Robust Route Planning for Sidewalk Robot Delivery

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Abstract

The last-mile delivery problem is a critical challenge in urban logistics due to high costs, traffic congestion, and environmental impacts. Sidewalk delivery robots offer a promising solution for urban areas, providing safer and higher-capacity alternatives to drones. However, their efficiency is significantly affected by unreliable travel times on sidewalks. This study addresses the robust shortest path problem (RSPP) for sidewalk robots, explicitly accounting for travel time uncertainty due to varying sidewalk conditions such as density and obstacles. We integrate optimization with simulation, using generated travel times to derive alternative uncertainty sets (budgeted, ellipsoidal, and SVC-based). This approach is applied to a realistic case study reproducing pedestrian patterns in Stockholm's city center (Sweden) and examines the economic efficiency of robust routing under various robot design and environmental factors. Results demonstrate that robust routing significantly improves operational reliability under variable sidewalk conditions compared to traditional methods.

1 Introduction

The last-mile delivery problem represents one of the most challenging aspects of modern logistics due to its cost, increased traffic congestion, and environmental impacts, particularly in urban areas (Boysen et al., 2021). The growing demand for e-commerce and the need for faster and more reliable deliveries have led to the development of innovative solutions such as drones and autonomous ground robots (Jennings and Figliozzi, 2019). Sidewalk robots represent a safer and higher-capacity solution than drones, while their efficiency can be significantly affected by factors like pedestrian traffic, terrain, road conditions, obstacles, and weather (Heimfarth et al., 2022). These external variables introduce substantial uncertainty in travel times. Existing approaches relying on static travel times for sidewalks, risk leading to suboptimal decisions in operational and strategic problems, such as routing, location, assignment, due to delays.

In this study, we address the robust shortest path problem (RSPP) for sidewalk delivery robots. Unlike "traditional´ shortest path problems, the RSPP explicitly accounts for travel time uncertainty, ensuring more reliable routing solutions under variable sidewalk conditions. To increase the realism, our approach integrates optimization with pedestrian simulation to generate sidewalk travel times. We investigate alternative uncertainty sets (budgeted, ellipsoidal, and kernel-based support vector) to solve the RSPP. We model the sidewalk robot navigation within a realistic urban pedestrian network based on Stockholm, Sweden, and provide a comprehensive analysis of how design-related factors (e.g., robot desired speed, size, maneuverability) affect the efficiency of both traditional routing and robust routing solutions.

2 Background

2.1 Sidewalk autonomous delivery robots

Sidewalk Autonomous Delivery Robots (SADRs), a subcategory of Autonomous Delivery Robots (ADRs), are pedestrian-sized robots designed to travel on sidewalks and deliver items autonomously (Srinivas et al., 2022). In this paper, the focus is specifically on SADRs, which are referred to as delivery robots for simplicity. The most common research focus for SADRs is the truck-robot routing problem. From an operational perspective, robots have notably slower speeds (5–10 km/h compared to 50–100 km/h) and can travel shorter distances (5–10 km compared to 10–30 km). These differences make robots particularly effective for delivering low-value items in densely populated urban environments (Simoni et al., 2020). Different ways to combine trucks and robots in delivery have been discussed by many researchers (Boysen et al., 2018; Liu et al., 2021; Chen et al., 2021). Several studies have also developed frameworks for robot-only delivery, where parcels are transported exclusively by a fleet of robots without relying on vehicles like trucks or vans. These frameworks often encompass various types of delivery robots, not limited to sidewalk robots (Ulmer and Streng, 2019).

Despite these advances, existing research assumes static or simplified travel conditions for the sidewalk robot routing problems, overlooking the impact of variable sidewalk travel times due to congestion and other factors. Robots, which typically travel at pedestrian speeds, may face significant delays when navigating crowded or obstructed sidewalks. The traditional shortest path considering length or fixed travel time may not perform effectively in all scenarios, especially when extreme conditions occur. This issue remains insufficiently addressed in current sidewalk network robot routing problems. This paper addresses this gap by incorporating variable travel times for sidewalk robot delivery in congested areas, providing a more realistic and robust approach for robot-based last-mile delivery operations.

