A tri-level model for the strategic game between the mobility-as-a-service (MaaS) platform and on-demand operators

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Short summary

This paper studies a multi-modal mobility system with a mobility-as-a-service (MaaS) platform, transportation network companies (TNCs), and mass transit (MT). The MaaS platform competes with TNCs and MT for travelers meanwhile cooperating with them to serve multi-modal trips. A tri-level model is formulated to capture the complex interactions among the stakeholders, where the MaaS platform designs service at the upper level, TNCs optimize their strategies at the middle level, and travelers make service choices following a nested logit (NL) model at the lower level. Numerical results show that a profit-maximizing MaaS platform can hardly survive in a market with excessive service capacity, whereas it becomes more appealing to long-distance travelers when travel time is prioritized over cost. On the other hand, when demand is high but insensitive to travel time, the MaaS platform may dominate the market by consolidating all service capacities of TNCs.
Keywords: mobility-as-a-service, mobility-on-demand, tri-level optimization, single-leader-multi-follower game

1 Introduction

Mobility-as-a-Service (MaaS) recently caught attention from both academia (Hörcher & Graham, 2020; Wong et al., 2020) and industry (Mohiuddin, 2021). The main idea of MaaS is to integrate multiple transport modes and payment systems into a seamless door-to-door travel solution for users. By promoting multi-modal trips via public transit, MaaS is also expected to relieve traffic congestion and thus bring positive impacts on the environment. Consequently, the research on MaaS is growing fast with a primary focus on its role in the urban transportation system (van den Berg et al., 2022), financial viability (Yao & Zhang, 2024), and operational strategies (Xi et al., 2024).

Within the MaaS framework, transport modes are typically divided into mobility-on-demand (MoD) and mass transit (MT). The former refers to ride-hailing and micro-mobility services provided by Transportation Network Companies (TNCs), while the latter includes classic public transit such as buses, metro, and trains. The inclusion of MoD is critical because it serves the first- and last-mile trips, connecting travelers' origins and destinations to MT stops. In other words, MoD helps consolidate spatially spreading travel demand onto MT. However, due to their high flexibility and convenience, current MoD services have become major competitors of MT and taken a massive market share (Zhu et al., 2021). Meanwhile, the competition among MoD operators (e.g., TNCs) has emerged in megacities and triggered heated debates (e.g., Zhang & Nie, 2021a; Cai et al., 2024). Therefore, a key challenge in MaaS design is how to manage the relationship between MoD and MT operators with different and most likely conflicting objectives. Furthermore, the MaaS platform itself would become a new player in the mobility market and maintain a so-called "coopetition" (Huang et al., 2024) relationship with both MT and MoD operators.

This study is motivated by the above challenges and aims to examine the principles of designing a MaaS system with MT and multiple competing TNCs. Although existing research has explored problems like the capacity assignment that matches multiple operators with demand (Xi et al., 2023; Yao & Zhang, 2024), bundle design and pricing (Ho et al., 2021; Hörcher & Graham, 2020), and the collaboration among service providers (Pantelidis et al., 2020), all from the MaaS platform's

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perspective, limited work has been dedicated to the understanding of the strategic responses of MT and TNC operators towards the entry and growth of MaaS platform. Specifically, the existing models often bypass the analysis of MT and TMCs by either fixing their operations (Liu & Chow, 2024) or simply excluding the non-MaaS option (Ding et al., 2023). A primary contribution of this study is to fill such a gap between analytical models and real practice.

In this study, we develop a tri-level optimization model that captures the complex interactions among travelers, MT and MoD operators, and a MaaS platform. At the upper level, the MaaS platform decides on the MaaS trip fare, the multi-modal trip composition, and the capacity purchase price. An oligopoly equilibrium among multiple TNCs forms the middle level, where the competing TNCs make decisions on whether to join MaaS, and if so, how much service capacity to sell to the MaaS platform, along with their trip fares. Finally, at the lower level, heterogeneous travelers choose among MaaS, TNC, and MT according to their preferences and trip attributes. Different from classic discrete choice models, the lower level also dictates an equilibrium because the demand for TNC trips in turn affects its utility.

The proposed modeling framework allows us to answer the following questions:

- How should the MaaS platform design the pricing and capacity purchase strategies to optimize its own objective, accounting for the potential reactions of TNCs and travelers?
- Whether and how would TNCs compete and cooperate with the MaaS platform, while competing with other TNCs and MT?
- How do travelers respond to the multi-modal travel option provided by the MaaS platform and compare it with TNC and MT services?

