Integrated demand-side management and timetabling for an urban transit system

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Short summary

The intelligent upgrading of metropolitan rail transit systems has enabled the implementation of demand-side management policies with integrated multiple operational strategies in real-world applications. However, the strong interdependence between supply and demand requires a coordinated approach that combines demand-side management policies with supply-side resource allocations to optimize the urban rail transit ecosystem. This study introduces a mathematical and computational framework to optimize train timetables, passenger flow control strategies, and trip-shifting plans via pricing policies. An integer linear programming model is proposed for this problem. To enhance computational efficiency, a Benders decomposition-based algorithm within the branch-and-cut method is developed. Computational results demonstrate that the proposed method outperforms the state-of-the-art commercial solver in computational efficiency. High-quality solutions, including optimal ones, are obtained at the root node with minimal branching, thanks to the novel decomposition framework and valid inequalities.
Keywords: Urban rail transit; Train scheduling; Trip booking; Trip shifting; Benders decomposition

1 Introduction

Nowadays, congestion within urban rail transit (URT) systems in metropolitan areas has become the norm in operations. The cause of congestion is the mismatch between the continuously evolving passenger demand and the relatively stable capacities of the URT system. To address this challenge, URT authorities and operators might consider several strategies. Initially, they could invest in technical and social strategies that involve infrastructure and operational improvements, i.e., to expand the number of lines and adjust train timetables to increase the frequency of trains (e.g., Robenek et al., 2018; Cacchiani et al., 2020; Binder et al., 2021; Xia et al., 2023, 2024). However, restricted land resources and continuing congestion both indicate that these measures alone are insufficient to accommodate the surge in passenger demand. For example, even though the headway of Beijing Metro Line 10 during the morning peak hour was lowered to one minute and 45 seconds, there were still many standing passengers inside trains. This highlights the necessity for *demand-side management methods*.

From the demand perspective, one approach is the implementation of a *passenger flow control strategy*, which controls the boarding rate at each station to maintain spare capacity for downstream stations, thereby optimizing capacity utilization. For instance, in 2019, the Beijing metro regularly enforced this strategy at 91 stations on weekdays. Another strategy is to use *congestion pricing*, which differentiates between peak and off-peak fares. This strategy has been effectively applied in various cities. In essence, congestion pricing aims to achieve *trip shifting*, i.e., encouraging passengers who intend to travel during peak periods to shift to less congested times, thereby balancing the demand at different time periods. Thereafter, we define *passenger directing* as imposing the passenger flow control strategy to limit boarding rates coupled with a pricing strategy designed to incentivize passengers to shift their departure times. Furthermore, a more moderate and emerging alternative within the urban rail transit system is the *trip booking* strategy, which allows limited passengers to reserve a travel slot one day in advance, thus bypassing queues outside the station for a passenger flow-controlled entry permit. The reservation system in this study allows passengers to reserve a time slot, which guarantees platform access and immediate train boarding, rather than requiring them to book specific seats. This operational strategy was adopted by the Beijing metro in 2020, and by 2021, it reportedly saved a cumulative 40,000 hours in passenger waiting times, with 88 percent approval from users. Given the fundamental supply-demand mismatch causing congestion, integrating these operation- and demand-side strategies is anticipated to be a more effective approach for operational management in the context of growing congestion in transportation around the world, which is the focus of this paper.

However, to our knowledge, the development of passenger flow control and trip booking strategies in practical operations currently stands apart from the production of train timetables and completely depends on the manual experience of operators. In particular, the trip booking strategy is an emerging method in intelligent URT systems, facilitated by technological advancements, yet it remains at the proof-of-concept stage in real-world operations. This stage calls for innovative approaches to realize its potential benefits.

While existing research has developed mathematical models and solution algorithms for both the passenger flow control problem and the joint optimization of train timetabling and passenger flow control problem, the integration of train timetabling, passenger directing, and trip booking problem is hardly addressed in the literature. Nevertheless, the experimental results from the related studies (e.g., Shi et al., 2018; Lu et al., 2022, 2023; Yuan et al., 2023) consistently indicate that although the number of detained passengers decreases with the implementation of the timetabling and passenger flow control policies compared to scenarios without them, the phenomenon of oversaturation persists. These findings highlight the necessity to collaboratively optimize additional demand management methods. On the other hand, the algorithms proposed for the joint optimization models primarily focus on solution efficiency and often fail to produce the exact solutions desired in the operational planning phase.

