On the extension of Electric Bus Charging Infrastructure considering Charging Scheduling and Energy Pricing

Dimitrios Rizopoulos*[1] and Konstantinos Gkiotsalitis[2]

[1]Ph.D. Candidate, Department of Transportation Planning and Engineering, National Technical University of Athens, Greece
[2]Assistant Professor, Department of Transportation Planning and Engineering, National Technical University of Athens, Greece

Short summary

The shift to fully electrified public transport is essential for sustainable urban mobility, but efficiently deploying charging infrastructure remains a major challenge. Especially in the case when cities already have electric bus networks, expanding fleets to additional routes introduces critical operational hurdles. In that regard, this study introduces a strategic planning model to optimize charging station placement, considering existing charging infrastructure at selected depots. The model utilizes multi-objective mixed-integer linear programming, integrating fleet operations, installation costs, two different types of chargers (slow/fast) and electricity pricing. By addressing these interdependent factors, the model minimizes capital expenditures, operational costs, and deadhead times, facilitating cost-effective and efficient infrastructure expansion. A real-world application in Limassol, Cyprus, highlights the model's utility in offering actionable insights for urban planners and policymakers, advancing the transition to sustainable public transport systems.
Keywords: charging station location; electric buses; operations research application.

1 Introduction

The transition toward electric mobility has become more crucial than ever. Although the uptake of various electric vehicle types has grown significantly in recent years, the level of adoption varies across cities, countries, and transportation sectors (Anastasiadou & Gavanas, 2022). In the realm of public transportation, there is a continuous movement to electrify vehicle fleets that facilitate public transit services, with particular emphasis on bus fleets in recent years (Kruchina, 2023).
A key factor driving the electrification of bus fleets is the urgent need to reduce Carbon Dioxide (CO_2) emissions and mitigate climate change, particularly in urban areas. While not specifically introduced for public transport, various initiatives and legislative frameworks support the transition to electric mobility by promoting cleaner transportation solutions. The European Green Deal[1] aspires to achieve climate neutrality within the EU by 2050, with a strong focus on reducing transport-related emissions. Complementing this goal, the Fit for 55 package[2], introduced as part of this overarching plan, aims to cut net greenhouse gas emissions by at least 55% by 2030. Furthermore, the Clean Bus Declaration Act, endorsed in 2016 by over 80 private and public entities, outlines a commitment and strategy for acquiring zero-emission buses, thereby advancing sustainable urban mobility targets for 2030 (Lu et al., 2023).
Despite the efforts on a political level, the transition to electric mobility brings new technical challenges, particularly in finding suitable locations for charging stations as well as scheduling the refueling process for electric public transport vehicles. While these challenges can be partly addressed by solely relying on traditional planning methods and domain expertise Bastarianto et al. (2023), several innovative planning approaches have been proposed in the scientific literature in the attempt to support the efforts made by practitioners and public transport authorities.
Early studies in this domain, including the ones by Jang et al. (2016) and Wang et al. (2016), proposed optimization models for electric bus charging infrastructure, focusing on minimizing initial infrastructure costs and determining optimal charger placements. Later, charger location was co-optimised together with charging scheduling, a tactical planning horizon problem. For

[1]The European Green Deal, Website
[2]The Fit-for-55 package, Website

example, Li et al. (2020) solved this integrated planning problem of regular charging electric bus scheduling and charging station placement using an Adaptive Genetic Algorithm, while McCabe & Ban (2023) introduced a MILP model that incorporated deadheading costs in the objective function of the integrated problem.

More recently, the prominent research direction has become the integration of the charging station location problem, charging scheduling as well as energy grid considerations. For instance, Zhou et al. (2022) addressed the integrated planning of charging station deployment and electric bus charging schedules, incorporating Time of Use (ToU) Tariffs and demand charges while focusing on fast charger deployment. Foda & Mohamed (2024) developed three optimization models to tackle the same integrated problem while accounting for the Total Cost of Ownership and Greenhouse Gas emissions. Finally, connected to this work are the ones by Liu et al. (2021, 2023) who, in their studies, account for Photovoltaic and Energy Storage Systems in combination with the integrated planning problem. To the authors' knowledge, no existing studies address the extension of electric bus charging networks, incorporating both slow and fast chargers, while considering charging scheduling, energy pricing constraints, and multiple objectives optimization.

To support the decision-making of public transport operators in their transition to electric mobility, this study presents a mathematical model for extending already established charging station networks for electric buses. The problem is formulated as Mixed Integer-Linear Programming (MILP) problem and integrates station placement, charging schedules, and energy pricing (ToU Tariffs and Peak Demand Charge). The model generates optimal solutions targeting two objectives: (i) minimizing bus deadhead times and (ii) reducing costs for charger installation and energy consumption. To accurately analyse the relationship between these two objectives, an ϵ-Constraint method is applied to approximate the Pareto optimal front of the problem.