The insights generated from this study also deliver effective guidance on designing MaaS systems in a competitive and cooperative mobility landscape.

2 Model formulation

In this study, we consider four major stakeholders, namely, the MaaS platform, transport network companies (TNCs), the mass transit (MT) provider, and travelers, and characterize their decision-making problems and interactions in a single-leader and multi-follower game (SLMFG). As the single leader, the MaaS platform proposes wholesale capacity prices to TNCs, designs the MaaS trip fare, and allocates MT and TNC capacities on MaaS trips. Following real-world practice, we assume the MaaS platform does not own service capacity (e.g., self-owned vehicles) but buys it from TNCs and MT with a limited budget. Given the MaaS platform's decisions, TNCs then decide on their participation levels, which are represented by their capacity sales to the MaaS platform, as well as their own trip fares. Since there are multiple TNCs in the market, each TNC's decision is made in anticipation of the others. We assume both TNCs and MT have fixed fleet sizes and their service capacities are expressed by total vehicle distances. For simplicity, we do not consider ride-pooling in this study but leave it in future research. Finally, travelers with different trip attributes and value of time choose among multi-modal trips offered by MaaS, and single-model trips served by TNCs or MT, with the objective of maximizing their own travel utilities.

In the remainder of this section, we will first describe the demand model with detailed specifications of travel time and costs for each mode, then establish the market equilibrium, and finally formulate the follower's problem among TNCs and the leader's problem for the MaaS platform.

Demand model

We classify travelers into a finite number of user classes, denoted by a set I. For travelers of type $i \in I$, the generalized cost of service m is specified as follows:

$$U_{im} = b_m + f_m(\ell_i) + VTT_i * t_m(\ell_i) + VWT_i * w_m(\ell_i), \tag{1}$$

where b_m is the service-specific disutility measured in monetary cost, ℓ_i is the class-specific average direct trip distance, $f_m(\cdot), t_m(\cdot), w_m(\cdot)$ are service-specific functions for trip fare, in-vehicle time, and waiting/access/transfer time, respectively, and VTT_i, VWT_i are class-specific value of in-vehicle and waiting/access/transfer times.

We adopt a nested logit (NL) model to specify the mode split. As illustrated in Fig. 1, travelers first choose among MaaS (M), TNC (T), and MT (P). Those choosing TNC then select a particular

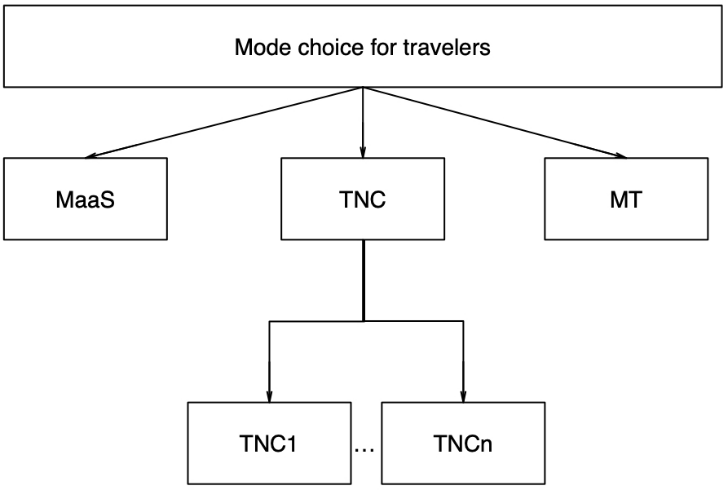

Figure 1: Mode split by nested structure

TNC from J, the set of TNCs. The conditional probability of class i choosing TNC $j \in J$ is given by

$$P_{ij|T} = \frac{\exp(-\theta_{iT} U_{ij})}{\sum_{j' \in J} \exp(-\theta_{iT} U_{ij'})}, \tag{2}$$

where $\theta_{iT} \in (0, 1]$ is the class-specific dispersion parameter within the TNC nest that accounts for the choice uncertainty among alternatives.
Accordingly, the composite generalized cost of TNC is evaluated as

$$I_{iT} = -\frac{1}{\theta_{iT}} \ln \left(\sum_{j \in J} \exp(-\theta_{iT} U_{ij}) \right). \tag{3}$$

Finally, the probability that a traveler in class i choosing service m is computed as

$$\text{MaaS and MT:} \quad P_{im} = \frac{\exp(-\theta_i U_{im})}{\exp(-\theta_i U_{iM}) + \exp(-\theta_i I_{iT}) + \exp(-\theta_i U_{iP})}, \quad m \in \{M, P\}, \tag{4}$$

$$\text{TNC:} \quad P_{im} = P_{ij|T} \frac{\exp(-\theta_i I_{iT})}{\exp(-\theta_i U_{iM}) + \exp(-\theta_i I_{iT}) + \exp(-\theta_i U_{iP})}, \quad m = j \in J. \tag{5}$$

