Decentralized Variable Speed Limit Bellman Control

Kevin Riehl*[1], Davide Pusino[1], Anastasios Kouvelas[1], and Michail Makridis[1]

[1]Traffic Engineering Group, Institute for Transport Planning and Systems, ETH Zürich, Switzerland

Short summary

Variable Speed Limit control can help avoid traffic jams before congestion forms. Vehicles upstream are required to decelerate at times in order to stop emerging congestion from propagating. This work proposes a fully decentralized, model-free, and infrastructure-free approach to Variable Speed Limit control, that employs connected vehicles as communication infrastructure, and as moving sensors and actuators. Dedicated Short Range Communication, consensus, and gossip algorithms, and a Bellman controller are components of this approach. The proposed method achieves significant improvements in traffic states, with up to 15% higher speeds, 5% lower density, and 8% higher flows. Significant improvements can be achieved at a compliance rate of at least 25% of all vehicles. Moreover, the approach is robust to gaps between platoons and recovers from periods of disconnection. The proposed method achieves traffic improvements similar to previous, centralized approaches, without the necessity of any infrastructure or model knowledge.
Keywords: Artificial Intelligence and Big data analytics; Automated and connected driving; Variable Speed Limit Control; Consensus Algorithm; Decentralized Control

1 Introduction

Traffic congestion on highways is a severe problem, as it causes longer travel times, increased fuel consumption, more emissions, reduced productivity, and increased frustration among drivers. While accidents, construction zones, and challenging weather conditions count amongst the most common causes of traffic congestion on highways Mahmud et al. (2012), traffic jams arise for many reasons. Bottlenecks, such as reduced number of lanes, and on- and off-ramps can cause congestion Daganzo (1997). Traffic congestion can also occur at the absence of any bottleneck, due to aggressive, non-cooperative human driving behavior such as lane-changing and car-following (phantom traffic jams) Sugiyama et al. (2008).

The slower-is-faster effect Gershenson & Helbing (2015) suggests vehicles upstream should decelerate (slow down) in order to enable conflict resolution downstream fast enough to avoid congestion forming. This idea is implemented in Variable Speed Limit control (VSL). VSL dynamically adjusts speed limits on highways to improve traffic flow and prevent congestion Fang et al. (2023). If implemented correctly, VSL can be an effective countermeasure against congestion on freeways, and significantly improve throughput and safety Khondaker & Kattan (2015). In practice, VSL can be challenging, due to complexities in real-time measurement of traffic & weather conditions, infrastructural requirements for sensors and electronic speed signs, public acceptance, and driver compliance Yuan et al. (2022).

The advent of connected and automated vehicles (CAVs) introduces new opportunities to this context and allows the employment of vehicles both as moving actuators to harmonize traffic flow and alleviate congestion and as moving sensors Du et al. (2023). In previous studies, real-time traffic state information (flow, density, speed) is used to inform upstream vehicles that comply with a state feedback controller to reduce their speed if necessary Du et al. (2023); Tajdari & Roncoli (2023). This approach however requires a communication and sensing infrastructure, accurate traffic state measurements, and model knowledge of the fundamental diagram.

Within this work, we propose an approach to VSL that is fully decentralized, model-free, and infrastructure-free. The approach requires nothing but connected vehicles, meaning the ability for vehicle-2-vehicle communication. The proposed method leverages Dedicated Short Range Communication (DSRC) Kenney (2011) to implement a decentralized communication infrastructure, and makes use of vehicles as moving sensors for decentralized speed estimation with a combination of discrete-time consensus Zhu & Martínez (2010) and gossip algorithms Boyd et al. (2005); Dimakis

et al. (2010). Finally, compliant vehicles that follow suggestions act according to a Bellman, bang-bang control law Bellman et al. (1956); Sonneborn & Van Vleck (1964), which does not require an understanding of the system model at any time.
The method is evaluated at the example of a highway bifurcation bottleneck. The results of conducted micro-simulations show, that the controller approach can achieve significant improvements in traffic (+15% speed, -5% density, and +8% flow) when compared with an uncontrolled situation, even though the decentralized communication does not guarantee recent and accurate speed estimates at all times. The results are consistent for different compliance rates, where at least 25% of vehicles must participate in the control to achieve a significant traffic improvement.
This work contributes to the literature on VSL control and demonstrates, that with connected vehicles an infrastructure-free, model-free, fully decentralized, vehicle-based approach can achieve traffic improvements similar to previous studies that require central communication and sensing infrastructure on highways. The implementation and simulation material can be found on the project's repository `https://github.com/DerKevinRiehl/decentralized_vsl/`.
The rest of this work is organized as follows. Section 2 outlines the communication infrastructure, sensing using consensus algorithms, and the Bellman control law of compliant vehicles. Section 3 presents the results of simulations, and discusses their implications. The work concludes with a summary of the main findings and elaborations on future works in section 4.