In this paper, we aim to close these gaps by presenting an integrated demand management and timetabling approach and an exact solution method. Three key layers of government, metro corporations, and passengers serve as the foundation for our analysis. Our main goal is to provide proof-of-concept insights to operators and governments through the optimal solution obtained by the exact solution. We don't consider a game theory method or a bi-level framework with passenger behaviors, since they both lead to models that are difficult to solve exactly in an acceptable time. We also leave out the sequential solution approach because it lacks feedback between decisions and often leads to suboptimal solutions. By optimizing timetables and passenger flow control strategies and providing discounts on ticket prices to encourage passengers to shift their trips, the metro corporation maximizes its passenger-oriented resource allocation. By shifting their travel plans and making reservations, passengers minimize waiting times. Meanwhile, the government makes up for the metro company's loss of fare revenue by providing additional subsidies.

2 Methodology

This paper formally addresses *the integrated optimization of trip booking, passenger directing, and train timetabling* (BDTT) problem, factoring in time-dependent passenger demand and time-varying reservation slot allocation plan. A framework of practical applications is summarized in Figure 1. Within the proposed framework, three travel pattern are available to a passenger, as shown in Figure 2. If a passenger successfully makes a reservation, he can travel at the scheduled time and directly enter the platform to board a train by paying the full ticket fare. Passengers with reservations are never detained. If a passenger does not have a reservation, he can either shift to the suggested arrival time or maintain his originally scheduled arrival time. For the former option, a passenger can enjoy a discount on the ticket price, whereas the latter option does not offer any discounts. Regardless of whether passengers without reservations choose to shift their travel time, they must queue outside the platform and wait for permission to enter and board a train, which may result in being stranded.

We formulate the BDTT problem as an integer linear programming (ILP) model that captures the interdependencies among train timetables, passengers with reserved trips, those without reservations, and passengers' trip-shifting plans. Our objective is to minimize the weighted sum of passengers' waiting time and additional governments' subsidies to encourage passengers to shift their departure times. We model the service fairness among passengers in terms of the possibility of boarding the first available train, both with and without reservations, and across different stations as constraints.

To solve this problem, we design a tailored decomposition framework in accordance with the mathematical properties of the proposed model and a Benders-decomposition-based solution method within the branch-and-cut method. In the developed decomposition approach, partial information about passenger dynamics is incorporated into the timetabling subproblem, which can significantly reduce the number of feasibility cuts. Furthermore, we introduce valid equalities and inequalities that not only strengthen the bounds at each node but also decrease the number of iterations producing inefficient feasibility cuts. These enhancements enable the proposed algorithm to solve real-world instances within reasonable computational times. We also implement a series of heuristic accelerating strategies to improve the solution quality at the root node.

3 Experiments and results

We validate the performance of our proposed methods based on two case studies. First, we evaluate the advantages of the proposed model and the performance at the root node of the solution method on a proof-of-concept case study. Thereafter, to gain more insights on the integrated optimization and the full performance of the algorithm, we conduct real-world instances based on the data of the Beijing metro system.

In Table 1, the results among various maximum values of shifting time are presented, where both the booking ratio and the service fairness factor are set to 50%, the discount on the ticket price is 80%, and the weight coefficients are designated as 1 and 5. These results include the percentage of passengers shifting departure times (SP), the average waiting time of passengers without reservations (AWT-WR), the number of detained passengers without reservations (DP), the percentage of additional government subsidies, and the maximum congestion experienced at stations during the operation of all trains.

It can be observed that when the demand management strategy encouraging passengers to shift trips through incentives is not implemented, at least 12 trains are required to serve all demand. However, if passengers are encouraged to shift their travel by up to 10 minutes, it becomes possible to meet all demand with just 11 trains. This finding leads us to conclude that a demand management strategy that promotes passenger shifts through incentives can effectively reduce fleet size.