Let D_i be the total number of travelers in class i, with the class-specific average direct trip distance ℓ_i, then the total demand expressed in travel distance of class i for service m is computed as

$$Q_{im} = D_i P_{im} \ell_i, \tag{6}$$

and the total demand for service m is

$$Q_m = \sum_{I \in I} Q_{im}. \tag{7}$$

In what follows, we specify each element in the generalized cost Eq. (1), i.e., the trip fare f_m, trip time t_m, and waiting/access/transfer time w_m of each service.

- **Trip fare**

  Following the real-world practice, we assume TNC trip fare is computed by vehicle travel distance while MT price is segment-based. As for MaaS trips, we adopt the idea of Yao & Zhang (2024) and compute the trip fare solely based on travelers' direct trip distance. Let f_m be the unit price of service m, then the trip fare for travelers in class i is given by

$$\text{TNC:} \quad f_{j|T}(\ell_i) = f_j \delta_T \ell_i, \tag{8}$$

$$\text{MT:} \quad f_P(\ell_i) = f_P n \ell_i, \tag{9}$$

$$\text{MaaS:} \quad f_M(\ell_i) = f_M \ell_i, \tag{10}$$

where δ_T is the road-network detour factor reflecting the ratio of vehicle travel distance to the direct trip distance (Yang et al., 2018), n is an exogenous parameter that gives the average number of segment per unit direct trip distance.

- **Trip time**

  The trip time for TNC and MT is computed with service-specific speed v_m, while for MaaS trips, a class-specific capacity allocation factor α_i is introduced to define how the trip is split between TNC and MT. The formulas of trip time for travelers in class i are as follows:

$$\text{TNC:} \quad t_{j|T}(\ell_i) = \frac{\delta_T \ell_i}{v_T}, \tag{11}$$

$$\text{MT:} \quad t_P(\ell_i) = \frac{\delta_P \ell_i}{v_P}, \tag{12}$$

$$\text{MaaS:} \quad t_M(\ell_i) = \frac{\delta_T \alpha_i \ell_i}{v_T} + \frac{\delta_P (1-\alpha_i)\ell_i}{v_P}, \tag{13}$$

  where δ_P is similarly defined as the transit-network detour ratio.

- **Waiting/access/transfer time**

  It has been both analytically and empirically demonstrated that the waiting time for TNC services, including both matching and pickup times, depends on the number of vacant vehicles in the market (Chen et al., 2019; Zhang & Nie, 2021b, e.g.,). We adopt the simple waiting time function proposed in (Zhou et al., 2022):

$$w_{j|T} = A(N_j^v)^{-k}, \tag{14}$$

  where A is a parameter that counts the exogenous factors in the matching process, N_j^v is the vacant vehicles available for the single-modal TNC trips and will be further specified later in the market equilibrium, and $k \in (0, 1]$ is the sensitivity parameter and shown to be 0.5 in regular e-hailing service without passenger competition in the matching proce (Zhang et al., 2019).

  The total access, egress, and transfer time for MT is computed as:

$$w_P(\ell_i) = 2\tau_a + \tau_t(n\ell_i - 1), \tag{15}$$

  where τ_a refers to the access and egress time, and τ_t is the time for each transfer. Recall that $n\ell_i$ gives the number of trip segments and thus the number of transfers is $n\ell_i - 1$.

  We assume the MaaS platform provides door-to-door trips and thus there is no access or egress time. However, travelers still need to wait for their first TNC trip (though the others can be well scheduled with almost zero waiting time) and experience transfer time in MT. Hence, the total waiting and transfer time is obtained as

$$w_M(\ell_i) = A(N_M^v)^k + \tau_t[n(1-\alpha_i)\ell_i - 1], \tag{16}$$

  where N_M^v is the vacant TNC vehicles available for the MaaS platform and will be specified later in the market equilibrium.

Market equilibrium

The travel demand specified above is endogenously determined by the market equilibrium because passenger waiting time for TNC services, a critical factor in the travel utility, is determined by the demand-supply relationship described by the following two flow conservation constraints:

$$(1-y_j)C_j = N_j^v \ell_0 + \sum_i Q_{ij}, \tag{17a}$$

$$\sum_j y_j C_j = N_M^v \ell_0 + \sum_i \alpha_i Q_{iM}, \tag{17b}$$

where C_j denotes the total service capacity of TNC $j \in J$, y_j is the fraction capacity of TNC j sold to the MaaS platform, ℓ_0 is the average vehicle travel distance per unit time (which is used to transfer vacant vehicle distance into vacant vehicle number).

