A Game-Theoretic Framework for Enhancing Cooperation in Network Design Problems

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Short summary

The decisions of self-interested operators may lead to suboptimal outcomes. In this study, we propose a game theoretical framework to enhance cooperation in multi-region network design problems. We propose two network design games to explore regional collaboration: In the Non-Cooperative Network Design Game with Inter-Regional Investment, regions allow cross-border investments while retaining profits generated locally; In Cooperative Network Design Game, the Co-Investment and Payoff-Sharing Mechanism is introduced, enabling regions to jointly optimize the network for better overall payoffs and fairly distribute the benefits. A case study on the Zurich-Wintertur bus network is conducted to demonstrate the efficiency of the proposed framework.

Keywords: Strategic Interaction, Network Design Problem, Non-Cooperative Game, Co-Investment, Payoff Sharing

1 Introduction

Transport networks are typically composed of multiple subnetworks, each characterized by distinct decision-makers. Due to the network inter-connectivity, the design of one subnetwork may affect the performance of others, and vice versa. In this context, it is essential to account for the *strategic interactions* among subnetwork designers. Medeiros (2019) concluded cross-border railway services tend to be overlooked in investments of countries, leading to a potential loss of travel demand and undermining the competitiveness of rail transport. Extensive research efforts have focused on single-region network design problems, while the understanding of strategic interaction in multi-region network design problems remains insufficient.
Game theory is a powerful tool that has been utilized to model strategic interactions in various multi-agent settings (see, e.g., Zardini et al. (2023, 2021) for previous work in our lab). Specifically, one can typically classify games into *cooperative* or *non-cooperative*. In the non-cooperative design, agents make decisions independently, optimizing their individual outcomes without communication. In cooperative games, instead, agents can choose to engage in joint efforts to enhance their collective outcomes in the competitive environment.
In this work, we establish a game-theoretic framework for the multi-region network design problem by leveraging both non-cooperative and cooperative game theory. The goal is to facilitate the modeling of multi-region network design problems, proposing mechanisms to align the interests of decision-makers, foster cooperation, and improve performance from the perspectives of sustainability, efficiency, and social welfare.

2 Methodology

Network design problem: Network, Travel Demand and Performance Matrices

Mobility Network We model the mobility network as an edge-labeled directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{L})$, where $\mathcal{V}$ is the set of vertices, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of directed edges and $\mathcal{L} : \mathcal{E} \to \mathcal{Z}$ is a mapping from the set of edges $\mathcal{E}$ to the set of edge labels $\mathcal{Z}$. Specifically, an element $z_e = (x_e, c_e, l_e, t_e) \in \mathcal{Z} = \{0, 1\} \times (\mathbb{N}_0 \cup \{\infty\}) \times \mathbb{R}^+ \times \mathbb{R}^+$ is characterized by the availability of the mobility service on edge x_e, the capacity on the edge c_e, the edge length l_e, and the travel time associated to the edge t_e. In this work, we consider two authorities. The graph $\mathcal{G}$ can be divided into two subgraphs $\mathcal{G}^1 = (\mathcal{V}^1, \mathcal{E}^1, \mathcal{L}^1)$ and $\mathcal{G}^2 = (\mathcal{V}^2, \mathcal{E}^2, \mathcal{L}^2)$ corresponding to two regions (denoted Region 1 and Region 2). The sets of edges for the subgraphs are defined as follows. The edge set of Region i is $\mathcal{E}^i = \{(u,v) \in \mathcal{E} | u, v \in \mathcal{V}^i\}$ for $i \in I = \{1,2\}$, and the region-connecting edge set is defined as $\mathcal{E}^c = \{(u,v) \in \mathcal{E} | u \in \mathcal{V}^i, v \in \mathcal{V}^j, i,j \in I, i \neq j\}$. This partition allows each regional network to be designed by regional authorities while maintaining the overall connectivity of the mobility network. To enable multimodal mobility choices, each regional subgraph $(\mathcal{G}^i)_{i\in\{1,2\}}$ contains a public transport (PT) network layer $\mathcal{G}^i_R$ and an alternative-mode network layer $\mathcal{G}^i_A, \mathcal{L}^i_A)$, which we assume represents an aggregated layer for other transportation modes.