The remainder of this paper is organized as follows. Section 2 presents our methodology and mathematical formulation of the problem. Section 3 provides the case study on Limassol, Cyprus. The article concludes with a summary of insights derived from this research.

2 Methodology

Problem statement

In response to the identified needs and respective research directions, we attempt to extend the definition and formulation of the integrated problem of the Charging Station Location Selection and Scheduling problem, as follows:

"*Given initial sets of pre-existing slow and fast charging stations located at various sites $\mathcal{V}$, along with a specified Time-of-Use tariff structure, peak demand charges, and a maximum available budget $\mathcal{B}$, determine the optimal placement of new slow and fast charging stations. The objective is to meet the charging demand of an electric bus fleet while minimizing installation costs, daily charging expenses, and deadhead time across a set of designated bus trips $\mathcal{K}$.*"

From the definition of the problem, one may deduce that two objectives are considered for the problem and the bus fleet operator (i.e. decision maker in our case):

1. *Objective 1:* The minimization of deadhead time, which is the travel time between the last stop of the bus line (end of itinerary) and the arrival time at the charging station location.
2. *Objective 2:* The minimization of the monetary cost of installing new chargers and the minimization of the daily operational charging costs stemming from the quantity of energy consumed according to the ToU Tariffs and demand charge cost (based on peak demand).

Problem Formulation

We consider a set of bus lines $\mathcal{L}$ which perform a set of $\mathcal{M}$ bus trips, out of which a subset $\mathcal{K}$ requires charging ($\mathcal{K} \subseteq \mathcal{M}$). These $|\mathcal{K}|$ vehicle trips must be assigned to a set of available charging stations $\mathcal{N}$ and scheduled to respective time slots $\mathcal{F}$. We consider that several chargers of two types, slow and fast, can be installed in a pre-defined set of candidate charging station locations $\mathcal{V}$. Several potential charging installation options $\mathcal{N}$ (i.e. chargers) may be available at a single physical location.

Our objective is to allocate $\mathcal{K} = \{1, 2, 3, ...\}$ vehicle trips requiring charging to the available chargers $\mathcal{N}$ and charging slots $\mathcal{F}$ in a manner that minimizes financial upfront investments and operational

costs, as well as vehicle deadhead times for charging purposes. The set of potential charger installations $\mathcal{N}$ can be divided into the set of slow charging options $\mathcal{N}_1$ and the set of fast charging options $\mathcal{N}_2$. Within these, the set $\mathcal{N}_3$ denotes the already installed slow chargers at any location within $\mathcal{V}$, while $\mathcal{N}_5$ represents the potentially new slow charging stations. Similarly, the set $\mathcal{N}_4$ corresponds to the already installed fast charging stations, and $\mathcal{N}_6$ represents the new potential fast charging stations.
Additionally, in this formulation, we assume three-time horizons. One is used for the time slots of the slow chargers, one for the time slots of the fast chargers, and one for demand charge time periods, as defined by an energy management authority or an energy grid operator. Each slow charger $j \in \mathcal{N}_1$ can be utilized multiple times throughout the day, resulting in a set of charging time slots $\mathcal{F}_1$ (i.e. first time horizon). Similarly, each charger $j \in \mathcal{N}_2$ results in a set of charging time slots $\mathcal{F}_2$ (i.e. second time horizon). Set $\mathcal{F}_3$ is used for the third time horizon, indicating the demand charge time periods, which are considered the time windows based on which the grid operator calculates its charges.
Given this first presentation of the problem's nomenclature, below, we provide the decision variables of the formulation:

- $x_j \in \{0,1\}$, where $x_j = 1$ if we decide to install charger $j \in \mathcal{N}$, and 0 otherwise. Notably, $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2 = \mathcal{N}_3 \cup \mathcal{N}_4 \cup \mathcal{N}_5 \cup \mathcal{N}_6$, indicating simultaneous selection of the charging location and type.
- $q_{kj} \in \{0,1\}$, where $q_{kj} = 1$ if trip $k \in \mathcal{K}$ is assigned to charger $j \in \mathcal{N}$, and 0 otherwise.
- $u^s_{kjf_1} \in \{0,1\}$, where $u^s_{kjf_1} = 1$ if trip k starts charging at time slot $f_1 \in \mathcal{F}_1$ at the *slow charger* $j \in \mathcal{N}_1$.
- $u^h_{kjf_2} \in \{0,1\}$, where $u^h_{kjf_2} = 1$ if trip k starts charging at time slot $f_2 \in \mathcal{F}_2$ at the *fast charger* $j \in \mathcal{N}_2$.