One can easily verify that Eqs. (17a) and (17b), along with the demand function Eq. (6), form a fixed point $x = F(x)$ with $x = (w_M(\ell_i), w_{j|T}(\ell_i))_{i\in I, j\in J}$. Since the traveler class does not affect the waiting time of single-modal TNC trips, the dimension of x is reduced to $|I| + |J|$. In other words, the fixed-point solution corresponds to the market equilibrium where no traveler has incentives to further change their mode choices. The existence of equilibrium can also be proved by evoking Kakutani's fixed-point theorem (Kakutani, 1941).

Follower's problem

In this study, we assume MT has sufficient service capacity and adopts a fixed pricing policy. Hence, the followers are referred to as TNCs that aim to maximize their profits by setting their participation levels and pricing strategies, under fixed service capacities. In other words, we focus the relatively short-term decisions (i.e., pricing and capacity sale) in this study, and separate them from long-term decisions (e.g., service capacity).
For each TNC $j \in J$, let C_j denote the service capacity, u_j is the unit operating cost. f_j is the fare rate per unit distance as defined in Section 2. To represent the participation level of TNCs in MaaS, we introduce $y_j \in [0, 1]$ as the fraction of service capacity that TNC j sells to the MaaS platform at a unit price p_T. Here, we assume the capacity purchase price is indifferent among TNCs, while our proposed model can be easily extended to consider heterogeneous capacity prices. Accordingly, the pricing and capacity planning problem for TNC j is formulated as

$$\max_{f_j, y_j} \quad \Pi_j = f_j Q_j + p_T y_j C_j - u_j C_j, \tag{18a}$$

$$s.t. \quad Q_j \leq (1 - y_j) C_j, \tag{18b}$$

$$f_j \geq 0, \tag{18c}$$

$$0 \leq y_j \leq 1. \tag{18d}$$

$$\tag{18e}$$

Objective (18a) consists of i) the revenue of non-MaaS trips $f_j Q_j$, ii) the revenue of capacity sale $p_T y_j C_j$, and iii) the operating cost $u_j C_j$ (which is constant and thus safely dropped when solving the problem). Constraint (18b) claims that the remaining service capacity must be sufficient to serve non-MaaS TNC trips, given demand Q_j solved from the market equilibrium Eq. (17). Constraints (18c) and (18d) define the feasible trip fare and MaaS participation level.

Leader's problem

As the leader, the MaaS platform decides on its fare rate f_M, capacity purchase price p_T, and capacity allocation ratio α_i for each traveler class i. For simplicity, we assume the MaaS platform has a fixed budget B_M for capacity purchase. The amount of TNC capacity is jointly determined by the price p_T proposed by the MaaS platform and the TNCs' participation levels $y_j, j \in J$, while the remaining budget is used to buy MT capacity at a fixed price p_0. The MaaS platform's design problem is thus formulated as

$$\max_{f_M, p_T, \alpha_i} \quad \Pi_M = f_M Q_M - B_M, \tag{19a}$$

$$s.t. \quad \sum_{i\in I} d_T(Q_M, C_M^T, C_M^P, \alpha_i) \leq C_M^T = \sum_{j\in J} y_j C_j, \tag{19b}$$

$$\sum_{i\in I} d_P(Q_M, C_M^T, C_M^P, \alpha_i) \leq C_M^P = \frac{B_M - p_T C_M^T}{p_0}, \tag{19c}$$

$$f_M \geq 0, \tag{19d}$$

$$p_T \geq 0, \tag{19e}$$

$$0 \leq \alpha_i \leq 1. \tag{19f}$$

Objective (19a) computes the MaaS profit as the revenue from MaaS trips minus the capacity purchase budget. Thanks to the assumption of a fixed budget, the problem reduces from profit maximization to revenue maximization. Constraints (19b) and (19c) dictate that the purchased TNC capacity C_M^T and MT capacity C_M^P should be sufficient to serve MaaS travel demand allocated to TNC and MT, respectively. Again, the MaaS demand Q_M there is determined by the market equilibrium Eq. (17). Finally, Constraints (19d)-(19f) describe the feasibility of decision variables.