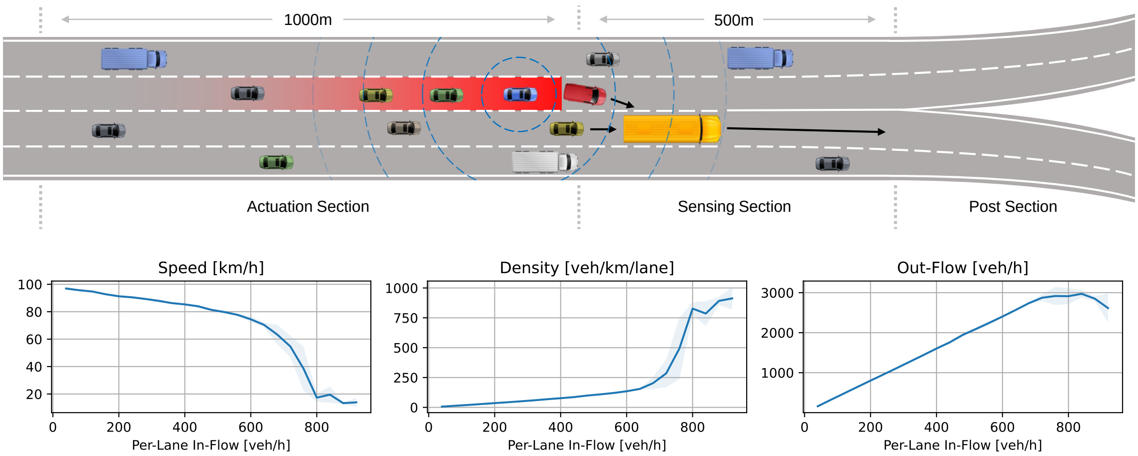

Figure 1: **Congestion Formation at Highway Bifurcation.** Impaired Speed, Density, and Out-Flow of ROI section (Sensing) for increasing per-lane in-flows.

2 Methodology

Vehicles & Freeway

We consider a population of n vehicles on a multi-lane freeway segment. The freeway segment consists of three sections: actuation section, sensing section, and post section, as shown in Fig. 1. Vehicles in the actuation section serve as mobile actuators, follow a control law, and actively reduce their speeds to enable decongestion in the bottleneck. Vehicles in the sensing section serve as mobile sensors and estimate the speed of this section v^{ROI}. This section is also called the region of interest (ROI), it is where the congestion due to the bottleneck (bifurcation) happens. While vehicles in the previous two sections communicate, vehicles in the post section do not participate in any communication, sensing, or actuation. Each vehicle i has a physical maximum speed $\tilde{v}_i^{max}$, a real speed v_i and a section s_i that represents the section it is located on.

Communication Infrastructure

Our method employs connected vehicles that communicate vehicle-2-vehicle, leveraging DSRC technology. We assume a share α of all vehicles is connected, and a maximum possible communication distance $d_c = 200$m. The communication takes place in rounds, and each round takes t_r time. During each round r, each vehicle sends out a message $m_{i,r}^{out}$ (in case it has a speed estimate available), and receives a set of messages $m_{i,r,k}^{in} \in \mathcal{M}_{i,r}^{in}$ from surrounding vehicles k. At the end of each round, the received messages are processed, and new messages to send out are prepared. Each vehicle's message consists of four fields: (A) "timestamp", (B) "section", (C) "value", and (D) "in-degree". Field (A) is populated with the current timestamp, field (B) is populated with the vehicle's current section, field (C) is populated with the vehicle's current estimate $\hat{v}_{i,r}^{ROI}$, and field (D) is populated with the vehicle's in-degree $\mathcal{N}_i$, which represents the number of received messages. These fields are used by the gossip and consensus algorithms during speed estimation.

