Transit Network Design and Frequency Setting accounting for Vulnerability

Laskaris G.*[1], Iliopoulou C.[2], Kepaptsoglou K.[1]

[1] Laboratory of Transportation Engineering, School of Rural, Surveying and Geoinformatics Engineering, National Technical University of Athens, Greece

[2] Department of Civil Engineering, University of Patras, Greece

SHORT SUMMARY

Transit network design in the phase of transit route design and frequency setting primarily aims on the minimization of passenger and operator related costs, neglecting vulnerability sourcing from its critical infrastructure. This study focuses on enhancing the resilience of public transit networks by incorporating vulnerability analysis into the transit network design problem. After estimating a set of candidate solutions with their critical infrastructure, we proceed to the optimization of frequencies of the solution that resulted to the lowest disruption cost. Simulated annealing is used to adjust the frequencies of the transit lines subject to fleet availability and capacity constraints. Applied to Mandl's network, the results shown that the final solution experiences significantly limited losses from disruption, while offering a comparable solution to the other candidate solutions that their objective is limited to cost optimization.

Keywords: Transit Network Design, Vulnerability Analysis, Transit Route Design, Frequency Setting

1. INTRODUCTION

Public transport networks are a major contributor to alleviate traffic and assist in transitioning towards sustainable mobility. However, they are exposed to vulnerability sourcing from endogenous and exogenous factors that drastically impact the operation and consequently the attractiveness and the performance of such systems. Transit network design ensures that the transit routes and the set frequencies can satisfy the demand and provide a reasonable alternative for commuting. Transit network design comprises transit route design, frequency setting, fleet allocation and driver rostering [1]. It is a dominant topic which has been thoroughly investigated and periodically reviewed in the literature [2], [3], [4]. The main objective of transit network design is the minimization of passenger and operator cost. Moreover, it accounts for constraints like network capacity or fleet size as well as recently introduced, for instance potential synergies with other forms of mobility and technologies such as modular vehicles [5] and charging of electric vehicles [6], [7]. Usually, the transit network design problem is complex and computationally expensive, therefore heuristics and meta-heuristics are utilized to solve the problem[4], [8].

As aforementioned, vulnerability is interwoven to transit operations. For transit networks, vulnerability is referred to as the magnitude of the effect of degradation of physical infrastructure on the connectivity and distances between transit stations [9], [10]. Vulnerabiliy analysis has been conducted in all types of transportation networks including public transport. Pan et al [11] reviewed vulnerability and resilience in transportation. Vulnerability has been studied using topological metrics such as centrality measures and graph theory [9], [12]. Furthermore, dynamic approaches have been used accounting for passengers' response to disruptions such as in the work of Cats and Jenelius [13], [14].

So far, vulnerability analysis has been studied and quantified post transit network design phase. A more robust transit network design may come with a significant cost for both passengers and operators. Therefore, this study attempts to further enhance transit network design solutions which are more resilient to potential failures of their critical infrastructure by optimizing their frequencies to minimize both passenger and operator cost. The methodology is applied on Mandl's network and it is assessed based on passenger and operator metrics. The remainder of the paper is organized as follows: in the next section the methodology is given, followed by the experimental setup and the discussion of the results obtained. In the last section the conclusions are drawn.

2. METHODOLOGY

Overview

The problem at hand refers to the optimal design of transit routes accounting for potential vulnerabilities sourcing from the critical infrastructure of the network. The proposed methodology involves two main steps: first a set of candidate networks is generated and their critical link is identified. Then a targeted attack is performed, and the impact of the attack is assessed based on the difference in passenger welfare. Network attacks can be either random or targeted and different in magnitude; targeted attacks in particular are imposed to specific network elements according to topological or operational attributes. During the second step, the frequencies of the candidate network that reported the minimum loss in passenger welfare are optimized. The final output is a transit network design resilient to potential disruption and optimized passenger and operator cost. The overall methodological framework is illustrated in Figure 1.