3 Tri-level formulation and solution procedure

With all stakeholders' problems specified in the previous section, we are now ready to present the tri-level optimization framework: the MaaS platform designs its service at the upper level, TNCs form a Nash equilibrium at the middle level by setting their participation levels in MaaS and competing for travelers with their pricing strategies, and finally, travelers reach a market equilibrium with their mode choices at the lower level.

The tri-level structure also reflects the solution procedure. Given trip fare rates $f_M, f_j, \forall j \in J$, capacity allocation ratios $\alpha_i, i \in I$, and TNC participation levels $y_j, j \in J$, the market equilibrium Eq. (17) is first solved using fixed-point iterations (Hu et al., 2024; Zhang & Nie, 2021b). Then, the middle-level Nash equilibrium (NE) among TNCs is solved given the market equilibrium as constraints, and similarly, the upper-level optimal MaaS service design is solved with the middle-level NE as constraints. In what follows, we present the detailed procedures of solving the middle- and upper-level problems.

Middle level: Nash equilibrium among TNCs

Our first step to solve the middle-level equilibrium is to relax the capacity constraint in each TNC's problem. Let $s_j = [f_j, y_j]^T$ denotes the strategy of TNC j and s_{-j} denotes the joint strategy of other TNCs. The Lagrangian of TNC $j \in J$ is given by

$$\mathcal{L}_j(s_j, \lambda_j, s_{-j}) = -f_j Q_j - p_T y_j C_j + u_j C_j + \lambda_j \Big(Q_j - (1 - y_j) C_j \Big), \tag{20}$$

where λ_j is the Lagrangian multiplier for the capacity constraint. Note that Eq. (20) also depends on s_{-j} because the travel demand Q_j is jointly determined by the decisions of all TNCs.

Given multipliers $(\lambda_j)_{j\in J}$, the equilibrium condition is then expressed as

$$\mathcal{L}_j(s_j^*, \lambda_j, s_{-j}^*) \leq \mathcal{L}_j(s_j, \lambda_j, s_{-j}^*), \quad j \in J. \tag{21}$$

Since $\mathcal{L}_j$ is differentiable, we may construct a variational inequality problem (VIP) such that any solution to the VIP corresponds to a Nash equilibrium (Cavazzuti et al., 2002). The VIP is to find $s^* \in \Omega$, such that

$$\langle -\nabla \mathcal{L}(s^*, \lambda), s - s^* \rangle \geq 0, \quad \forall s \in \Omega, \tag{22}$$

where $s = (s_j)_{j\in J}$, $\Omega = \{(f, y) | f \geq 0, 0 \leq y \leq 1\}$, $\nabla \mathcal{L}(s) = (\nabla_{s_j} \mathcal{L}_j(s_j, \lambda_j, s_{-j}))_{j\in J}$.

The evaluation of $\nabla \mathcal{L}(s, \lambda)$ at each feasible solution s, however, requires the sensitivity of market equilibrium, denoted by $\frac{\partial x^*}{\partial s}$. Following the previous studies (e.g. Zhang & Nie, 2021b), we apply the implicit function theorem (Krantz & Parks, 2002) and derive the sensitivities by solving the following equality system:

$$\frac{\partial x^*}{\partial s} = \nabla_s F(x^*, s) + \nabla_x F(x^*, s) \frac{\partial x}{\partial s}, \tag{23}$$

where $\nabla_x F(x, s)$ and $\nabla_s F(x, s)$ denote the partial derivatives of the fixed-point $F(\cdot)$.

While the VIP (22) is defined with given multipliers $\lambda = (\lambda_j)_{j\in J}$, when solving the NE among TNCs, we implement the basic differential multiplier method (BDMM) (Platt & Barr, 1987) and update both s and λ simultaneously. This largely improves the computational efficiency compared to creating a double-loop to iterate s and λ separately.

Upper level: optimal MaaS service design

Let (s^*, λ^*) denotes the middle-level equilibrium solution, which induces a lower-level equilibrium x^*, and define $g = [p_T, f_M, \alpha]^T$ as the MaaS platform's strategy with $\alpha = (\alpha_i)_{i\in I}$. Similar to the TNC's problem, we first write the Lagrangian as follows:

$$\begin{aligned} \mathcal{L}_{\mathcal{M}}(g, \mu) &= -f_M Q_M + \mu_1 \left(\sum_{i\in I} d_{Ti} - C_M^T \right) + \mu_2 \left(\sum_{i\in I} d_{Pi} - C_M^P \right) \\ &= -f_M \sum_{i\in I} D_i P_{iM} \ell_i + \mu_T \left(\sum_{i\in I} \alpha_i D_i \ell_i P_{iM} - \sum_{j\in J} y_j C_j \right) \\ &\qquad + \mu_P \left(\sum_{i\in I} (1 - \alpha_i) D_i \ell_i P_{iM} - \frac{B_M - p_T \sum_{j\in J} y_j C_j}{p_0} \right), \end{aligned} \tag{24}$$