2.2 Robust shortest path problem

The real-world network conditions are often uncertain due to fluctuating travel times, changing road conditions, or unpredictable delays. To address these uncertainties, the robust shortest path problem (RSPP) extends the classical SPP by incorporating uncertainty into the model, ensuring that the chosen path remains effective even under adverse conditions. Commonly used types of uncertainty sets include convex hull (Kasperski and Zielinski, 2016), intervals (Chassein and Goerigk, 2015), ellipsoid (Ben-Tal and Nemirovski, 1998), budgeted uncertainty (Goerigk and Schöbel, 2016) and permutohull (Bertsimas and Brown, 2009). In recent years, novel approaches based on machine learning have moved beyond traditional uncertainty modeling. Shang et al. (2017) proposed a Support Vector Clustering (SVC) to construct uncertainty sets, allowing for the dynamic adjustment of uncertainty sets to better align with observed variability and real-world trends. In this paper, we implement a kernel-based SVC combined with the TSC algorithm to specifically address the robot delivery RSPP and compare its performance with two other well-known methods through realistic sidewalk robot delivery simulation scenarios.

3 Methodological Approach

In this study, the performance of three main RSP approaches in the context of sidewalk robot delivery is systematically evaluated with the pedestrian simulation data. The framework of the integration of simulation and optimization is shown below.

In RSPP, the road network segment costs are not precisely known. Based on the set of observations of costs, the uncertainty set U can be modeled. The RSPP is then denoted as:

$$\min\{\max_{\mathbf{u}\in U}\mathbf{u}^T\mathbf{x} : \mathbf{x} \in X\} \tag{1}$$

which means finding the shortest path in X considering the worst case in U. In this paper, three different types of uncertainty sets—Budgeted uncertainty, Ellipsoidal uncertainty, and Kernel-based SVC uncertainty— are employed to solve the RSPP for sidewalk robot navigation.

The Budgeted uncertainty approach (Bertsimas and Sim (2003)) for robust discrete optimization involves defining each entry u_j, $j \in [n]$ within the interval $[\underline{c}_j, \underline{c}_j + d_j]$. In the case of sidewalk robot navigation, u_j represents the robust cost at certain segment j, $\underline{c}_j$ is the minimum observed robot

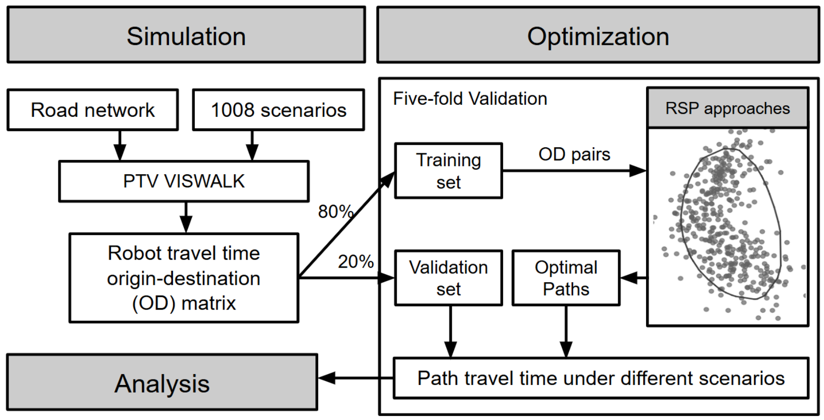
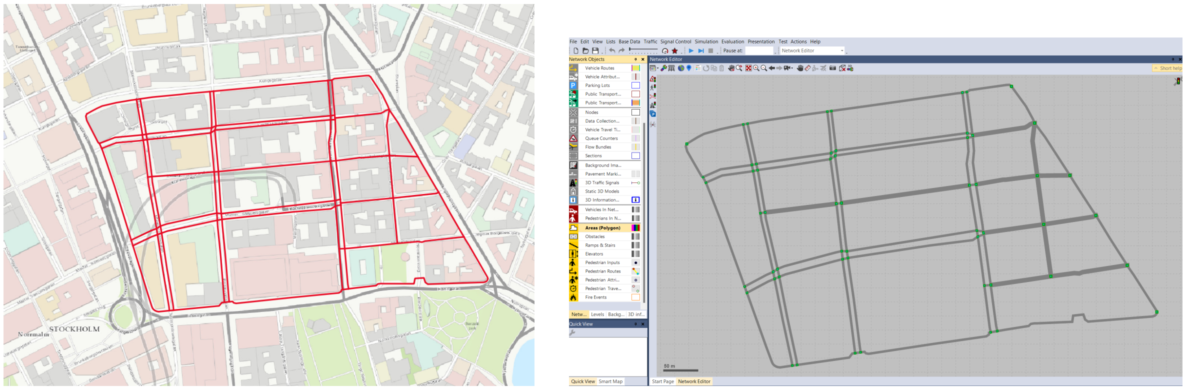