Speed Estimation & Communication Protocol

Vehicles in the sensing section estimate the speed state of the ROI v^{ROI} using a discrete-time average consensus algorithm. A homogeneous traffic state across all lanes is implicitly assumed [1]. Vehicles in the sensing section determine their estimate of the ROI speed $\hat{v}_{i,r}^{ROI}$ as the weighted average of their own speed and the received estimates from surrounding vehicles in the sensing section as follows:

$$\hat{v}_{i,r}^{ROI} = \frac{1}{\mathcal{N}_i + 1}(v_i + \sum_{m_{i,r,k}^{in} \in \mathcal{M}_{i,r}^{in}} m_{i,r,k}^{in}["value"]) \tag{1}$$

Vehicles in the actuation section communicate their received estimates via a gossip algorithm with each other, and therefore back-propagate information on received estimates from the sensing section. They determine their speed estimates as described in the three following cases.
(i) If vehicle did not receive any message from surrounding vehicles in the communication round, it will stick to its previous estimate:

$$\hat{v}_{i,r}^{ROI} = \hat{v}_{i,r-1}^{ROI} \tag{2}$$

(ii) If received at least one message from surrounding vehicles of the sensing section, it will consider only those p messages from the sensing section and determine its speed estimate as the average of the received speed estimates:

$$\hat{v}_{i,r}^{ROI} = \frac{1}{p}(\sum_{m_{i,r,k}^{in} \in \mathcal{M}_{i,r}^{in}} m_{i,r,k}^{in}["value"]) \tag{3}$$

(iii) If received messages from surrounding vehicles of the actuation section only, it will determine its estimate as the most recent available estimate from all received messages. If the resulting estimate from any of the cases above exceeds a certain maximum considered estimation age a_{max}, the vehicle forgets previous estimates and does not possess an estimate. Vehicles that just entered the sensing section will reset their speed estimate to their current speed.
Vehicles in the sensing section always have a speed estimate of the ROI (at least determined by their own speed), while vehicles in the actuation section do not always have a speed estimate.

Control Law

We assume a share γ of all connected vehicles is compliant. Each connected and compliant vehicle i follows a Bellman, two-point, bang-bang control law Bellman et al. (1956); Sonneborn & Van Vleck (1964), based on its speed estimate $\hat{v}_i^{ROI}$, to control its maximum speed v_i^{max} as follows:

$$v_i^{max} = \begin{cases} \tilde{v}_i^{max} & \text{if} \quad \hat{v}_i^{ROI} \geq v_{thr} \\ \tau \times \tilde{v}_i^{max} & \text{if} \quad \hat{v}_i^{ROI} < v_{thr} \end{cases} \tag{4}$$

If the speed estimate $\hat{v}_i^{ROI}$ drops below a certain threshold speed v_{thr}, vehicles will consider a reduced maximum speed, which is defined by the speed reduction factor $\tau \in [0, 1]$.

Simulation Design

Highway Map with Bifurcation Bottleneck: Fig. 1 exemplifies the congestion formation at a highway bifurcation with four lanes. As vehicles need to perform their lane changes before the

[1] In most countries, outer lanes on highways are used for higher speed travelling compared to the middle lanes. However, our approach employs the average speed across all lanes in the ROI, as the highway segment is before a bifurcation. Yet, this assumption does not affect how the approach works.

bifurcation, conflicts can occur, slowing down vehicles upstream and causing increasing congestion. The red vehicle for example needs to change lanes, but as other vehicles on that lane, such as the yellow bus, prevent it from lane changing, it must decelerate, which causes congestion on the vehicles upstream in its lane. For higher levels of in-flowing traffic, the average speed and out-flow in the region of interest (sensing section) drops, while the density grows.