Travel Demand A travel request is defined as $r_m = (o_m, d_m, \alpha_m, \theta_m) \in \mathcal{R} = \mathcal{V}_A \times \mathcal{V}_A \times \mathbb{N}_0 \times \Theta$, characterized by the origin o_m, the destination d_m, the number of trips α_m and the type of trips $\theta_m \in \Theta$. The trips can classified into intra-regional trips and inter-regional trips based on origins and destinations. Travelers choose between a PT-prioritized route and an alternative-mode-based route (the alternative route) by evaluating the utility of both routes (u^R_m and u^A_m):

$$u^A_m = -\sum_{e\in\mathcal{E}^A_m} (\gamma_{\text{vot}} \frac{l_e}{v_A} + \gamma_{\text{A}} l_e), \quad \forall m \in \mathcal{M}, \tag{1}$$

$$u^R_m = -\sum_{e\in\mathcal{E}^R_m} l_e x_e (\frac{\gamma_{\text{vot}}}{v_R} + \gamma_{\text{t}}) - (1-x_e) \sum_{a\in\mathcal{E}_R} \mathbb{1}_{\mathcal{E}^a_e} l_a (\frac{\gamma_{\text{vot}}}{v_A} + \gamma_{\text{A}}), \quad \forall m \in \mathcal{M}, \tag{2}$$

where x_e denote the service availability on edge e, γ_{vot} the value of time, γ_{a} and γ_{t} the distance-based prices for alternative modes and PT service, respectively, l_e the travel distance on edge e, v_a and v_t the average speeds of alternative modes and public transport, and $\mathbb{1}_{\mathcal{E}^a_e}$ indicate whether edge e belongs to the alternative-mode edges when PT service on edge a is unavailable. For request r_m, a proportion $p_m \in [0,1]$ of trips choose the PT-prioritized route, determined by:

$$p_m = \frac{e^{u^R_m}}{e^{u^A_m} + e^{u^R_m}}, \quad \forall m \in \mathcal{M}, \tag{3}$$

Performance Matrices The performance of a transportation network is assessed network performance based on the CO_2 emissions, total travel costs, and profitability generated within its own region, denoted by J^e_i, J^c_i, and J^p_i respectively. The network design of region i and other region(s)$-i$ is represented as $h = (h_i, h_{-i})$:

$$f_i(h) = \omega_0 J^e_i(h) + \omega_1 J^c_i(h) - \omega_2 J^p_i(h) \tag{4}$$

where $\omega_0, \omega_1, \omega_2 > 0$ are weights. The system emission J^e_i accounts for both the PT service and alternative services. γ^R_m and γ^A_m denote the CO_2 emission unit for PT and alternative services, respectively. The total travel cost J^c_i is the travel cost within the region i. In Eq. (7), J^p_i measures the gap between the revenue from the PT service and the construction cost.

$$J^e_i(h) = \sum_{e\in\mathcal{E}^i_R} \gamma^R_m l_e y_e + \sum_{e\in\mathcal{E}^i_A} \gamma^A_m l_e y_e, \tag{5}$$

$$J^c_i(h) = \sum_{e\in\mathcal{E}^i_R} l_e y_e (\frac{\gamma_{\text{vot}}}{v_R} + \gamma_R) + \sum_{e\in\mathcal{E}^i_A} l_e y_e (\frac{\gamma_{\text{vot}}}{v_A} + \gamma_A), \tag{6}$$

$$J^p_i(h) = \sum_{e\in\mathcal{E}^i_R} \gamma_R l_e y_e - \sum_{e\in\mathcal{E}^i_R} (c^b l_e x_e + c^k l_e s_e), \tag{7}$$

The variable $Y = (y_e)_{e\in\mathcal{E}}$ represents the served flow on edges. Flows on public transport edges $\mathcal{E}_R$ depend on elastic demand and are constrained by capacity c_e, while flows on alternative edges $\mathcal{E}_A$ account for alternative route demand and unserved public transport demand. Specifically, $y_e(h)$ behaves as follows:

$$y_e(h) = \begin{cases} \min\left\{ \sum_{m\in\mathcal{M}} \mathbb{1}_{\mathcal{E}^R_m(e)} \alpha_m p_m, c_e \right\}, & \text{if } e \in \mathcal{E}_R, \\ \sum_{m\in\mathcal{M}} \mathbb{1}_{\mathcal{E}^A_m(e)} \alpha_m \hat{p}_e - \sum_{a\in\mathcal{E}^{\mathcal{R}}} \mathbb{1}_{\mathcal{E}^a_m(e)} y_a(h), & \text{otherwise,} \end{cases} \tag{8}$$

where $\hat{p}_e$ represents the maximum proportion of travelers using public transport when all edges of the route are fully covered. The boolean indicator functions $\mathbb{1}_{\mathcal{E}^R_m(e)}$ and $\mathbb{1}_{\mathcal{E}^A_m(e)}$ indicate whether edge e is part of the edge set of the PT-prioritized route and the alternative route, respectively.