Figure 1: Framework for Simulation and Optimization

travel time of segment j, and d_j denotes the deviation between maximum travel time and minimum travel time. The degree of conservatism is controlled by the parameter Γ, which restricts at most $\Gamma \in \{1, 2, ..., n\}$ to take values in the interval, while the remaining entries take the minimum values. The Ellipsoidal uncertainty (Ben-Tal and Nemirovski, 1998, 1999) encompasses various reasonable types of ellipsoids and the intersections of finitely many ellipsoids. They make the corresponding robust convex program a tractable problem. An ellipsoidal uncertainty set has a parameter $\lambda \geq 0$ to control the size of the ellipsoid. The Kernel-based SVC uncertainty approach (Shang et al., 2017) adopts an unsupervised machine learning algorithm, the Support Vector Clustering (SVC) with the Weighted Generalized Intersection Kernel (WGIK), which utilizes Support Vectors (SV) to define boundaries in feature space, grouping data points into clusters by finding the smallest sphere that encloses them. A regularization parameter $v \in (0, 1]$ is introduced to control the conservatism degree of the uncertainty set. It is an upper bound on the fraction of outliers and a lower bound on the fraction of SVs. This approach not only manages correlated uncertainties, resulting in asymmetric uncertainty sets, but it also features adaptive complexity, embodying a nonparametric approach. The convex polyhedral uncertainty set generated can ensure the robust counterpart problem of the same type as the deterministic problem. To tackle the challenges of computationally intensity and difficulty in efficiently finding solutions, the Two-Stage Clustering with Dimensional Separation (TSD-DS) algorithm proposed by Roytvand Ghiasvand et al. (2024) is employed during application.

The navigation of sidewalk robots in a pedestrian environment is simulated using the Social Force Model (SFM), which has been extended and implemented in the commercial software PTV VISWALK (PTV, 2023). SFM, one of the most prominent models for pedestrian dynamics, was originally proposed by Helbing and Molnar (1995). It describes a pedestrian's motion in the form of an acceleration or deceleration, resulting from a number of different forces act on the pedestrian such as social, psychological, and physical forces. To date, no study has specifically calibrated SFM parameters for sidewalk robots within VISWALK. In this work, we draw on the parameters proposed by Truong and Ngo (2017), who developed an extended SFM for mobile robot navigation in dynamic and crowded environments. Four parameters are decided to change: τ, A_{soc_iso}, B_{soc_iso} and λ. The relaxation time parameter *Tau* is widely considered to be one of the most influential parameters (Gruden et al., 2022; Shi et al., 2021) as it defines the time required for a pedestrian to adjust their current speed and direction to align with their desired speed and direction. Given the conservative design of sidewalk robots — expected to react more slowly and cautiously than humans — and the calibrated τ value of 1.5 for wheelchairs and stretchers reported by Castro-Quispe et al. (2020), the default τ value is set to 0.8. The last three parameter values are identical to those used for human repulsive forces in the extended SFM proposed by Truong and Ngo (2017). In addition, the desired speed of sidewalk robots is set to a constant value 5 km/h with no deviation, inspired by the robot specs of the Starship Robot (Starship, 2023). The sidewalk robot is set to the size of an adult male.