Traffic simulation: We conduct time-discrete micro-simulations of the road network using SUMO Lopez et al. (2018). Traffic is spawned randomly by sampling a Bernoulli distribution, and picking a random lane origin-destination combination with a uniform distribution. The fleet composition on the highway is assumed to be mixed traffic consisting of the following four vehicle types: cars (55%, max. speed 200 km/h), delivery vehicles (22%, max. speed 200 km/h), omnibuses (11%, max. speed 85 km/h), and trucks (12%, max. speed 130 km/h). We assume a Krauss car-following model Krauß (1998); Krauß et al. (1997), and an Erdmann lane-changing model Erdmann (2015). The maximum speed on the highway is assumed to be 100 km/h (which can be violated/exceeded by aggressive driving behavior up to 25 km/h). Traffic simulation experiments are run for 6000s of simulation time, with an additional 1000s warm-up time, and repeated 20 times to determine average and standard deviation for traffic fundamentals (speed, density, flow), communication, and control-related statistics.

3 Results and discussion

In this section, we discuss dedicated experiments, that were conducted for the design and evaluation of the communication system and controller. Afterwards, the control performance is analyzed, followed by a discussion on convergence guarantees of the consensus algorithm.

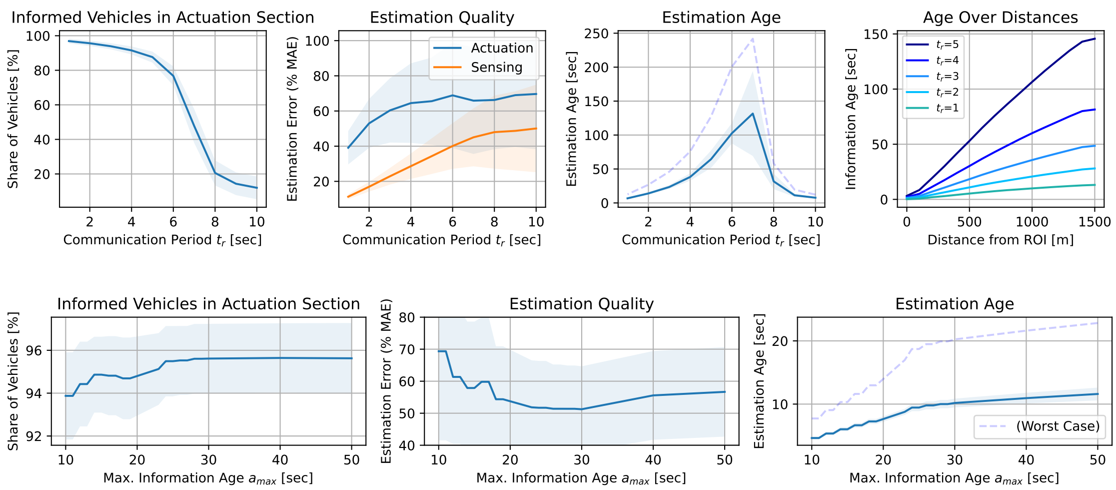

Figure 2: **DSRC Communication Design.** Communication Period Duration t_r, Maximum Estimation Age a_{max}, and their effect on ROI Speed Estimation Quality. Plots generated for per-lane in-flows of 720 veh/h.

Communication Design

Fig. 2 describes the decentralized communication system design. The first row shows the effects of varying the communication period t_r on the ROI speed estimation quality of vehicles in the actuation and sensing section. The mean absolute estimation error (MAE) is used to quantify the quality of vehicles' estimates.

The estimation quality of vehicles in the sensing section is consistently higher, as these vehicles actively participate in the average consensus algorithm when compared with the vehicles in the actuation section. Vehicles in the actuation section receive information via the gossip algorithm, and the most accurate and recent estimates back-propagate over time from vehicle to vehicle upstream. This results in outdated (aged) estimate information the further actuated vehicles are away from the ROI upstream.

For shorter periods t_r (more frequent communication), we observe consistent communication im-

provements. The share of vehicles in the actuation section that have an estimate available can grow to almost 100%, the age of estimates decreases for all distances of vehicles in the actuation section, and the estimation quality for both actuated and sensing vehicles can be significantly improved.
Smaller maximum estimation ages a_{max} that are considered by vehicles in the actuation section, can help to reduce the estimates' age at the cost of the share of vehicles that still have an estimate available and the estimation quality.
Therefore, we finally consider the following parameter combination for the communication infrastructure: every $t_r = 2$s speed estimation, and $a_{max} = 30$s maximum information age. This parameter combination achieves sufficiently high estimation quality, and broad information distribution, while not costing too much DSRC bandwidth, and not demanding too frequent communication from vehicles.