Multi-Region Network Design Game

In this study, we introduce a game-theoretic framework to model the multi-regional network design problem. The framework assumes that each decision-maker is rational and self-optimizing. We

propose two types of network design games to explore potential collaboration:
a) **Non-Cooperative Network Design Game with Inter-Regional Investment**: In this model, regions allow cross-border investments while retaining profits generated locally.
b) **Cooperative Network Design Game**: The co-investment and payoff-sharing mechanism is introduced, enabling regions to jointly optimize the network for better overall payoffs and fairly distribute the benefits.

Non-cooperative Network Design Game with Inter-regional Investment The action is defined to be $h_i := \{x_e, s_e\}_{e \in \mathcal{E}^{xi'}} \in H_i$. The payoff function is a mapping $f_i : H \to F$, where $f_i(H_i, H_{-i})$ represents the payoff for operator i . Let $\mathcal{E}^{xi'} \in \mathcal{E}_R$ and $\mathcal{E}^{pi'} \in \mathcal{E}_R$ represent the design space and payoff space, respectively. By extending the design space $\mathcal{E}^{xi'} \supset \mathcal{E}_R^i$, the agent is allowed to invest in facilities outside its territory. Simoutnouly, by restricting the payoff space to the edge set within its territory, $\mathcal{E}^{pi'} = \mathcal{E}_R^i$, the agent's costs and payoffs are limited to those generated within its own network. The optimization model for single agent can be written as:

$$
\begin{aligned}
\min_{H_i^t} \quad & f_i(\mathcal{X}_i, \mathcal{X}_{-i}) && \text{(9a)}\\
\text{s.t.} \quad & \sum_{e \in \mathcal{E}_R^i} c^b l_e x_e^t + c^k l_e s_e^t \leq B_i^{\mathrm{t}}, && \text{(9b)}\\
& (X_e, S_e) = (x_e + x_0, s_e + s_0) && \text{(9c)}\\
& X_e \leq S_e \leq X_e \Omega, && \text{(9d)}\\
& c_e = \kappa S && \text{(9e)}\\
& \text{Eq. (1)} - (8),
\end{aligned}
$$

where B_i^t indicates the budget of regional authority i at the design stage t. X_e and S_e represent the connectivity and service frequency of the edge e. The existing infrastructure is denoted by $\mathcal{X}^0 = (x_0, s_0)_{e \in \mathcal{E}_R^i}$. Constraints in Eq. (9b) are the budget constraints, and Constraints in Eq. (9d) and Eq. (9e) are the relation restriction on edge construction, service frequency, and edge capacity.

Definition 1 (Nash Equilibrium of the Non-cooperative Network Design Game)**.** A strategy profile (h_i, h_{-i}) is a Nash Equilibrium of the interactive network design process if, for all regional authorities $i \in I$, $f_i(h_i, h_{-i}) \leq f_i(\hat{h}_i, h_{-i}) \ \forall \ \hat{h}_i \in H_i$.

Proposition 1. (The Existence of Pure NE of Relaxed Non-Cooperative Network Design Game) The pure nash equilibrium exists for the non-cooperative network design game with the following properties:

1. $x_e \in [0, 1]\,, \quad s_e \in [0, s_{\max}]\,, \quad \forall e \in E_R$;
2. $\Delta_e = \sum_{a \in \mathcal{E}_A^i} \mathbb{1}_{\mathcal{E} a_e}((\frac{\gamma_{\text{vot}}}{v_R} + \gamma_m^R) - (\frac{\gamma_{\text{vot}}}{v_A} + \gamma_A + \gamma_m^A)) \leq 0, \quad \forall e \in E_R$,

Proof. According to condition 1, the decision variables are relaxed to be continuous, and the action space H is compact and convex, with $f_i(h_i, h_{-i})$ continuous in h_i. With condition 2, f_i is convex in h_i for fixed h_{-i}. By the Maximum Theorem and Kakutani's Fixed Point Theorem, the relaxed non-cooperative network design game has at least one pure Nash equilibrium. □