A second observation is there is an inverse relationship between the elasticity in departure time adjustments and the reduction of the number of detained passengers. Specifically, as passengers' shifting behavior is encouraged and the duration of maximum shifting time extends, the number of detained passengers is remarkably reduced. For example, when 11 trains are operated, the number of detained passengers decreases from 211 to 32 as the maximum shifting time changes from 10 to 25 minutes. However, there is no linear connection between the maximum allowable shifting time and the average waiting time. This is because the optimal solution is a trade-off between the waiting time and additional government subsidies.

These results indicate that the integration of directing and timetabling policies could increase the effectiveness of the rail transit system. This improvement is demonstrated by a considerably reduction in the waiting time of passengers without doing any serious harm to the operator's perspective.

Furthermore, to examine the effectiveness of the proposed algorithm, we define the following three variants:
(i) **BD** uses the optimality and feasibility cuts.
(ii) **TCBD** extends BD by including the cut loop stabilization at the root node and the tailing off strategy.
(iii) **TTCBD** extends TCBD by including the tree search strategy.

Figure 3 presents an overall comparison among Gurobi, BD, TCBD, and TTCBD in terms of the convergence trends of upper and lower bounds and the optimality gap over time with respect to

Table 1: Results among various values of maximum allowable shifting time. Abbreviations: SP= Shifting passengers; AWT-WR = Average waiting time of passengers without reservations; DP = Detained passenger; MW = Maximum number of passengers waiting at stations.

# of Trains	Maximum shifting time (min)	SP (%)	AWT-WR (min)	# of DP	Additional subsidy (%)	MW
11	0	Infeasible	–	–	–	–
	5	Infeasible	–	–	–	–
	10	21.14	2.48	211.00	2.03	312.00
	15	26.25	1.91	32.00	2.52	236.00
	20	22.48	1.93	32.00	2.16	225.00
	25	22.75	1.92	32.00	2.19	236.00
12	0	0	2.53	202.00	0	245.00
	5	14.22	1.93	0	1.37	231.00
	10	15.64	1.87	0	1.50	227.00
	15	16.28	1.84	0	1.56	212.00
	20	12.76	1.93	0	1.23	231.00
	25	14.00	1.90	0	1.35	214.00

the instance based on the Beijing metro. Looking to Gap displayed in Figure 3(a), it is evident that TTCBD outperforms the other algorithms without the tree search strategy. Specifically, TTCBD finds a solution with a Gap of less than 5% within 2,200 seconds. In comparison, both TCBD and BD reach solutions with similar quality after 4,000 seconds, whereas Gurobi does not converge to a comparable solution within 5,000 seconds. Figure 3(b) provides detailed results of upper and lower bounds over time for these solution methods. Notably, the initial lower bound obtained by TTCBD is highest. The significant improvement in performance of TTCBD can be attributed to the incorporation of a tree search strategy, which facilitates strong branching and in-depth exploring for the lower bound. The second observation is that Gurobi, BD, and TCBD can quickly find a good upper bound, while the lower bound slowly increases. All in all, this experiment illustrates that TTCBD outperforms the other alternatives.

4 Conclusion

We study the integrated optimization of booking, directing and timetabling on oversaturated urban rail trainsit lines. We introduce an ILP model and a Benders-decomposition-based solution approach within a branch-and-cut framework. Computational results show that our solution method outperforms commercial solvers in terms of computational efficiency. Our integrated optimization approach reduces the fleet size for operators and decreases the waiting time of passengers, thereby validating the effectiveness of our proposed methods.