Furthermore, several additional dependent variables are considered regarding the charging scheduling of the electric buses in the third time horizon $\mathcal{F}_3$:

- $UD^s_{jf_3} \in \{0,1\}$, where $UD^s_{jf_3} = 1$, if any trip $k \in \mathcal{K}$ is charging at the slow charger $j \in N_1$ during demand charge time period f_3.
- $UD^h_{jf_3} \in \{0,1\}$, where $UD^h_{jf_3} = 1$, if any trip $k \in \mathcal{K}$ is charging at the fast charger $j \in N_2$ during demand charge time period f_3.
- $IP^{\max}$, representing the maximum power consumed across all demand charge time periods $f_3 \in F_3$.
- $EC^s_{kjf_1} \in R_{\geq 0}$, representing the amount of energy transferred from the slow charging station $j \in N_1$ to the bus of trip $k \in \mathcal{K}$ at time slot $f_1 \in F_1$.
- $EC^h_{kjf_2} \in R_{\geq 0}$, representing the amount of energy transferred from the fast charging station $j \in N_2$ to the bus of trip $k \in \mathcal{K}$ during demand charge time period $f_2 \in F_2$.
- $DC^s_{kjf_3} \in R_{\geq 0}$, representing the amount of energy transferred from the slow chargers $j \in N_1$ to the bus of trip $k \in \mathcal{K}$ during demand charge time period $f_3 \in F_3$.
- $DC^h_{kjf_3} \in R_{\geq 0}$, representing the amount of energy transferred from the fast chargers $j \in N_2$ to the bus of trip $k \in \mathcal{K}$ during demand charge time period $f_3 \in F_3$.
- $DEC_{f_3} \in R_{\geq 0}$, representing the energy consumption at all chargers, slow and fast, during each charging time slot $f_3 \in F_3$.

To include the aggregate monetary costs, we consider the ToU Tariffs T_{f_3} (price per kWh at charging slot $f_3 \in F_3$). The Demand Charge Rate DCR applies to the peak power usage $IP^{\max}$ during the demand charge period. The fixed cost of installing a charger $j \in \mathcal{N}$ is b_j, with a total budget $b^{\max}$.
Given these parameters, the aggregate monetary costs for the extension and operation of the CS network are:

$$\text{TOUC} = \sum_{f_3}^{F_3} (T_{f_3} \cdot DEC_{f_3}) \tag{1}$$

$$\text{DCC} = \text{DCR} \cdot \text{IP}^{\max} \tag{2}$$

$$\text{CSI} = \sum_{j \in \mathcal{N}_5 \cup \mathcal{N}_6} x_j b_j \tag{3}$$

Objective functions

The first objective function minimizes deadhead travel time y_k for each electric bus trip requiring charging from set $|\mathcal{K}|$ and is expressed as follows:

$$\text{minimize } \mathcal{O}_1 = \sum_{k \in \mathcal{K}} y_k = \sum_{j \in N} t_{kj} q_{kj} \text{ for all } k \in \mathcal{K} \tag{4}$$

Where parameter t_{kj} represents the bus' deadhead time from the last stop of trip k to charger j. Based on equations (1) to (3), the second objective function representing the monetary cost associated with the installation and operation of the CS network is expressed as:

$$\text{minimize } \mathcal{O}_2 = DCC + TOUC + CSI \tag{5}$$

Modeling of the CS network extension problem and the Charging Scheduling sub-problem

According to the latest research directions, a standard set of constraints is considered for the CS network extension problem:

$$x_j = 1 \qquad \forall j \in \mathcal{N}_3 \cup \mathcal{N}_4 \tag{6}$$

$$q_{kj} \leq x_j \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N} \tag{7}$$

$$\sum_{k \in \mathcal{K}} q_{kj} \geq x_j \qquad \forall j \in \mathcal{N} \tag{8}$$

$$\sum_{j \in \mathcal{N}} q_{kj} = 1 \qquad \forall k \in \mathcal{K} \tag{9}$$

$$\sum_{j \in \mathcal{N}} x_j b_j \leq b^{\max} \tag{10}$$

Similarly, Constraints (11) to (18) are considered for modeling the charging scheduling problem:

$$\sum_{f_1 \in \mathcal{F}_1} u^s_{kjf_1} + \sum_{f_2 \in \mathcal{F}_2} u^h_{kjf_2} \leq q_{kj} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N} \tag{11}$$

$$\sum_{f_1 \in \mathcal{F}_1} \sum_{j \in \mathcal{N}_1} u^s_{kjf_1} + \sum_{f_2 \in \mathcal{F}_2} \sum_{j \in \mathcal{N}_2} u^h_{kjf_2} = 1 \qquad \forall k \in \mathcal{K} \tag{12}$$