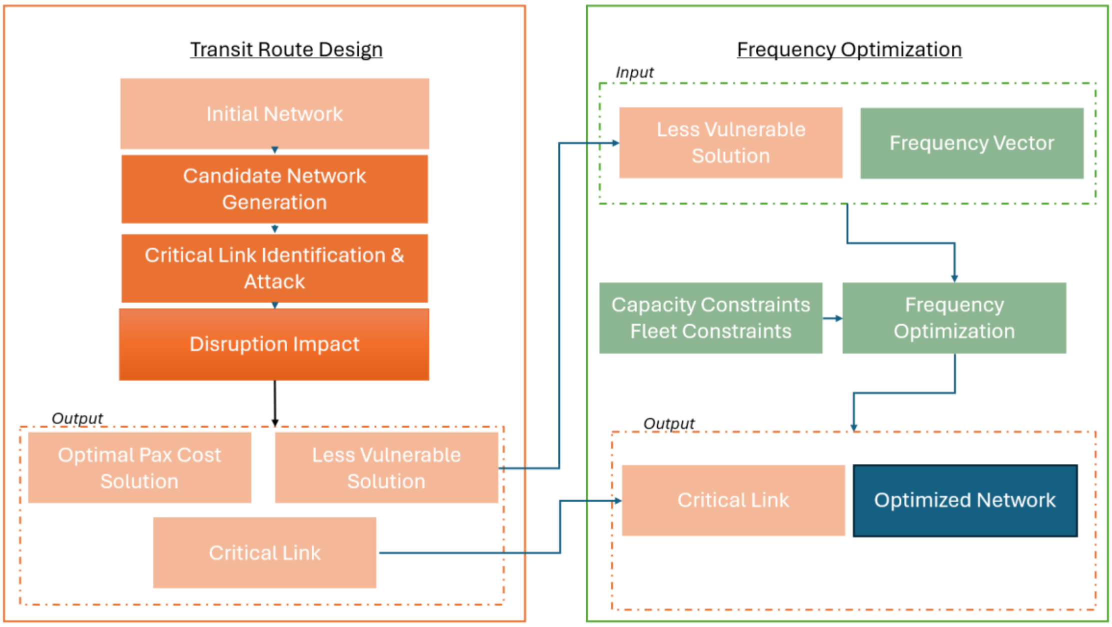


Figure 1 Methodology Overview

Transit Route Design

The transit network is defined as a directed graph *G(S,E)*, where *S* the set of stops and *E* the set of transit links. Each link has the following characteristics: length, riding time and capacity. The transit network has *L* transit lines, each one having a transit route *r* and a frequency *f*. Passenger demand is given in a set of origin-destination pairs.

The optimization problem aims to find the optimal transit route design by minimizing the passenger related cost and can be defined as:

$$\hat{R} = Z\left(R\right) \tag{1}$$

Where *R*, the set of transit routes.

The objective function *Z* aims to optimize passenger welfare. Passenger welfare Z^{pax} consists of all different time components and number of transfers, weighted by passengers' perceptions thereof:

$$Z^{pax} = \sum_{o=1}^{N}\sum_{d=o}^{N}\left(\beta^{wait}t^{wait} + \beta^{inveh}t^{inveh} + \beta^{transfer}n^{transfer}\right) \tag{2}$$

Passenger betweenness centrality is used to identify the importance of each link based on connectivity, weighted with passenger flows since passengers will experience the impact of the disruption. Passenger betweenness centrality of a link *e* is the ratio of the passenger flow traversing the link over the total passenger flow. Passenger betweenness centrality is expressed as follows:

$$PBC = \frac{q_e}{\sum_{o \in O}\sum_{d \in D} Q} \tag{3}$$

Where,

q^e	passenger flow in link *e*;
$\sum_{o \in O}\sum_{d \in D} Q$	total passenger flow in the network.

Passenger betweenness centrality is calculated for all links of the network. The link with the highest value is considered the most critical and it is removed to demonstrate a disruption.