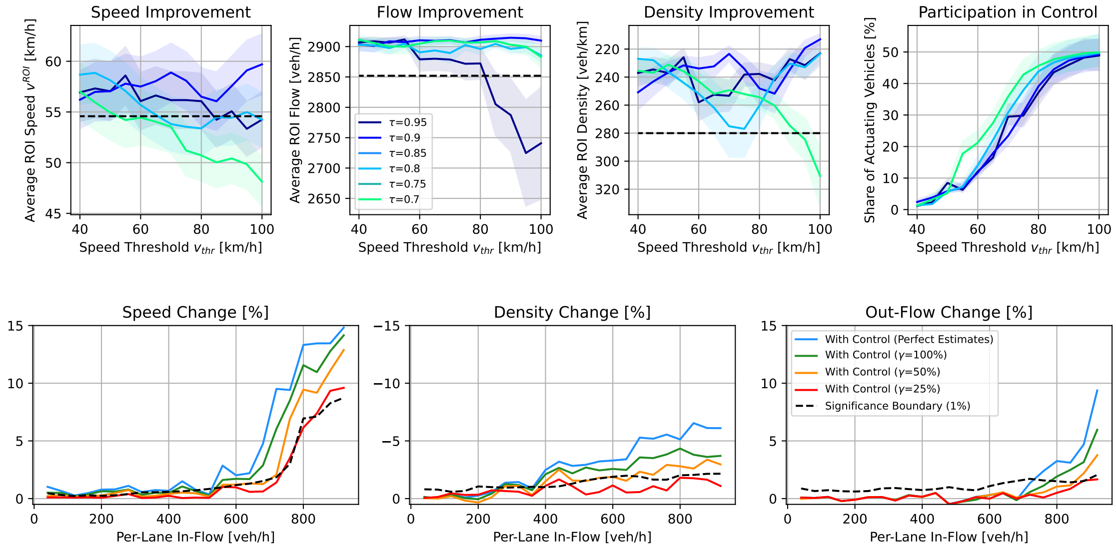

Figure 3: **Two-point Controller Design.** Speed Control Threshold v_{thr}, Speed Control Factor τ, Compliance Ratio γ, and their effect on ROI traffic improvement. Plots in the first row show traffic improvement for per-lane in-flows of 720 veh/h.

Control Design

Fig. 3 (top row) describes the decentralized two-point controller design. The speed threshold v_{thr} and speed reduction factor τ are varied, with effects on the participation of vehicles in the actuation section, and with mixed findings on improvements in speed, flow, and density. The higher v_{thr}, the more vehicles participate in reducing their speed. Depending on τ improvements in speed, flow, and density can be achieved. Finally, we consider the following parameter combination for the control law: $v_{thr} = 80$ km/h and $\tau = 0.9$. This parameter combination achieves the largest and most consistent improvements across all three traffic fundamentals.
Finally, the control performance is evaluated for different per-lane in-flows, and different compliance rates γ. The results can be found in Fig. 3 (bottom row). Significant improvements can be achieved beginning from an in-flow beginning from 720 veh/h. The ROI speed can be increased by 15%, the density can be reduced by 5%, and the out-flow can be increased by 8%. The improvements resemble those of prior works using VSL in a similar context Du et al. (2023); Tajdari & Roncoli (2023).
Even though vehicles in the proposed decentralized method have ROI speed estimates that are sometimes outdated or inaccurate, they achieve comparably similar performance improvements to vehicles with perfect estimates (blue line). The more vehicles comply with the control law (larger γ), the better the control performance. Based on our simulations, at least $\gamma = 25\%$ compliance rate is necessary to achieve significant improvements. This is consistent with prior findings employing a centralized sensing and communication approach Du et al. (2023); Tajdari & Roncoli (2023).