$$\sum_{k \in \mathcal{K}} u^s_{kjf_1} \leq 1 \qquad \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{13}$$

$$\sum_{k \in \mathcal{K}} u^h_{kjf_2} \leq 1 \qquad \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{14}$$

$$(1 - u^s_{kjf_1})M + u^s_{kjf_1} c^s_{f_1} \geq (\tau_k + t_{kj}) q_{kj} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{15}$$

$$(1 - u^h_{kjf_2})M + u^h_{kjf_2} c^h_{f_2} \geq (\tau_k + t_{kj}) q_{kj} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{16}$$

$$-(1 - u^s_{kjf_1})M + u^s_{kjf_1} c^s_{f_1} \leq (p^s_k + t_{kj}) q_{kj} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{17}$$

$$-(1 - u^h_{kjf_2})M + u^h_{kjf_2} c^h_{f_2} \leq (p^h_k + t_{kj}) q_{kj} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{18}$$

The start times for charging slots at slow and fast charging stations are given by $c^s_{f_1}$ and $c^s_{f_2}$, while p^s_k and p^h_k are the latest allowable start times for charging for each bus k.

Precalculated parameters for the modelling of energy consumption

Given the provided State of Charge (SOC_k) at each bus line's final stop and the travel distances to all existing or potential charger locations, these inputs enable the precalculation of energy quantities that are potentially transferable to electric buses based on the charging decisions. Under this assumption, we define the following precomputed parameters:

- The $EC_{kj}^{\max} = SOC_k^{\max} - SOC_k + e \cdot d_{kj}$, which is the energy (kWh) that can be transferred from a charger j to the bus k during any time slot, if the bus k charges at charger j.
- CP^s, which is the charging power in kilowatts (kW) for slow chargers.
- CP^h, which is the charging power in kilowatts (kW) for fast chargers.
- $SME^s = hr_1 \cdot CP^s$, which is the maximum energy (in kWh) that can be transferred from a *slow charger* to the bus during a slow charger time slot f_1.
- $SME^s = hr_2 \cdot CP^h$, similarly to the aforementioned pre-calculated parameter, it represents the maximum energy (in kWh) that can be transferred from a *fast charger* to the bus during a fast charger time slot f_2.
- CC_{kj}^s, which is a matrix of parameters, which take the value of 1 if the bus serving trip $k \in \mathcal{K}$ charges for the whole duration of any charging slot f_1 at charging outlet $j \in \mathcal{N}_1$ ($EC_{kj}^{\max} \leq SME^s$). The parameter takes the value of 0 if the bus charges up to its full battery capacity ($SOC_k^{\max} = SOC_k - e \cdot d_{kj} + EC_{kj}^{\max}$).
- CC_{kj}^h, similarly to the previously mentioned pre-calculated parameter, CC_{kj}^h takes the value of 1 if the bus serving trip $k \in \mathcal{K}$ charges for the whole duration of any slot f_2 at charging outlet $j \in \mathcal{N}_2$ ($EC_{kj}^{\max} \leq SME^h$).
- $PEC_{kj}^s = SME^s \cdot CC_{kj}^s + EC_{kj}^{\max} \cdot (1 - CC_{kj}^s)$, is the pre-calculated parameter for the amount of energy that can be transferred from *slow charger* $j \in \mathcal{N}_1$ to bus k, given its SOC_k at the last stop of the bus line, the battery capacity of the vehicle $SOC_k^{\max}$ and the minimum travel distance d_{kj}, is assigned to the *slow charger* j.
- $PEC_{kj}^h = SME^h \cdot CC_{kj}^h + EC_{kj}^{\max} \cdot (1 - CC_{kj}^h)$, similarly to the previously mentioned pre-calculated parameter, PEC_{kj}^h represents the amount of energy that can be transferred from *fast chargers* $j \in \mathcal{N}_2$ to any bus k.

Time-of-Use Tariffs modelling

Given the aforementioned pre-calculated parameters PEC_{kj}^s and PEC_{kj}^h the MILP model can utilise constraints (19) to (24) to optimise the variables $EC_{kjf_1}^s$ and $EC_{kjf_1}^h$:

$$EC_{kjf_1}^s \leq M \cdot u_{kjf_1}^s \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{19}$$

$$EC_{kjf_2}^h \leq M \cdot u_{kjf_2}^h \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{20}$$

$$EC_{kjf_1}^s \geq (-M) \cdot (1 - u_{kjf_1}^s) + PEC_{kj}^s \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{21}$$

$$EC_{kjf_1}^s \leq M \cdot (1 - u_{kjf_1}^s) + PEC_{kj}^s \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_1 \in \mathcal{F}_1 \tag{22}$$

$$EC_{kjf_2}^h \geq (-M) \cdot (1 - u_{kjf_2}^h) + PEC_{kj}^h \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{23}$$

$$EC_{kjf_2}^h \leq M \cdot (1 - u_{kjf_2}^h) + PEC_{kj}^h \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_2 \in \mathcal{F}_2 \tag{24}$$

Given $EC_{kjf_1}^s$ and $EC_{kjf_2}^h$, we still need to calculate the exact energy consumed per billable time period f_3. To achieve that, we must further consider the following model parameters.