After the removal of the critical link, the impact of the disruption is expressed as the change in passenger welfare. The states of the network pre and post disruption are compared by the changes in all passenger travel time components. With W_0 denoting passenger welfare for a transit network under normal operation and W_{dis} the passenger welfare under disruption, the impact of the disruption on a network is expressed as follows:

$$\Delta W = W_{dis} - W_0 \tag{4}$$

Simulated Annealing is used to reconfigure the transit routes of the network. In each cycle, transit routes are sampled based on the feasibility constraints. The value of objective function of each candidate transit network is calculated, followed by the calculation of passenger betweenness centrality for each link, the identification of the transit link with the highest value and its removal. After the link removal, the passenger welfare loss due to disruption is estimated. The calculation of the objective function for the base and the disrupted network requires the assignment of users to their shortest paths. All three values (objective function value, critical link and welfare loss) are registered, and the iterative procedure terminates when the given number of replications (cycles) has been reached.

Frequency Optimization

After generating the set of candidate transit networks, the one resulting to the minimum loss in passenger welfare is selected. Following an iterative procedure and based on the assignment results, the frequency of each line is adjusted in order to minimize the value of the objective function. The objective function is defined as the sum of the passenger Z^{pax} and the operator cost Z^{oper} :

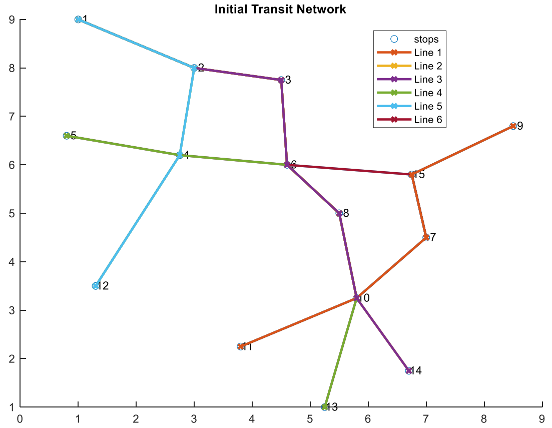
$$Z(r,f)=\theta_1 Z^{pax}(r,f)+\theta_2 Z^{oper}(r,f) \tag{5}$$

Where r is the set of routes and f is the set of the corresponding frequencies of each candidate transit network. θs can be set to reflect the relative importance attached to each component. Simulated annealing is used to adjust the frequencies of the lines. During each iteration a frequency is sampled from a given vector of frequencies for each transit line. Then the feasibility of the solution is checked with two constraints. The first constraint is the fleet size. The candidate solution is not feasible if it violates the given fleet availability. The second constraint ensures that the capacity of each link of the network will not be exceeded. If the candidate solution satisfies the constraints, then the objective function value is calculated. The final score of each solution is registered and the frequency vector with the lowest value of the objective function is chosen as the final solution.

3. RESULTS AND DISCUSSION

Experimental Setup

We applied the methodology on Mandl's network [15] with 15 nodes and 21 links. The initial network comprises six transit lines, which are shown in Figure 2 accompanied with their characteristics. In the first phase, all candidate sets of transit routes are estimated together with their critical link. Then in the second phase, the network that results in the lowest loss in passenger welfare is used as an input and during a second iterative procedure its frequencies are optimized.



Transit Line id	**Transit Route**	**Frequency (min)**	**Travel Time (min)**	**Route Length (m)**
1	[1,2,3,6,8,10]	3	23	11040
2	[5,4,6,8,10,13]	5	28	13440
3	[1,2,3,6,15,7]	7,5	18	8640
4	[9,15,7,10,11]	6	22	10560
5	[2,3,6,8,10,14]	7,5	23	11040
6	[1,2,4,12]	30	21	10080

Figure 2 Mandl's Network

The optimization procedure is implemented in Mathworks MATLAB. For the network studied 100 replications were conducted for the transit route design and additional 100 replications for the frequency optimization. The whole procedure took about 2h on a laptop with Intel(R) Core(TM) i7 at 4.00GHz and 8GB of RAM.

Results

All candidate solutions resulting from the first phase of the problem (transit route design) are depicted in Figure 3. The initial network is indicated in the graph with a green triangle, and it can be observed that a potential attack can increase passenger cost by 26.11%. The network with the lowest impact after the removal of its critical link is indicated with a red square, while the network with the lowest passenger cost with a red diamond.