Control Performance & Impediments

Fig. 4 presents the control in action for two different in-flow scenarios. When speed drops and density rises without control, one can especially well observe the stabilization introduced by the decentralized, Bellman controller, with its implications on the traffic state.
The participation in actuation and estimation quality depends on the scenario. The estimation quality (measured as relative mean absolute error of the ROI speed estimates) is essentially higher before capacity (left scenario) when compared with at capacity (right scenario). Moreover, before capacity, the control causes actuation of vehicles when it is necessary to decongest, while at capacity an almost permanent actuation of compliant vehicles can be observed.
The left scenario showcases another challenge of the decentralized approach between 2000s and 2400s. In this interval, the share of vehicles upstream that have an estimate drops significantly, and estimation quality worsens substantially. Vehicles tend to cluster in platoons. If a gap between two groups of vehicles is larger than the maximum communication distance d_c, it acts like an information barrier preventing the flow of information upstream. This causes the communication system, consensus, and gossip algorithm to fail due to the lack of connectivity. Even though the communication and estimation fail during that interval, and therefore no actuation takes place, there are no significant impairments of the traffic state during that time, and the communication system recovers.
In addition to that, the two scenarios of Fig. 4 demonstrate the correlation of estimation quality, actuation, and the ROI traffic state. In times of a dropping speed and rising density in the ROI, actuation starts to increase after some delay and stays on a high activity level until the traffic state recovers. When estimates worsen, the effects of control on traffic state improvements are disturbed. What's more, the results of Fig. 4 highlight, that this approach not only improves the speed on average but also homogenizes the traffic over time.

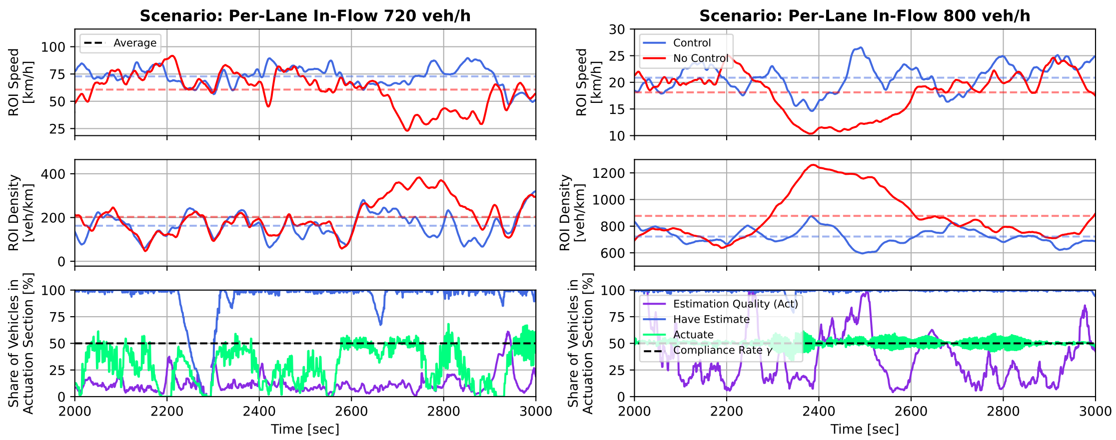

Figure 4: **Controller effect on two congestion scenarios.** Speed, density, and estimation metrics are shown for high (720 veh/h) and very high (800 veh/h) inflows. The controller ($\gamma = 50\%$) can significantly reduce congestion in both cases.

Convergence Guarantees & Influence on Control

Generally, there is no convergence guarantee for the time-discrete average consensus algorithm, as there is an underlying, time-varying communication network topology (switching topology). The vehicles in the sensing section permanently change, due to vehicles entering from the actuation section, and due to vehicles leaving into the post section. Moreover, the underlying average changes over time as well (as the vehicles change their speed continuously).
However, for small communication periods t_r (very frequent communication, $t_r < 0.5$s) the topology can be assumed to be constant over time, and the underlying speed average to be constant for short time periods. Similarly, the estimation quality in the actuation section will improve as well, and the estimation age will shrink to negligible delays with more frequent communication. Therefore, convergence can be guaranteed if and only if the vehicle-2-vehicle communication network graph is strongly connected, as each vehicle includes its own estimate (presence of self-loops). The connectivity of the graph will depend on the density of vehicles in the sensing section and the