(a) Network in QGIS

(b) Network in VISWALK

Figure 2: Refined Pedestrian network in Norrmalm, Stockholm

4 Analysis

The sidewalk robot navigation in a complex urban pedestrian network is simulated using PTV VISWALK. The input data for the VISWALK simulation comprises road network geometry, pedestrian demand, and behavioral parameters, and obstacles, calibrated to reflect real-world pedestrian and robot behavior. The simulation duration is set as 900 seconds (15 minutes). The constructed pedestrian origin-destination matrices from 10 a.m. to 10 p.m (12 hours) across different days of the week (7 days) are combined with 12 obstacle scenarios, to constitute 1008 (7 days * 12 hours * 12 obstacle scenarios) different simulation scenarios. For each scenario, robot travel times on sidewalks can be summarized into a travel time origin-destination (OD) matrix that is employed in the SPP and RSPP.

The road network used in our analysis is a section of Norrmalm, located in central Stockholm, Sweden. This area was selected due to its high pedestrian traffic and complex interactions between obstacles and pedestrians. The network has 99 segments, including 65 sidewalks and 34 crossings (Figure 2). Note that each segment is bidirectional for pedestrians and robots, thus the robot travel time OD matrix is 198*198. The pedestrian daily traffic volumes data were obtained from *Stockholms miljöbarometer*[1], administered by the Environmental Administration of Stockholm. It was further refined based on the distribution of pedestrian volumes across different days of the week and hours of the day. To enhance the realism of the simulation, various obstacles were introduced based on actual street layouts, specifically focusing on sidewalks, to reflect real-world barriers for pedestrians and sidewalk robots. Obstacles, represented as squares with side lengths randomly varying between 1 and 1.4 meters, were placed at random locations on randomly selected sidewalk segments across 12 scenarios with varying percentages of road coverage (0%, 10%, 20%, 30%, 40%, and 50%), ensuring diversity through randomized positions and sizes.

We first verify that the optimal paths between fixed node pairs differ across various scenarios. Only five OD pairs out of 500 random OD pairs maintain the same path under all scenarios. Then the three RSP methods, each defined by a scaling parameter with 10 possible values, are derived to determine the robust shortest paths for a set of OD pairs. The conventional optimal paths under the free flow scenario are also obtained as a benchmark for comparison. A five-fold validation approach is used to assess robustness, with 80% scenarios for training and 20% scenarios for validation, sampled from the travel time matrix. The performance of all methods is evaluated for every fold, and the average performance across all folds is computed for analysis. This method ensures a comprehensive and rigorous assessment of the RSP under diverse conditions.

To achieve a balanced evaluation of all methods, we adopt three performance criteria: the average travel time over all OD pairs and all scenarios, the average of the worst-case travel time among scenarios of all OD pairs, the average of the average value of the worst 5% of travel time among scenarios of all OD pairs. The performance of various methods under different parameter settings is depicted in Figure 3. The parameter values are marked next to the points. Note that all values have been normalized using the free flow travel time of the benchmark paths (the point marked with black star)

[1] https://miljobarometern.stockholm.se/trafik/gangtrafik/flodeskarta-for-gangtrafik/

