, are quantified using these ratios: 1. CAV advantage (τ/ρ): If > 1, CAVs outperform HDVs post-M-day. 2. Effect of changing to CAV (τ_b/ρ): If > 1, switching reduces travel time. 3. Effect of remaining HDV (τ_b/τ): If > 1, remaining HDV improves travel time. 4. Perceived effect (u_b/u): If > 1, HDVs perceive better times.

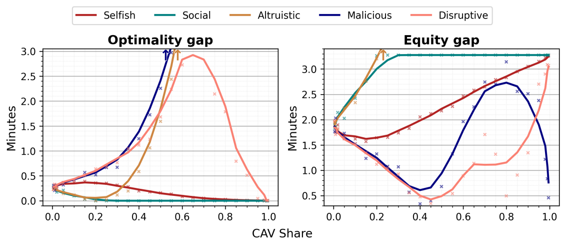

Figure 5: Optimality and equity gaps are analyzed for different CAV shares and strategies. The social strategy achieves a 0 optimality gap at high CAV shares but with significant equity gaps. The selfish strategy is similar. Altruistic strategies result in large gaps, while malicious and disruptive strategies show varying levels of both gaps.

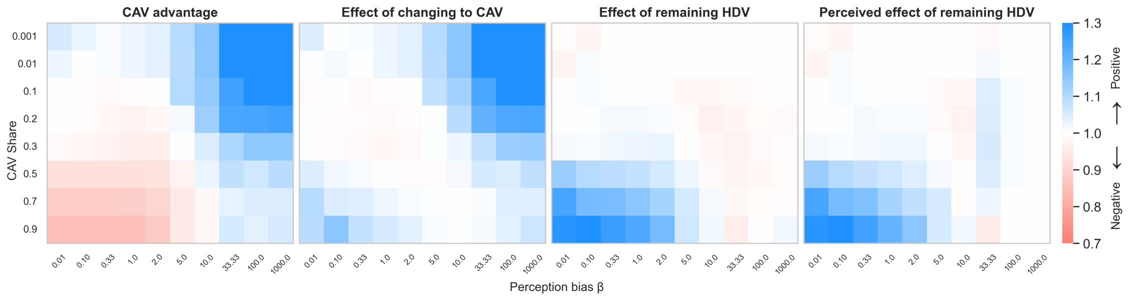

Figure 6: Positive and negative impacts of introducing CAVs under the selfish strategy, varying fleet shares, and human preference bias are shown in Fig. 4. CAV advantage is highest with high human bias and low fleet shares, reversing for high shares and low bias. Changing to CAV is almost always beneficial. Remaining HDV benefits mainly at low bias and high fleet shares but can be slightly negative otherwise.

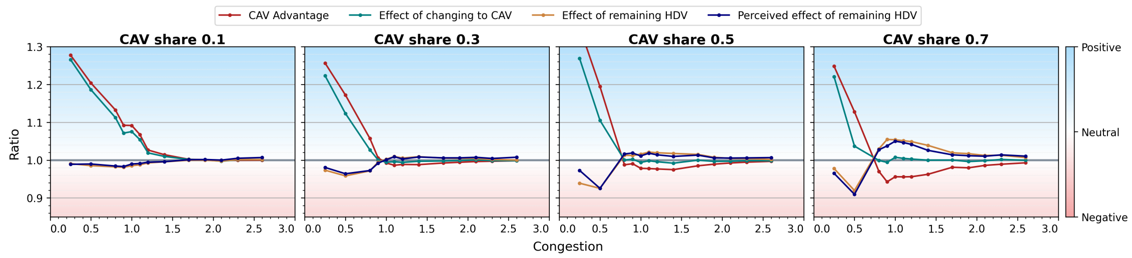

Figure 7: Outcomes of replacing HDVs with CAVs under the selfish strategy and varying congestion levels (C) are shown in Fig. 4. At low congestion, remaining HDV has negative effects, worsening with higher CAV shares, while switching to CAV is positive. High congestion levels make agents indifferent. Intermediate congestion shows negative CAV advantage but positive effects for remaining HDV, increasing with CAV share.

3 Discussion

In this research we provide evidence for the existence of certain phenomena emerging from HDV-CAV interaction in the context of route choice which are of paramount significance for the performance of future urban systems. Our abstract models deliberately reduce the complexity of the problem in order to highlight these typical phenomena, which are likely to be even more pronounced in real urban mobility systems. To achieve this, we abstract reality at three main levels: network topology, traffic flow, human route-choice decision process.