where $\mu = [\mu_T, \mu_P]^T$ are multipliers associated with the TNC and MT capacity constraints. Here, the constant capacity purchase budget B_M is dropped as it has no impact on the solution.
Note that $\mathcal{L}_\mathcal{M}$ can also be seen as a function of (g, s, x), where the relationship between g and (s, x) is characterized by the equilibrium conditions presented in the previous sections. Accordingly, given the multipliers μ, the Lagrangian gradient is computed as

$$\nabla \mathcal{L}_\mathcal{M}(g, s^*, x^*, \mu) = \nabla_g \mathcal{L}_\mathcal{M}(g, s^*, x^*, \mu) + \nabla_s \mathcal{L}_\mathcal{M}(g, s^*, x^*, \mu)\frac{\partial s^*}{\partial g} + \nabla_x \mathcal{L}_\mathcal{M}(g, s^*, x^*, \mu)\left(\frac{\partial x^*}{\partial s^*}\frac{\partial s^*}{\partial g} + \frac{\partial x^*}{\partial g}\right), \tag{25}$$

where $\frac{\partial s^*}{\partial g}$ is the sensitivity of middle-level TNC equilibrium with respect to the MaaS platform's decisions, and $(\frac{\partial x^*}{\partial s^*}, \frac{\partial x^*}{\partial g})$ is the sensitivity of lower-level market equilibrium with respect to both the MaaS platform' and TNCs' decisions.
In Section 3, we discuss how to obtain $\frac{\partial x^*}{\partial s}$. The same approach is used to solve $\frac{\partial x^*}{\partial g}$. To derive $\frac{\partial s^*}{\partial g}$, we differentiate the first-order condition $\nabla \mathcal{L}(s^*, x^*, \lambda^*) = 0$, which yields

$$\nabla^2_{ss}\mathcal{L}(s^*, x^*, \lambda^*)\frac{\partial s^*}{\partial g} + \nabla^2_{sx}\mathcal{L}(s^*, x^*, \lambda^*)\frac{\partial x^*}{\partial g} = 0, \tag{26}$$

and solve $\frac{\partial s^*}{\partial g}$ from the linear system Eq. (26). It is worth noting that the first-order condition $\nabla \mathcal{L}(s^*, x^*, \lambda^*) = 0$ is sufficient here because s^* can be proved to be an interior solution in our setting thanks to the logit-based demand model. The proof is omitted due to the word limit.

4 Results and discussion

We conduct numerical experiments to explore the optimal strategies of the MaaS platform and two TNCs, as well as the corresponding profits and market shares, under various market conditions and traveler heterogeneity.
The experiment settings are summarized in Table 1. We first consider two demand levels. In the low-demand scenario, the total demand is below the total capacity of TNCs, thus it represents a less competitive market condition. In contrast, the total demand in the high-demand scenario exceeds the total capacity of TNCs, leading to a highly competitive market. Besides, we consider travelers to be different in their trip distance and value of time.

Main Scenario Settings	**Long Distance**	**Short Distance**
Capacity of TNC 1 (km)	24,000	
Capacity of TNC 2 (km)	32,000	
Trip distance (km)	50	10
High demand (pers.)	1000	1500
Low demand (pers.)	500	1500
High time value (CHF/h)	VTT: 10; VWT: 5	VTT: 15; VWT: 5
Low time value (CHF/h)	VTT: 3; VWT: 2	VTT: 5; VWT: 2
Transit capacity price (CHF/km)	0.05	
Budget (k CHF)	12	

Table 1: Experiment settings.

*VTT: Value of Travel Time; VWT: Value of Waiting Time.

Table 2 reports the main findings from the experiments, which are grouped into four scenarios: i) high demand, low time-value (HL), ii) high demand, high time-value (HH), iii) low demand, low time-value (LL), and iv) low demand, high time-value (LH).
In HL, the MaaS platform emerges to be the dominant player that purchases all capacities from TNCs at a price higher than their single-modal trip fare rate. Although the TNC capacity purchase price is even higher than the MaaS trip fare rate, the much lower MT capacity price helps drag down the operation cost and makes MaaS trips still profitable. While all travel demand is captured by the MaaS platform, a higher fraction of TNC capacity is allocated to short-distance trips ($\alpha_2 = 0.67$), likely to ensure a higher time efficiency.