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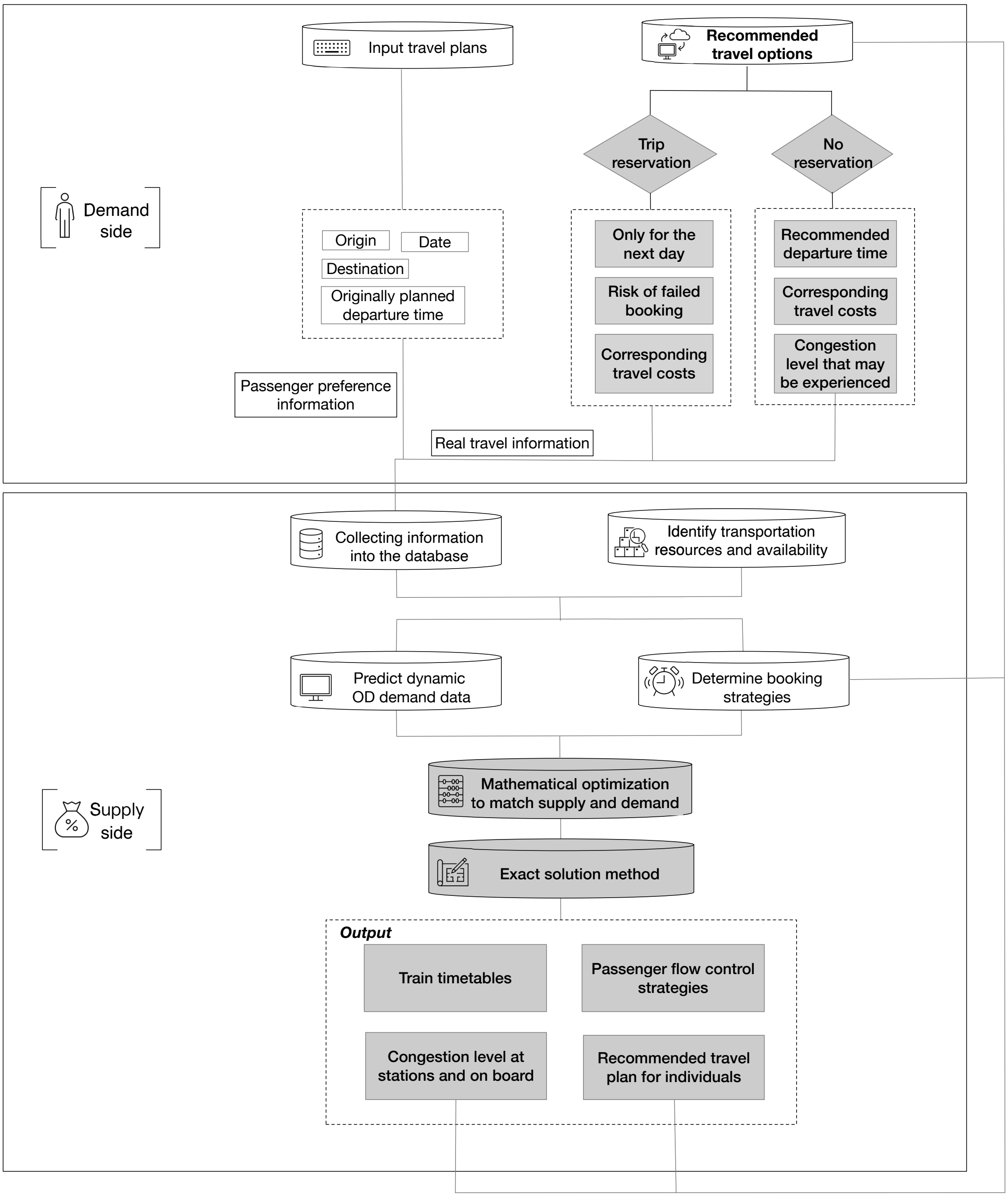


Figure 1: A framework for practical applications.