- $DCP^s = \frac{hr_1 \cdot CP^s}{r^s} = hr_3 \cdot CP^s$, which corresponds to the maximum amount of energy per demand charge period f_3 for *slow chargers*,
- $DCP^h = \frac{hr_2 \cdot CP^h}{r^h} = hr_3 \cdot CP^h$, which corresponds to the maximum amount of energy per demand charge period f_3 for *fast chargers*,
- $FCS_{kj}^s = \left\lfloor \frac{EC_{kj}^{\max}}{DCP^s} \right\rfloor$, which corresponds to the number of time periods f_3 within any respective charging time slot f_1 that consume energy equal to DCP^s, out of the total number of r^s slots within any f_1 that is chosen for the charging of k at j,

- $FCS_{kj}^{h} = \left\lfloor \frac{EC_{kj}^{\max}}{DCP^{h}} \right\rfloor$, similarly to the aforementioned pre-calculated parameter, FCS_{kj}^{h}, represents the number of time periods f_3 within any respective charging time slot f_2 that consume energy equal to DCP^h,
- $RE_{kj}^{s} = EC_{kj}^{\max} - FCS_{kj}^{s} \cdot DCP^{s}$, which is the amount of energy transferred from a charger $j \in \mathcal{N}_1$ to a bus k during the time period $f_3 = FCS_{kj}^{s} + 1$,
- $RE_{kj}^{h} = EC_{kj}^{\max} - FCS_{kj}^{h} \cdot DCP^{h}$, which is the amount of energy transferred from a charger $j \in \mathcal{N}_2$ to a bus k during the time period $f_3 = FCS_{kj}^{h} + 1$,

Given the pre-calculated values for CC_{kj}^{s} or CC_{kj}^{h}, when they are equal to 1, the following constraints should be used for the values of $DC_{kjf_3}^{s}$ and $DC_{kjf_3}^{h}$:

$$DC_{kjf_3}^{s} \leq M \cdot EC_{kjz_{f_3}^{s}}^{s} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_3 \in \mathcal{F}_3 \tag{25}$$

$$DC_{kjf_3}^{h} \leq M \cdot EC_{kjz_{f_3}^{h}}^{h} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_3 \in \mathcal{F}_3 \tag{26}$$

$$DC_{kjf_3}^{s} \geq (-M) \cdot (1 - CC_{kjz_{f_3}^{s}}^{s}) + \frac{EC_{kjz_{f_3}^{s}}^{s}}{r^s} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_3 \in \mathcal{F}_3 \tag{27}$$

$$DC_{kjf_3}^{s} \leq \frac{EC_{kjz_{f_3}^{s}}^{s}}{r^s} + M \cdot (1 - CC_{kjz_{f_3}^{s}}^{s}) \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_3 \in \mathcal{F}_3 \tag{28}$$

$$DC_{kjf_3}^{h} \geq (-M) \cdot (1 - CC_{kjz_{f_3}^{h}}^{h}) + \frac{EC_{kjz_{f_3}^{h}}^{h}}{r^h} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_3 \in \mathcal{F}_3 \tag{29}$$

$$DC_{kjf_3}^{h} \leq \frac{EC_{kjz_{f_3}^{h}}^{h}}{r^s} + M \cdot (1 - CC_{kjz_{f_3}^{h}}^{h}) \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_3 \in \mathcal{F}_3 \tag{30}$$

For the case when CC_{kj}^{s} or CC_{kj}^{h} is equal to 0, then the following constraints should hold:

$$DC_{kjf_3}^{s} \geq (-M) \cdot CC_{kj}^{s} + DCP^{s} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_3 \in \mathcal{F}_3 : f_3 \leq FCS_{kjz_{f_3}^{s}}^{s} \tag{31}$$

$$DC_{kjf_3}^{s} \leq DCP^{s} + M \cdot CC_{kj}^{s} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_3 \in \mathcal{F}_3 : f_3 \leq FCS_{kjz_{f_3}^{s}}^{s} \tag{32}$$

$$DC_{kjf_3}^{s} \geq (-M) \cdot CC_{kj}^{s} + RE_{kj}^{s} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_3 \in \mathcal{F}_3 : f_3 = FCS_{kj}^{s} + 1 \tag{33}$$