The competitive landscape substantially shifts when travelers have high values of time, e.g., during peak hours when time efficiency is prioritized over cost. In HH, we observe travelers are more willing to choose the more flexible TNC services. As a result, the market is more balanced between TNCs and the MaaS platform, while leaving less than 10% of the market to MT. Interestingly, MaaS in this setting is specialized in serving long-distance travelers and captures nearly 40% of the demand. This result indicates the particular advantage of MaaS in striking a balance between travel cost and time. Excluding short-distance trips from its service also allow the MaaS platform to allocate more TNC capacity to long-distance trips ($\alpha_1 = 0.63$) compared to HL. However, the MaaS platform's profit drops significantly compared to HL, largely due to the almost doubled TNC capacity purchase price.

The MaaS platform becomes less likely to survive when the TNC service capacity is sufficient to serve all travelers in the market. Regardless of travel distance, travelers uniformly choose TNC as their preferred mode of transportation. As shown in Table 2, in both LL and LH, TNCs and the MaaS platform cannot reach an agreement on the capacity purchase. Accordingly, the MaaS platform uses all budget to buy transit capacity as per the assumption while generates zero revenue, which yields a negative profit of -12k CHF. The absence of MaaS trips in the low-demand scenarios reflects the key challenge of implementing MaaS. Both travelers and operators must benefit from the integration of mobility services. Otherwise, a self-contained MaaS system is hardly achievable. Particularly, in LL and LH, the market reduces to a duopoly of two TNCs, where the TNC with a larger fleet takes the majority of market.

	TNC 1		**TNC 2**		**MaaS**			
High Demand	**Low time-value**							
- Optimal strategy	Fare 0.48	Participation $\sim$1	Fare 0.48	Participation $\sim$1	Fare 0.6	Capacity price 0.7	α_1 0.55	α_2 0.67
- Profits (k CHF)		14.4		19.2		26.5		
- Market share %		~ 0		~ 0		98.7		
- Long distance %						99.8		
- Short distance %						95.2		
	High time-value							
- Optimal strategy	Fare 1.18	Participation 0.20	Fare 1.18	Participation 0.24	Fare 0.96	Capacity price 1.30	α_1 0.63	α_2
- Profits (k CHF)		24.8		33.5		6.5		
- Market share %		27.5		34.8		29.6		
- Long distance %		25.0		31.5		38.5		
- Short distance %		35.6		45.6		0		
Low Demand	**Low time-value**							
- Optimal strategy	Fare 0.68	Participation ~ 0	Fare 0.67	Participation ~ 0	Fare 0.87	Capacity price 0	α_1	α_2
- Profits (k CHF)		8.6		11.5		~ -12		
- Market share %		40.5		54.6		0		
- Long distance %		40.0		56.6				
- Short distance %		41.3		51.1				
	High time-value							
- Optimal strategy	Fare 1.21	Participation ~ 0	Fare 1.21	Participation ~ 0	Fare 0.90	Capacity price 0	α_1	α_2
- Profits (k CHF)		17.7		24.2		~ -12		
- Market share %		41.4		56.7		0		
- Long distance %		39.7		58.1				
- Short distance %		44.2		54.2				

Table 2: Main results

5 Conclusion

This paper proposes a tri-level framework to model an aggregate multi-modal transportation system with a MaaS platform, multiple TNCs, and MT. As the "leader" at the upper level, the MaaS platform designs MaaS trip fare, capacity purchase prices, and capacity allocation ratios; as "followers", TNCs then decide on their participation levels, i.e., the fraction of service capacity sold to the MaaS platform, along with their own fare rates; and at the lower level, travelers choose among all travel options following an NL model. We establish the equilibrium conditions at the middle and lower levels and develop a gradient-based algorithm to solve the tri-level optimization. A set of numerical experiments are conducted with different market settings and traveler characteristics. Our results show that the MaaS platform is more likely to survive in a high-demand market and tends to become monopoly when travelers' value of time is relatively low. On the other hand, when travelers prioritize travel time over cost, MaaS trips are more appealing to long-distance travelers.

References

Cai, Z., Chen, Y., Mo, D., Liu, C., & Chen, X. M. (2024). Competition and evolution in ride-hailing market: A dynamic duopoly game model. *Transportation Research Part C: Emerging Technologies*, *164*, 104665.

Cavazzuti, E., Pappalardo, M., Passacantando, M., et al. (2002). Nash equilibria, variational inequalities, and dynamical systems. *Journal of optimization theory and applications*, *114*(3), 491–506.