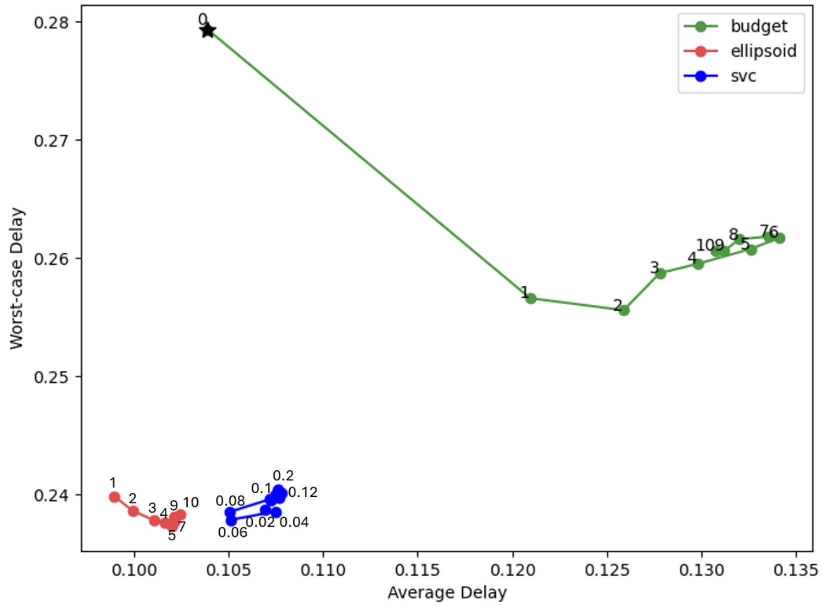


Figure 3: Trade-off between average delay and worst-case delay

to minimize the impact of varying distances between different OD pairs. If we denote the travel time of one robust path of one OD pair under one scenario as T_i, the free flow travel time of the benchmark path of that OD pair as T_{ff}, then the normalized value, namely *Normalized Delay*, of that OD pair under that scenario is $(T_i - T_{ff})/T_{ff}$.

Figure 3 illustrates the trade-off between the average delay and the worst-case delay, which varies with changes in the parameters of different methods. On both axes, lower values indicate reduced average and worst-case travel delay for a given SP method, signifying better performance. Robust paths typically exhibit longer average delay than conventional paths because they incorporate uncertainty into their models to ensure reliability under adverse conditions. However, they can significantly reduce travel delay in the most challenging situations. As shown in the figure, all data points for RSP methods are positioned below the benchmark, representing lower worst-case travel delay. The values of worst-case delay for Ellipsoidal and SVC uncertainty are significantly lower than that of the benchmark and the Budgeted uncertainty.

The average and worst delay for different OD pairs instead of taking average among them is further investigated, considering that the robust methods may not always perform well for each OD pair. The parameters providing the lowest worst-case value for each robust method in Figure 3 are chosen, i.e., the parameter of budgeted uncertainty is set to 2, the parameter of ellipsoidal uncertainty is set to 6, and the parameter of SVC uncertainty is set to 0.06. The results of these three robust methods as well as the conventional SP method, used as benchmark, are represented in the form of box plots in Figure 4. Each point refers to the worst-case travel delay of all validation scenarios for a given OD pair using a particular SP or RSP method. There are 100 points (OD pairs) for each box plot. The upper boundary of the box represents the 3rd percentile (75%), while the lower boundary corresponds to the 1st percentile (25%). The green line inside the box indicates the median, and the red line represents the mean value of the data set. Here, all RSP methods show a more concentrated distribution compared to the conventional method. The distribution of Ellipsoidal uncertainty is comparatively lower and has a lower average worst value than the other two RSP methods.

The RSP approaches significantly improve travel times in worst-case scenarios compared to the traditional shortest path method, with only a modest increase in average travel time of all scenarios. Among these, the Ellipsoidal uncertainty is considered to provide robust optimal paths with the most effective trade-off in average delay and worst-case delay. It performs well for scenarios beyond the training set, which means superior resilience to overfitting. In addition, the data-driven SVC method, while more complex and effective for datasets with large distribution differences across dimensions, does not outperform other RSP methods in the sidewalk delivery problem.

The advantages of RSP, compared with the standard SP model, are examined through a sensitivity