$$DC_{kjf_3}^{s} \leq RE_{kj}^{s} + M \cdot CC_{kj}^{s} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_1, \forall f_3 \in \mathcal{F}_3 : f_3 = FCS_{kjz_{f_3}^{s}}^{s} + 1 \tag{34}$$

$$DC_{kjf_3}^{h} \geq (-M) \cdot CC_{kj}^{h} + DCP^{h} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_3 \in \mathcal{F}_3 : f_3 \leq FCS_{kjz_{f_3}^{h}}^{h} \tag{35}$$

$$DC_{kjf_3}^{h} \leq DCP^{h} + M \cdot CC_{kj}^{h} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_3 \in \mathcal{F}_3 : f_3 \leq FCS_{kjz_{f_3}^{h}}^{h} \tag{36}$$

$$DC_{kjf_3}^{h} \geq (-M) \cdot CC_{kj}^{h} + CC_{kj}^{h} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_3 \in \mathcal{F}_3 : f_3 = FCS_{kjz_{f_3}^{h}}^{h} + 1 \tag{37}$$

$$DC_{kjf_3}^{h} \leq CC_{kj}^{h} + M \cdot CC_{kj}^{h} \qquad \forall k \in \mathcal{K}, \forall j \in \mathcal{N}_2, \forall f_3 \in \mathcal{F}_3 : f_3 = FCS_{kjz_{f_3}^{h}}^{h} + 1 \tag{38}$$

Based on these constraints, we can then calculate the energy consumption in kWh per demand charge period f_3 for slow chargers, fast chargers, and the whole charging stations network:

$$DEC_{f_3} = \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}_1} DC_{kjf_3}^{s} + \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}_2} DC_{kjf_3}^{h} \qquad \forall f_3 \in \mathcal{F}_3 \tag{39}$$

Modeling of Peak demand charge

The Peak Demand charge, distinct from ToU tariffs, is an additional cost for commercial electricity usage. It is incorporated into the formulation via equation (2) and the second objective function $\mathcal{O}_2$. Its calculation depends on the variables $UD_{jf_3}^{s}$ and $UD_{jf_3}^{h}$, defined by constraints (40) and (41):

$$M \cdot \sum_{k \in \mathcal{K}} u_{kjz_{f_3}^{s}}^{s} \geq UD_{jf_3}^{s} \geq m \cdot \sum_{k \in \mathcal{K}} u_{kjz_{f_3}^{s}}^{s} \qquad \forall j \in \mathcal{N}_1, \forall f_3 \in \mathcal{F}_3 \tag{40}$$

$$M \cdot \sum_{k \in \mathcal{K}} u_{kjz_{f_3}^{h}}^{h} \geq UD_{jf_3}^{h} \geq m \cdot \sum_{k \in \mathcal{K}} u_{kjz_{f_3}^{h}}^{h} \qquad \forall j \in \mathcal{N}_2, \forall f_3 \in \mathcal{F}_3 \tag{41}$$

Based on this, the dependent variable $IP^{\max}$ needs to be calculated according to the following constraints:

$$IP^s_{f_3} = \sum_{j \in \mathcal{N}_1} (CP^s \cdot UD^s_{jf_3}) \qquad \forall f_3 \in \mathcal{F}_3 \tag{42}$$

$$IP^h_{f_3} = \sum_{j \in \mathcal{N}_2} (CP^h \cdot UD^h_{jf_3}) \qquad \forall f_3 \in \mathcal{F}_3 \tag{43}$$

$$TIP_{f_3} = IP^s_{f_3} + IP^h_{f_3} \qquad \forall f_3 \in \mathcal{F}_3 \tag{44}$$

$$IP^{\max} \geq TIP_{f_3} \qquad \forall f_3 \in \mathcal{F}_3 \tag{45}$$

Location capacity and coverage

Given the state of charge SOC_k for each trip $k \in \mathcal{K}$ that has completed its operations and necessitates charging, there is a set of feasible charger locations that trip k can access without its state of charge dropping below $SOC^{\min}_k$. This coverage constraint is formally expressed in constraint (46).

$$SOC_k - e \cdot q_{kj} \cdot d_{kj} \geq SOC^{\min}_k \qquad \forall k \in \mathcal{K} \tag{46}$$

In addition, each candidate location can have up to a specific number of chargers, with constraint (47) formally expressing these limitations.

$$\sum_{j \in \mathcal{N}: \theta_j = v} x_j \leq Cap_v \qquad \forall v \in \mathcal{V} \tag{47}$$

Parameter θ_j is utilized for the mapping of the charging outlet $j \in N(N1 \cup N2 \; and \; N1 \cap N2 = 0)$ to the physical location $v \in \mathcal{V}$.