In all these aspects our models seem to be more restrictive for CAVs than real world and yet we were able to clearly reveal the disturbing phenomena. Hence, the results might be even sharper when, as it is in real cities, the network topology and traffic flow are complex, humans are even less optimal or homogenous and advanced machine learning is used to optimize CAV strategies. On the other hand, in the real world, human drivers might have better access to information facilitated by new technologies. Moreover, we modeled travel times by simple analytical BPR functions which are easily optimized by machines. In reality, CAVs will not have such precise information about the system and their optimization is likely to be based on reinforcement learning [2] and high performance computing.

The advantages of CAVs visible in our experiments can be summarized as follows.

- Advantage by collective decision taking, e.g. strategies that improve the average travel time of the fleet or of the system, which are hardly possible if every agent, like humans, is independent.

- Advantage by better access to information and information sharing, e.g. perfect understanding of the characteristics of the traffic system.
- Advantage by advanced processing and optimization capabilities, e.g. human behaviour modeling, human choice prediction.
- Advantage by lack of perception error, i.e. decisions based on *actual* as opposed to *perceived* travel times.
- Advantage by instantanous adaptation, which allows the machines to keep the system out of equilibrium and exploit slower human drivers' adaptation, as is the case for malicious and disruptive strategies.

These sources of advantage enable more efficient CAV routing decisions. In the default selfish case the CAVs outperform human drivers by selecting on average faster routes for small CAV market shares, Figs. 2, 4. For large market shares, the impact is more complex. Namely, the CAVs obtain better travel times than travel times of HDVs they replaced, however the driving conditions for HDVs improve even more, Fig. 1. This is due to the fact that they bring the system closer to optimum which involves different travel times on routes. The tipping poing is around 25%, Fig. 4, for the default moderate congestion levels and spread of human preferences. System-wise, collective strategies of CAVs, even if they are selfish may reduce the mean travel time (see optimality gap, Fig. 5), reducing e.g. CO_2 emissions and noise [15]. Other strategies, notably malicious and altruistic for large CAV shares, may increase the optimality gap, reducing the liveability of cities and sustainability of urban driving.

To summarize, CAV fleets will transform urban traffic systems. One of the aspects in which this will manifest itself will be route choice. The impact on the human drivers and urban welfare will depend on the strategies CAVs are allowed to adopt. For instance, for the outright malicious CAV fleet strategy, the driving conditions will deteriorate for everyone. At the other end of the spectrum, the altruistic strategy might bring huge benefits to the HDVs which remain in the system.

Non-standard strategies aside, however, our results indicate that even in the most straightforward scenarios with modest shares of CAVs minimizing their collective travel time the remaining human drivers might become disadvantaged as a side-effect. Do we want this?

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References

[1] Ahmad, F., Al-Fagih, L. (2024) Travel behaviour and game theory: A review of route choice modeling behaviour, Journal of Choice Modelling 50, 100472, https://doi.org/10.1016/j.jocm.2024.100472.

[2] Akman, O., Psarou, A, Varga, Z., Jamróz, G., Kucharski, R. (2024). Impact of Collective Behaviors of Autonomous Vehicles on Urban Traffic Dynamics: A Multi-Agent Reinforcement Learning Approach. EWRL 17(2024).

[3] Arthur, W. B. (1994). Inductive Reasoning and Bounded Rationality. The American Economic Review, 84(2), 406–411.

[4] Avila, A. M., & Mezić, I. (2020). Data-driven analysis and forecasting of highway traffic dynamics. Nature communications, 11(1), 2090.

[5] Bacharach, M. (1994) The epistemic structure of a theory of a game. Theor Decis 37, 7–48. https://doi.org/10.1007/BF01079204

[6] Ben-Akiva, M., Bierlaire, M. (1999). Discrete Choice Methods and their Applications to Short Term Travel Decisions. In: Hall, R.W. (eds) Handbook of Transportation Science. International Series in Operations Research and Management Science, vol 23. Springer, Boston, MA. https://doi.org/10.1007/978-1-4615-5203-1 2

[7] Bogers, E. A. I., Bierlaire, M. and Hoogendoorn, S. P. (2007). Modeling Learning in Route Choice. Transportation Research Record, 2014(1), 1-8. https://doi.org/10.3141/2014-01

[8] Çolak, S., Lima, A., & González, M. C. (2016). Understanding congested travel in urban areas. Nature communications, 7(1), 10793.

[9] Daganzo, C. F. (1979). Multinomial probit: the theory and its application to demand forecasting. New York: Academic Press.

[10] Bureau of Public Roads (1964). Traffic assignment manual. U.S. Dept. of Commerce, Urban Planning Division.