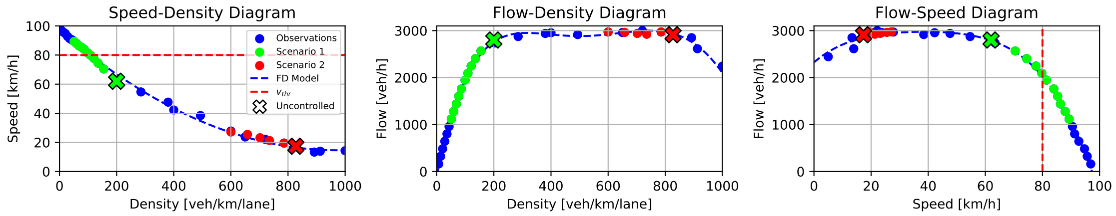

Figure 5: **Fundamental Diagram of Highway Bifurcation & Control**. The fundamental diagram of roads puts the traffic state fundamentals speed, density, and flow into relation with each other. Based on observations, typically, a negative decay between speed & density, and a u-shaped relationship between flow & density, and flow & speed can be observed. The FD models are estimated with a polynomial fit throughout the observations.

maximum communication distance d_c.
The width of the four-lane highway (16m) is negligible when compared to its section length (500m and 1000m), and therefore a single-lane highway can be assumed in the following discussion. We consider that the arrival of vehicles in the ROI is equivalent to sampling every second, an independent Bernoulli random variable of probability p. Assuming a constant speed of v for all vehicles, the time delay between two consecutive vehicles entering the ROI can be described as a geometrically distributed, random variable T, where p equals the vehicle flow per second:

$$P(T = t) = (1 - p)^{t-1}p \tag{5}$$

The distance between two consecutive vehicles can therefore also be described as a geometrically distributed random variable D:

$$P(D = d) = (1 - p)^{\frac{d}{v}-1}p \tag{6}$$

Given the previously mentioned assumptions, the convergence of the consensus algorithm can be guaranteed if the communication graph between the vehicles is strongly connected, which is the case if and only if every distance D_i between two consecutive vehicles is smaller or equal than the maximum communication distance d_c. The number of vehicles inside the ROI N can be determined as the product of density ν and length of the sensing section l_s. The convergence guarantee is as follows:

$$P(\text{Strongly Connected}) = P(\bigcap_{i=1}^{N-1} D_i \leq d_c) = (P(D_i \leq d_c))^{N-1} = (1 - (1-p)^{\frac{d_c}{v}-1})^{N-1} \tag{7}$$

Flow p and speed v can be determined by the relationships of the Fundamental Diagram (Fig. 5) for a given density ν. The dependence of the graph's strong connectedness on density is shown in Fig. 6. The cumulative distance distribution in Fig. 6 (left) empirically supports the assumption of the single lane (empiric distribution with four lanes from simulations follows the theoretical, geometric distribution with one lane).
Depending on the density of vehicles, different distance distributions exist, as shown in Fig. 6 (middle). The distance distributions will determine the probability that all vehicles that lie on the sensing section will form a strongly connected, vehicle-2-vehicle communication topology, as shown in Fig. 6 (right). The geometric distribution of distances is also reflected in the connectivity of the graph depending on the vehicle density. Beginning from a density of as low as 200 veh/km more than 99% of all vehicles have a communication distance to their neighbours that is lower than d_c, and therefore span a strongly connected graph. This critical density of 200 veh/km is easily reached on highways in practice, long before traffic congestion is an issue (per lane in-flow of around 720 veh/h), see Fig. 1. It is especially at this critical density that the controller is observed to cause significant improvements to the traffic state, see Fig. 4.
In terms of control, a minimum compliance rate $\gamma = 25\%$ suffices, as there are four lanes. For this compliance rate, on average, each lane will have at least one vehicle that slows down, forcing the vehicles behind it to slow down as well. This is, why for this minimum γ we can observe significant improvements in the traffic situation in Fig. 3. Of course, aggressive drivers in a hurry could try

to avoid these vehicles by lane-changing maneuvers, which gets increasingly harder for larger γ. A discussion for speed threshold v_{thr} and speed reduction factor τ is more challenging though, and must be selected for a specific road context. The Bellman controller with v_{thr} and τ will cause an oscillating between two operating points around the mean speed with control, which can be observed in Fig. 4 and Fig. 5. The amplitudes and frequency of oscillations depend on the traffic state (position on fundamental diagram, FD), τ, v_{thr}, and γ. The share of vehicles that actuate depends on v_{thr} and the current speed/density, as shown in Fig. 4 (left) and Fig. 5. Once the speed drops below v_{thr}, an increase of the actuating vehicles up to the compliance rate γ can be observed (with some delays).