Cooperative Network design Game: Co-investment and Payoff-sharing Mechanisms The Co-investment mechanism enables shared funding of public transport services, where regions combine their budgets and design the network with the goal of minimizing the total of their individual objectives. Payoff-sharing mechanism is designed to distribute the payoffs generated through the co-investment mechanism. In this work, we apply the Nash Bargaining Solution concept for the payoff mechanism modeling. Payoff-sharing mechanism can be formulated as the following optimization problem :

$$
\begin{aligned}
\max_{q_i \in \mathcal{Q}} \quad & \prod_{i \in I} (q_i - F_i^{ne}) && \text{(10a)}\\
\text{s.t.} \quad & \sum_{i \in I} q_i = \sum_{i \in I} F_i^c. && \text{(10b)}
\end{aligned}
$$

where $q_i \in \mathcal{Q}$ denotes the distributed payoff to authority i through the payoff-sharing mechanism. F_i^c denotes the payoff resulting from the co-investment mechanism. F_i^{NE} is the payoff the region can get if they do not apply cooperative network design, which is the Nash Equilibrium of non-cooperative network design. The constraints in Eq. (10b) restrict the sharable value to the amount generated by the co-investment mechanism.

3 Case Study

We use the networks of Zurich and Winterthur as a case study (in Figure 1), which include the bus networks in Zurich and Winterthur as well as the connecting rail lines operated by SBB. One-day travel demand data in the study area is simulated by MATSim based on population data provided by the Swiss Bureau of Statistics. To reduce computational time, the total number of requests is scaled down by a factor of ten.

Figure 2 compares the centralized design with the Nash equilibria of the non-cooperative network design game under varying budgets and allocations (μ represents the budget proportion for Zurich vs. Winterthur). As shown in Figure 2a, the centralized design achieves the lowest total cost, favoring Zurich due to its higher travel demand (Figures 2b and 2c). However, while the centralized approach minimizes system cost, it is less feasible as self-optimizing agents prioritize their own interests and it fails to ensure fair resource distribution.

Figure 3 compares network design game solutions under a budget split of $\mu = 8 : 2$, with 50% allocated via 1) interregional investment mechanism and 2) co-investment and payoff sharing mechanism. The y-axis shows the cost ratio relative to the Nash equilibrium. Co-investment with payoff sharing reduces costs by 38%, while interregional investment lowers system costs by 36% (40% for Zurich, 29% for Winterthur). This demonstrates that both proposed models effectively align regional interests and simultaneously optimize the overall network. The mechanisms not only unlock the inter-city transport demand but also stimulate local trips.

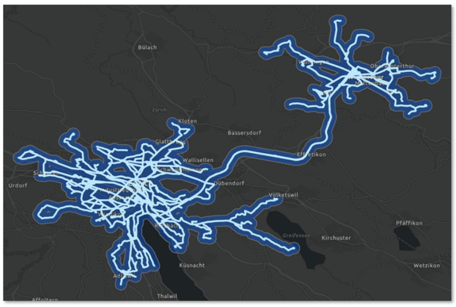


Figure 1: Study Area: Zurich and Wintertur

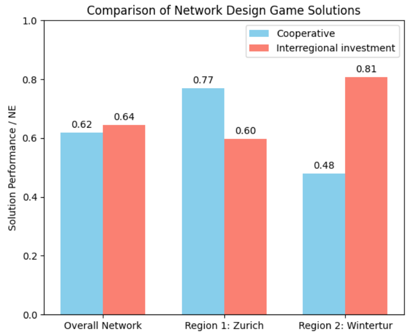
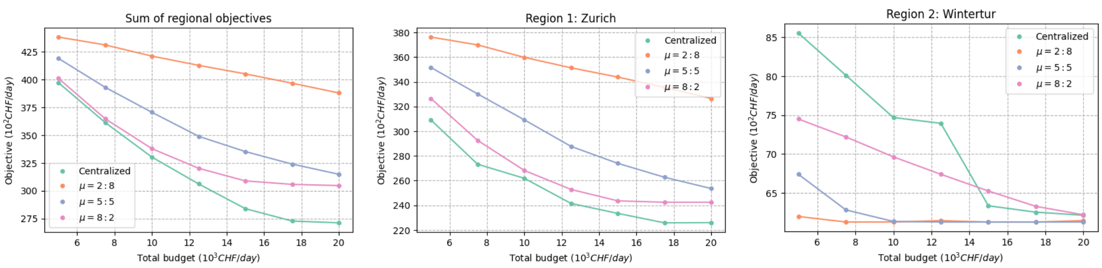


Figure 3: Network Design Solutions



(a) Overall network

(b) Zurich

(c) Wintertur

Figure 2: The Comparison to Centralized Network Design with varying budget distribution.

4 Acknowledgements

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