[11] Cantarella, G. E. and Cascetta, E. (1995) Dynamic Processes and Equilibrium in Transportation Networks: Towards a Unifying Theory. Transportation Science 29(4):305-329.

[12] Cantarella, G., Watling, D., De Luca, S., Di Pace, R. (2019). Dynamics and Stochasticity in Transportation Systems: Tools for Transportation Network Modelling. Elsevier.

[13] Cascetta, E. (2009) Transportation Systems Analysis. Second Edition. Springer Optimization and Its Applications, Springer.

[14] Cascetta, E. (1989). A stochastic process approach to the analysis of temporal dynamics in transportation networks, Transportation Research Part B: Methodological, 23(1), 1-17, https://doi.org/10.1016/0191-2615(89)90019-2.

[15] Choudhary, A., & Gokhale, S. (2016). Urban real-world driving traffic emissions during interruption and congestion. Transportation Research Part D: Transport and Environment, 43, 59-70.

[16] Daganzo, C.F, Sheffi, Y. (1977) On Stochastic Models of Traffic Assignment. Transportation Science, INFORMS, vol. 11(3), pages 253-274, August.

[17] Davis, G. A. and Nihan, N. L. (1993). Large Population Approximations of a General Stochastic Traffic Assignment Model. Operations Research, 41(1), 169–178.

[18] Di, X., Liu, H., Pang, J. and Ban, X. (2013). Boundedly Rational User Equilibria (BRUE): Mathematical Formulation and Solution Sets. Procedia - Social and Behavioral Sciences 80, 231–248.

[19] Duan, J., Zeng, G., Serok, N., Li, D., Lieberthal, E. B., Huang, H. J., & Havlin, S. (2023). Spatiotemporal dynamics of traffic bottlenecks yields an early signal of heavy congestions. Nature communications, 14(1), 8002.

[20] Erev, I., & Roth, A. E. (1998). Predicting how people play games: Reinforcement learning in experimental games with unique, mixed strategy equilibria. American economic review, 848-881.

[21] Fosgerau, M., Emerson, M., de Palma, A., Shum, M. (2020). Discrete choice and rational inattention: a general equivalence result. International Economic Review Vol. 61, No.4, November.

[22] Fosgerau, M., Paulsen, M., & Rasmussen, T. K. (2022). A perturbed utility route choice model. Transportation Research Part C: Emerging Technologies, 136, 103514.

[23] Friesz, T. L., Bernstein, D., Mehta, N. J., Tobin, R. L., and Ganjalizadeh, S. (1994). Day-To-Day Dynamic Network Disequilibria and Idealized Traveler Information Systems. Operations Research, 42(6), 1120–1136

[24] Gao, S., Frejinger, E. Ben-Akiva, M. (2010) Adaptive route choices in risky traffic networks: A prospect theory approach, Transportation Research Part C: Emerging Technologies, 18 (5) 727-740, https://doi.org/10.1016/j.trc.2009.08.001.

[25] Gawron, C. (1998). Simulation-based traffic assignment – computing user equilibria in large street networks. Ph.D. Dissertation, University of Köln, Germany

[26] Geroliminis, N., & Daganzo, C. F. (2008). Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings. Transportation Research Part B: Methodological, 42(9), 759-770.

[27] Glimcher, P. W. (2022). Efficiently irrational: deciphering the riddle of human choice. Trends in cognitive sciences, 26(8), 669-687.

[28] Harker, P. T. (1988). Multiple equilibrium behaviors on networks. Transportation science, 22(1), 39-46.

[29] Harsanyi, J. C. (1973). Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. International journal of game theory, 2(1), 1-23.

[30] Horowitz, Joel L. (1984). The stability of stochastic equilibrium in a two-link transportation network. Transportation Research Part B: Methodological, Elsevier, vol. 18(1), pages 13-28, February.

[31] Ji, Y., & Geroliminis, N. (2012). On the spatial partitioning of urban transportation networks. Transportation Research Part B: Methodological, 46(10), 1639-1656.

[32] Kashmiri, F. A., Lo, H. K. (2022). Routing of autonomous vehicles for system optimal flows and average travel time equilibrium over time. Transportation Research Part C: Emerging Technologies, 143, 103818

[33] Leclercq, L., Geroliminis, N. (2013) Estimating MFDs in Simple Networks with Route Choice, Procedia - Social and Behavioral Sciences, 80, 99-118, https://doi.org/10.1016/j.sbspro.2013.05.008.