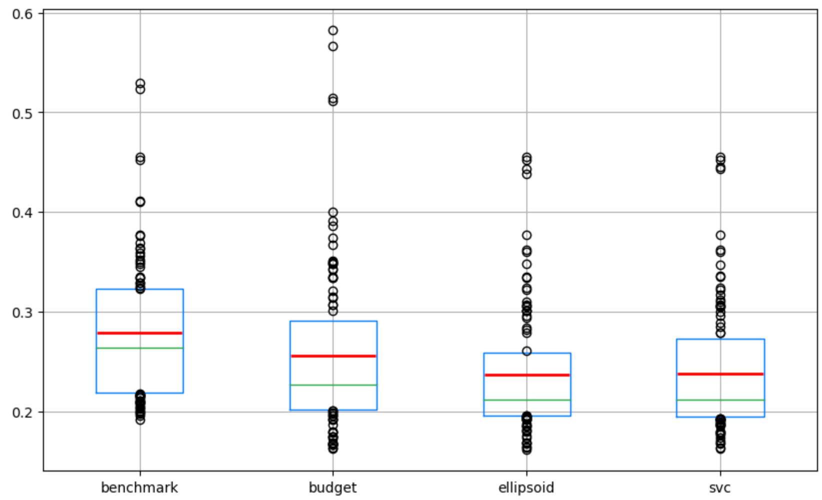


Figure 4: Distribution of worst-case delay of 100 OD pairs with SP and RSP methods

analysis in the context of on-demand food delivery, with respect to several sidewalk robot design elements. The main idea is to analyze the Willingness-To-Pay (WTP) of the standard SP model and the most efficient RSP model, which is the ellipsoidal approach in our case, according to different robot design features, such as robot speed or size. In this context, WTP refers to the monetary value consumers assign for delivery speed and reliability, calculated as $WTP = VOT * T + VOR * \sigma$, based on de Jong et al. (2014). Here, the VOT (Value of Time) represents the monetary values that consumers are willing to assign on savings of waiting time at their locations. The VOR (Value of Reliability) captures the monetary values placed on reducing delivery time variability. The variables T and σ denote the average delivery time and the standard deviation of delivery time respectively.

Table 1: Willingness-To-Pay (WTP) and Improvement for Different Robot Design Features

Robot Design Features	**WTP for Standard SP**	**WTP for RSP**	**Imporvement (%)**
Speed 5 km/h	20.131	19.293	4.16
Speed 7.5 km/h	12.569	12.367	1.61
Speed 10 km/h	8.988	8.972	0.18
Width 50 cm	20.457	19.529	4.54
Width 75 cm	20.753	19.747	4.85
Width 100 cm	21.121	20.048	5.08
Conservative behavior	23.170	21.615	6.71
Normal behavior	20.131	19.293	4.16
Aggressive behavior	17.647	17.596	0.29

The analysis considers varying robot design features, such as desired speed, width, and moving behaviors—categorized as conservative, normal, and aggressive—represented through the parameters of the Social Force Model (SFM). The WTP for both the standard SP and RSP models, as well as the percentage improvement achieved by RSP over SP, are shown in Table 1. The results show that the RSP models have higher economic values than the SP model in most cases across different robot design features. Their advantages are more pronounced for slower, wider, and more conservative robots.

5 Conclusion

By addressing the variability in sidewalk travel times and incorporating robust optimization techniques, this study advances the understanding of efficient and reliable last-mile delivery systems using autonomous robots. We explore the RSP problem for sidewalk robot navigation, marking the first study of its kind in the literature. Three RSP approaches with different uncertainty sets are employed, including Budgeted uncertainty, Ellipsoidal uncertainty, and a data-driven SVC-based uncertainty approach. The SVC approach, grounded in unsupervised machine learning, requires minimal assumptions and parameter tuning, enabling an endogenous balance between conservatism and efficiency. The performance of the RSP approaches is evaluated against the standard SP approach to shed light on the trade-offs between robustness and efficiency. The results demonstrate that RSP approaches consistently outperform the standard SP approach in sidewalk robot navigation. However, the kernel-based SVC approach, while more sophisticated and effective for datasets with significant distributional differences across dimensions, does not exhibit superior performance compared to other data-driven RSP methods in the context of sidewalk delivery. Furthermore, systematic sensitivity analyses reveal that the RSP models offer greater economic value compared to the SP model, with this advantage being more significant for robots that are slower, wider, and exhibit more conservative behaviors. Further research is required to improve the kernel-based SVC approach, focusing on enhancing its applicability and effectiveness in RSPP by investigating optimized kernel functions and ensuring scalability for large and complex datasets.

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