The bi-objective Mixed Integer-Linear Programming model

Considering the above, the problem of the Optimal Extensions of Electric Bus Charging Infrastructure, when considering charging scheduling, two types of chargers (slow/fast) and energy pricing, is formulated as follows:

$$\begin{aligned} &(\tilde{Q}) : \\ &\min \quad \mathcal{O}_1, \mathcal{O}_2 \\ &\text{s.t.:} \quad \text{Constraints (6) to (47)} \end{aligned}$$

The utilisation of the ϵ-Constraint method

Our problem and respective formulation have two objectives, $\mathcal{O}_1$ and $\mathcal{O}_2$. To effectively study their relationship, we extend the methodology to utilize the ϵ-Constraint method that helps us estimate the Pareto optimal front for the bi-objective problem under investigation. The ϵ-Constraint method is utilised by solving m mathematical programs $\tilde{Q_m}$ based on the values of set $\epsilon = \epsilon_1, \epsilon_2, ..., \epsilon_m$ as the upper bound for Objective function $\mathcal{O}_1$:

$$(\tilde{Q_m}) : \tag{48}$$

$$\min \quad \mathcal{O}_2$$

$$\text{s.t.} \quad \mathcal{O}_1 \leq \epsilon_m \tag{49}$$

$$\text{Constraints (6) to (47)}$$

3 Results and discussion

Next, we present an application of the ϵ-constraint method and the MILP model in the urban centre of Limassol, Cyprus and bus lines 3, 4, 5 (and 5B), 7, 9 (and 9A), and 11. The data about ToU Tariffs and demand charges in Limassol have been acquired after direct communication with the Electricity Authority of Cyprus.
The network of bus lines considered for Limassol, Cyprus, is depicted in Figures 1 and 2. As for the initial set-up of the chargers, we consider that four slow chargers are already established. One

is located at candidate charging location #2, and three are located at candidate charger location #1. At most, up to three slow and three fast chargers are allowed at each candidate location (i.e. blue markers of Figure 2).

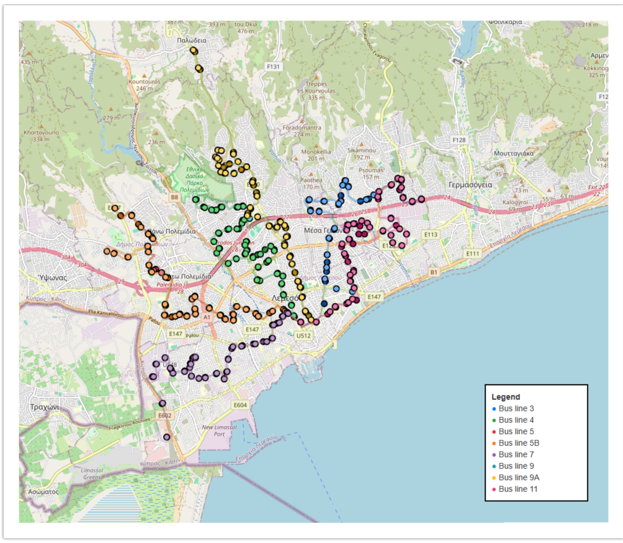

Figure 1: The bus lines considered for the case study in Limassol, Cyprus.

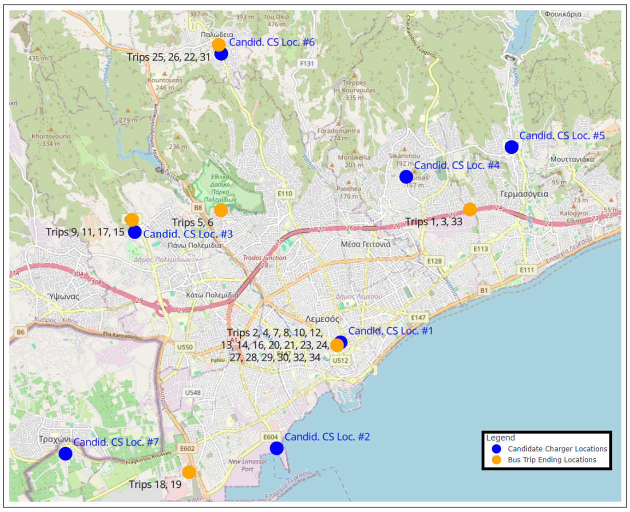

Figure 2: The candidate charger locations (blue markers) and the bus trip services' last stops (orange markers) for the Limassol, Cyprus case study.

Using the ϵ-constraint method, the MILP model is solved for various ϵ values, with the results being shown in Figures 3 and 4. Figure 3 displays $\mathcal{O}_2$ (in dollars) on the y-axis against ϵ values on the x-axis. Figure 4 plots all solutions, with $\mathcal{O}_1$ (deadhead time in minutes) on the x-axis and $\mathcal{O}_2$ on the y-axis.