[34] Li, Q., Chen, Z., Li, X. (2022). A review of connected and automated vehicle platoon merging and splitting operations. IEEE Transactions on Intelligent Transportation Systems, 23(12), 22790-22806.

[35] Li, J., Wang, Z., & Nie, Y. (2024). Wardrop equilibrium can be boundedly rational: A new behavioral theory of route choice. Transportation Science. https://doi.org/10.1287/trsc.2023.0132

[36] Mahmassani, H.S., Chang, G. (1987) On Boundedly Rational User Equilibrium in Transportation Systems. Transportation Science 21(2):89-99.

[37] Matejka, F. and McKay, A. (2015). Rational Inattention to Discrete Choices: A new foundation for the Multinomial Logit Model. American Economic Review 105, 272-98.

[38] McFadden D. Conditional logit analysis of qualitative choice behavior Front. Econ. (1974), pp. 105-142

[39] Nash, John (1951) Non-Cooperative Games. The Annals of Mathematics 54(2):286-295.

[40] Von Neumann, J., and Morgenstern, O. (1944). Theory of games and economic behavior. Princeton University Press.

[41] Olmos, L. E., Çolak, S., Shafiei, S., Saberi, M., & González, M. C. (2018). Macroscopic dynamics and the collapse of urban traffic. Proceedings of the National Academy of Sciences, 115(50), 12654-12661.

[42] Patriksson, M. (2015). The traffic assignment problem: models and methods. Courier Dover Publications.

[43] Prieto-Curiel, R., & Ospina, J. P. (2024). The ABC of mobility. Environment International, 185, 108541.

[44] Rosenthal, R.W. (1973) A class of games possessing pure-strategy Nash equilibria. Int J Game Theory 2, 65–67. https://doi.org/10.1007/BF01737559

[45] Saberi, M., Hamedmoghadam, H., Ashfaq, M., Hosseini, S. A., Gu, Z., Shafiei, S., ... & González, M. C. (2020). A simple contagion process describes spreading of traffic jams in urban networks. Nature communications, 11(1), 1616.

[46] Smith, M., Hazelton, M. L., Lo, H. K., Cantarella, G. E., and Watling, D. P. (2013). The long term behaviour of day-to-day traffic assignment models. Transportmetrica A: Transport Science, 10(7), 647–660. https://doi.org/10.1080/18128602.2012.751683

[47] R.S Sutton, A.G. Barto *Reinforcement Learning. An introduction. 2nd edition.* The MIT Press 2018.

[48] Thurstone, L. L. (1994). A law of comparative judgment. Psychological review, 101(2), 266.

[49] Van Vuren, T., Watling, D. (1991). A multiple user class assignment model for route guidance. Transportation research record, 22-22.

[50] Wang, Z., Bian, Y., Shladover, S. E., Wu, G., Li, S. E., & Barth, M. J. (2019). A survey on cooperative longitudinal motion control of multiple connected and automated vehicles. IEEE Intelligent Transportation Systems Magazine, 12(1), 4-24.

[51] Wardrop, J. G. (1952). Some Theoretical Aspects of Road Traffic Research. Proceedings of the Institution of Civil Engineers. 1 (3): 325–362 .

[52] Watling, D., Cantarella, G. (2015) Model Representation & Decision-Making in an Ever-Changing World: The Role of Stochastic Process Models of Transportation Systems, Networks and Spatial Economics, Springer, vol. 15(3), pages 843-882, September.

[53] Watling, D., Rasmussen, T., Prato, C., Nielsen, O. (2018) Stochastic user equilibrium with a bounded choice model, Transportation Research Part B: Methodological 114, 254-280, https://doi.org/10.1016/j.trb.2018.05.004.

[54] Whitehead, D. (2008). The El Farol bar problem revisited: Reinforcement learning in a potential game.

[55] Yang, H. (1998). Multiple equilibrium behaviors and advanced traveler information systems with endogenous market penetration. Transportation Research Part B: Methodological, 32(3), 205-218.

[56] Yang, H., Zhang, X., Meng, Q. (2007) Stackelberg games and multiple equilibrium behaviors on networks, Transportation Research Part B: Methodological, 41 (8) 841-861, https://doi.org/10.1016/j.trb.2007.03.002.

[57] Zhang, K., & Nie, Y. M. (2018). Mitigating the impact of selfish routing: An optimal-ratio control scheme (ORCS) inspired by autonomous driving. Transportation Research Part C: Emerging Technologies, 87, 75-90.

[58] Zhu, S., & Levinson, D. (2015). Do people use the shortest path? An empirical test of Wardrop's first principle. PloS one, 10(8), e0134322.