Chen, H., Zhang, K., Nie, M., & Liu, X. (2019). A physical model of street ride-hail. *Available at SSRN 3318557*.

Ding, X., Qi, Q., Jian, S., & Yang, H. (2023). Mechanism design for mobility-as-a-service platform considering travelers' strategic behavior and multidimensional requirements. *Transportation Research Part B: Methodological*, *173*, 1–30.

Ho, C. Q., Hensher, D. A., & Reck, D. J. (2021). Drivers of participant's choices of monthly mobility bundles: Key behavioural findings from the sydney mobility as a service (maas) trial. *Transportation Research Part C: Emerging Technologies*, *124*, 102932.

Hörcher, D., & Graham, D. J. (2020). Maas economics: Should we fight car ownership with subscriptions to alternative modes? *Economics of Transportation*, *22*, 100167.

Hu, M., Szeto, W., & Wang, Y. (2024). Cooperative design of feeder bus and bike-sharing systems. *Transportmetrica A: Transport Science*, 1–34.

Huang, W., Ding, Y., & Jian, S. (2024). Strategic coopetition among transportation service providers considering supply–demand congestion effects and asymmetric bargaining power. *Transportation Research Part B: Methodological*, *188*, 103043.

Kakutani, S. (1941). A generalization of brouwer's fixed point theorem.

Krantz, S. G., & Parks, H. R. (2002). *The implicit function theorem: history, theory, and applications*. Springer Science & Business Media.

Liu, B., & Chow, J. Y. (2024). On-demand mobility-as-a-service platform assignment games with guaranteed stable outcomes. *Transportation Research Part B: Methodological*, *188*, 103060.

Mohiuddin, H. (2021). Planning for the first and last mile: A review of practices at selected transit agencies in the united states. *Sustainability*, *13*(4), 2222.

Pantelidis, T. P., Chow, J. Y., & Rasulkhani, S. (2020). A many-to-many assignment game and stable outcome algorithm to evaluate collaborative mobility-as-a-service platforms. *Transportation Research Part B: Methodological*, *140*, 79–100.

Platt, J., & Barr, A. (1987). Constrained differential optimization. In *Neural information processing systems*.

van den Berg, V. A., Meurs, H., & Verhoef, E. T. (2022). Business models for mobility as an service (maas). *Transportation Research Part B: Methodological*, *157*, 203–229.

Wong, Y. Z., Hensher, D. A., & Mulley, C. (2020). Mobility as a service (maas): Charting a future context. *Transportation Research Part A: Policy and Practice*, *131*, 5–19.

Xi, H., Li, M., Hensher, D. A., Xie, C., Gu, Z., & Zheng, Y. (2024). Strategizing sustainability and profitability in electric mobility-as-a-service (e-maas) ecosystems with carbon incentives: A multi-leader multi-follower game. *Transportation Research Part C: Emerging Technologies*, *166*, 104758.

Xi, H., Liu, W., Waller, S. T., Hensher, D. A., Kilby, P., & Rey, D. (2023). Incentive-compatible mechanisms for online resource allocation in mobility-as-a-service systems. *Transportation Research Part B: Methodological*, *170*, 119–147.

Yang, H., Ke, J., & Ye, J. (2018). A universal distribution law of network detour ratios. *Transportation Research Part C: Emerging Technologies*, *96*, 22–37.

Yao, R., & Zhang, K. (2024). Design an intermediary mobility-as-a-service (maas) platform using many-to-many stable matching framework. *Transportation Research Part B: Methodological*, 102991.

Zhang, K., Chen, H., Yao, S., Xu, L., Ge, J., Liu, X., & Nie, M. (2019). An efficiency paradox of uberization. *Available at SSRN 3462912*.

Zhang, K., & Nie, Y. M. (2021a). Inter-platform competition in a regulated ride-hail market with pooling. *Transportation Research Part E: Logistics and Transportation Review*, *151*, 102327.

Zhang, K., & Nie, Y. M. (2021b). To pool or not to pool: Equilibrium, pricing and regulation. *Transportation Research Part B: Methodological*, *151*, 59–90.

Zhou, Y., Yang, H., Ke, J., Wang, H., & Li, X. (2022). Competition and third-party platform-integration in ride-sourcing markets. *Transportation Research Part B: Methodological*, *159*, 76–103.

Zhu, Z., Xu, A., He, Q.-C., & Yang, H. (2021). Competition between the transportation network company and the government with subsidies to public transit riders. *Transportation Research Part E: Logistics and Transportation Review*, *152*, 102426.