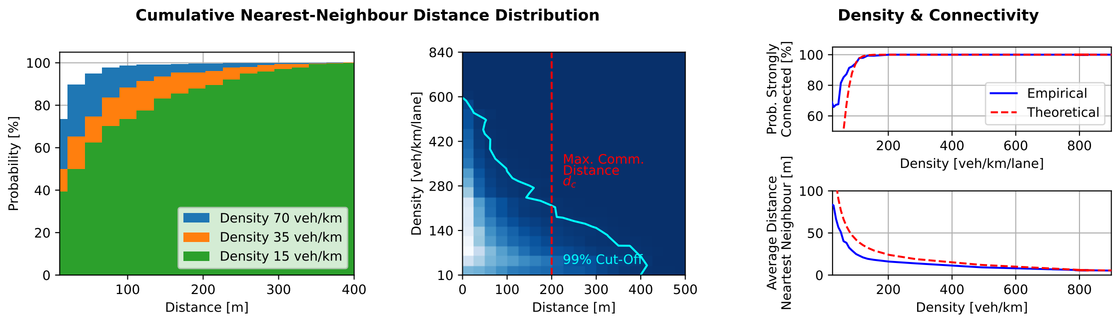

Figure 6: **Distance & Connectivity**. The convergence of the consensus algorithm depends on the strong connectedness of the communication graph. The graph's connectivity depends on the density of vehicles and the maximum communication distance. Convergence and strong connectedness can only be guaranteed if the distance of vehicles to their nearest neighbours is smaller than the maximum communication distance. The theoretic convergence guarantees match the empiric observations for sufficiently large density.

In the other scenario of Fig. 4 (right), when speed is permanently below v_{thr}, the share of actuating vehicles is permanently around the compliance rate γ. Fig. 5 demonstrates the role of τ; in both scenarios, the speed oscillates with an amplitude of around τ due to the controller. Due to the higher decay in scenario 1 (720 veh/h), the oscillations are larger when compared with scenario 2 (800 veh/h). In this case, tuning v_{thr} and τ is comparable to varying the controller's aggressiveness (low v_{thr} and τ) that acts with a strong control input as soon as a slowdown is estimated, or a more robust controller (high v_{thr} and τ) that will only act when needed (Fig 3).
To summarize, the consensus is guaranteed to achieve perfect estimates (assuming highly frequent communication) beginning from a critical vehicle density of around 200 veh/km. The two-point controller achieves significant improvements and transitions the traffic state to slightly oscillating, improving traffic conditions along the fundamental diagram.

4 Conclusions

This work set out to study a fully decentralized, infrastructure-free approach to VSL. It relies on connected vehicles that serve as a decentralized communication, sensing, and actuation infrastructure. Compliant vehicles/drivers that follow the suggestions decelerate slightly to mitigate congestion formation downstream and reach their destinations faster.
The results of simulation experiments demonstrate that the method succeeds in mitigating congestion, achieving up to 15% higher speed, 5% lower density, and 8% higher outflow compared to the uncontrolled scenario. These achievements are similar to those of previous works that require infrastructure due to their centralized approach. The proposed VSL approach acts robust to inaccurate, outdated estimates of the mean speed in the ROI, and reliably recovers from disruptions due to gaps between platoons. A compliance rate of at least 25% is observed to be necessary to achieve significant improvements.
This work's findings and chosen parameters are limited to the simulation case study. The approach itself is limited by distances and the connected vehicles' communication technology: (i) If the communication distance is too short, the decentralized communication infrastructure lacks sufficient connectivity for the consensus and gossip algorithms; (ii) if the actuation section is too long, estimation quality suffers from aged information; (iii) if the sensing section too short, estimation

quality suffers as the time that vehicles participate in the consensus algorithm is too short.
Future research could delve into exploring more realistic simulations inspired by real-world bottlenecks (i.e. lane merging, on-ramps), assess decentralized speed estimation on a lane level, investigate the robustness of parameter optimization in different real-world contexts (i.e. varying bottlenecks, distances, communication technologies), elaborate on methods for parameter estimation in real-world contexts, and studying the effect of platoons, longer actuation areas, and shorter communication distances d_c on the control performance. Moreover, the design of effective incentive mechanisms to encourage more drivers to comply (i.e. rewards, gamification, information on decongestion benefits) might be an interesting question to study.

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