According to the results, four Pareto optimal solutions exist to the problem under study. Pareto's optimal solutions for Limassol differ in terms of the number of chargers they are proposing, the type of charger, and the charging schedule. The first Pareto solution proposes the installation of eight chargers overall, one of which is fast. That means that the model solution proposes the installation of three new slow chargers and one fast in addition to the already existing ones. The solution derives a deadhead time of $\mathcal{O}_1 = 42.67$ minutes and a cost of $\mathcal{O}_2 = 150893.34$ dollars.

Pareto optimal solutions two, three and four provide a considerable decrease in monetary cost $\mathcal{O}_2$, but an increase in deadhead time $\mathcal{O}_1$. Particularly, Pareto solution two proposes a network of seven slow chargers and results in a deadhead time of $\mathcal{O}_1 = 49.23$ minutes and a cost of $\mathcal{O}_2 = 100903.86$ dollars. Pareto solution three proposes six slow chargers and achieves a deadhead time of $\mathcal{O}_1 = 87.61$ at a cost of $\mathcal{O}_2 = 70904.74$ dollars. Finally, Pareto solution #4 reduces the monetary cost $\mathcal{O}_2$ compared to #3, but increases deadhead time substantially. It proposes the

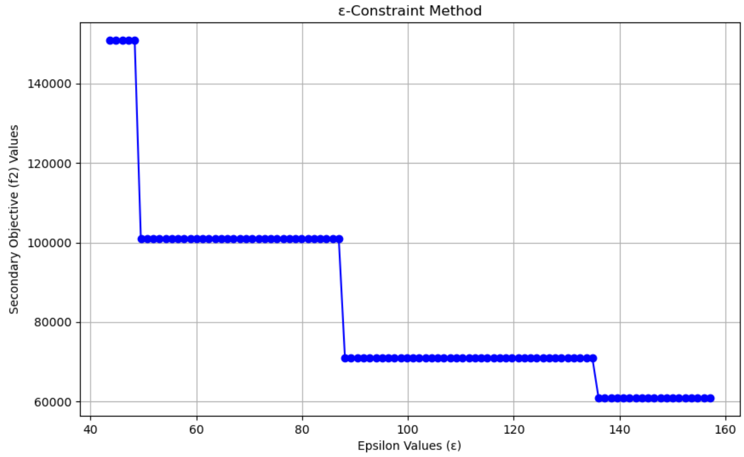

Figure 3: Results of the e-constraint method for the case study of the Limassol, Cyprus area.

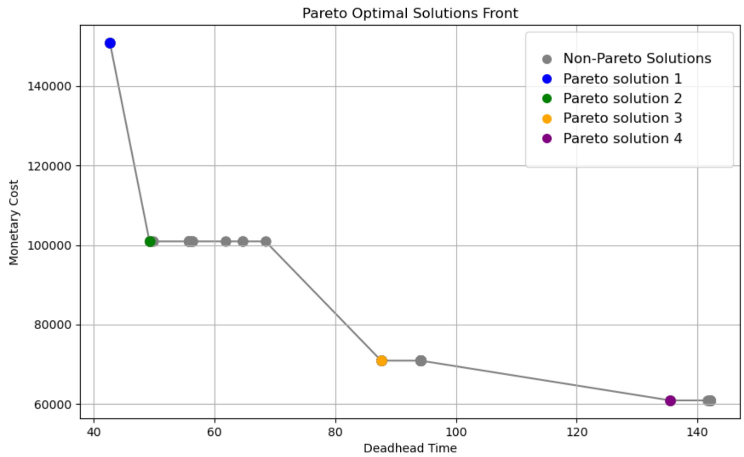

Figure 4: The approximation of the Pareto optimal front of solutions for the case study on the network of Limassol, Cyprus.

installation of five overall chargers (i.e. only one extra fast charger), with the deadhead time being calculated at $\mathcal{O}_1 = 135.51$ minutes, while the cost is $\mathcal{O}_2 = 60899.07$ dollars.
Depending on the objectives of the public transport operator, one of the four Pareto-optimal solutions may be selected for the strategic planning of the extension of the charging stations network in Limassol, Cyprus.

4 Conclusions

The approximation of the Pareto front for the bi-objective formulation of the charging station network in Limassol, Cyprus, offers critical insights into the dynamics of the problem. Notably, four non-dominated solutions have been identified, each representing a trade-off between competing objectives, thereby enabling a deeper posterior analysis based on the specific priorities of decision-makers. These solutions reflect diverse configurations of charging station networks, balancing factors such as the number and type of chargers, their locations, and the associated operational efficiencies. The solution diversity that emerges from this analysis underscores the complexity of the optimization landscape and highlights the potential for stakeholders to tailor solutions to specific operational and economic goals.

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