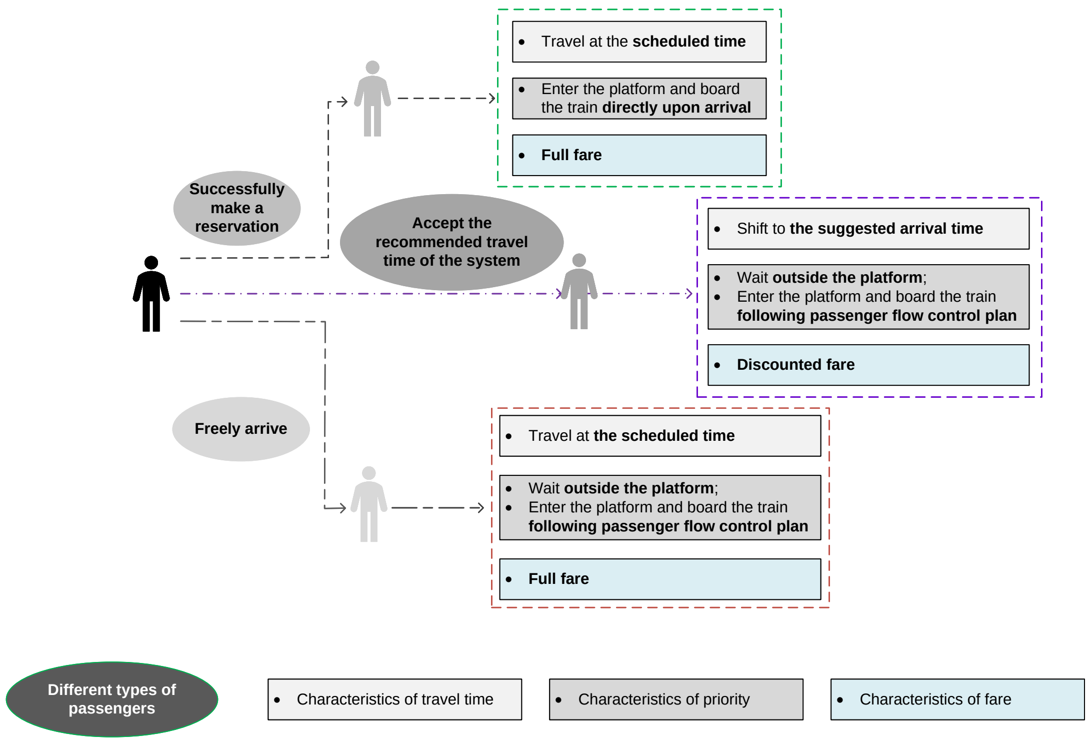
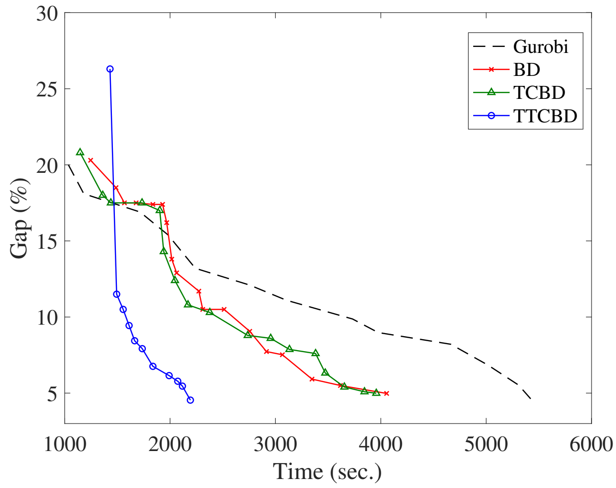
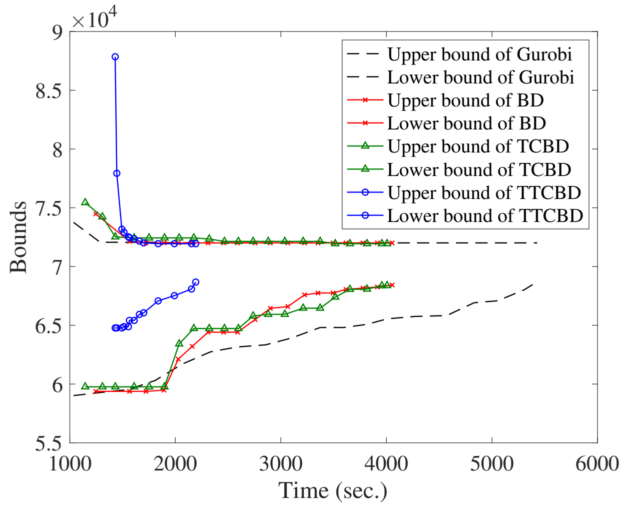


Figure 2: Illustration of passengers' types, travel time, priority, and fare characteristics. Notes: Passengers with reservations are never detained.



(a) Gap



(b) Upper and lower bounds

Figure 3: Convergence trends of the upper and lower bounds and the Gap.