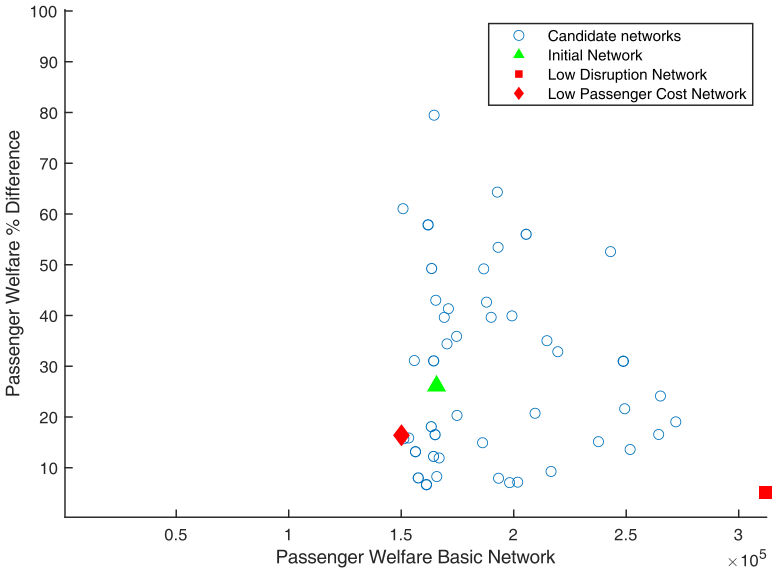


Figure 3 Passenger Welfare and % difference from the disrupted network

As shown in Table 1, the low impact of the attack on the low disruption network results in extremely high passenger cost, in terms of waiting time, in vehicle time and number of transfers. Therefore, the frequencies of the transit lines of this network are optimized.

Table 1 Passenger Welfare of the initial and two candidate solutions

	Base Network			Disrupted Network			Critical Link
	Waiting Time (sec)	Riding Time (sec)	Average Number of Transfers	Waiting Time (sec)	Riding Time (sec)	Average Number of Transfers	
Initial Network	272.10	670.47	0.85	225.20	778.28	0.63	[6,8]
Low Disruption	577.62	1407.93	2.75	1338.91	1414.52	4.32	[14,1]
Low PaxCost	85.27	622.17	0.23	387.92	674.76	1.07	[10,8]

The network with the low disruption cost is used as an input for frequency optimization. The results are summarized in Table 2. Overall, there is a significant reduction in the objective function value by 10.74%, It can be observed that the optimal allocation of frequencies results in a substantial improvement in waiting time and number of transfers. The riding time is roughly similar pre- and post-optimization. The explanation lies in the structure of the network. The majority of the transit routes traverse a single corridor that connects two areas of the network and there are no alternative shorter routes. In addition, the optimized network requires less vehicles for operation and this corresponds to reduction in both vehicle-hours and vehicle-kilometers.

Table 2 Passenger Welfare of the Low Disruption network before and after optimization

	Passenger Cost			**Operator Cost**			
	Waiting Time (sec)	**Riding Time (sec)**	**Average Number of Transfers**	**Vehicle * Hours**	**Vehicle * Kilometers**	**Number of Vehicles Needed**	**Objective Function**
Initial Network	577.62	1407.93	2.75	57.27	1649.28	88.00	311815.12
Optimized Network	236.19	1407.88	0.38	44.00	1267.20	68.00	278320.01
% Difference	-59.11%	0.00%	-86.21%	-23.17%	-23.17%	-22.73%	-10.74%

Figure 4 illustrates the difference in frequency in each segment of the network. As observed, the optimization boosts the frequencies in the shared transit corridor, that serves the majority of the stops. This result is reflected also by the reduction in waiting times shown in Table 2. In only three segments, the frequency is reduced and in four segments the frequency remained the same. All these segments can be found at the edges of the network, where there is no need for high frequencies, as they can be served by a local/feeder transit line.

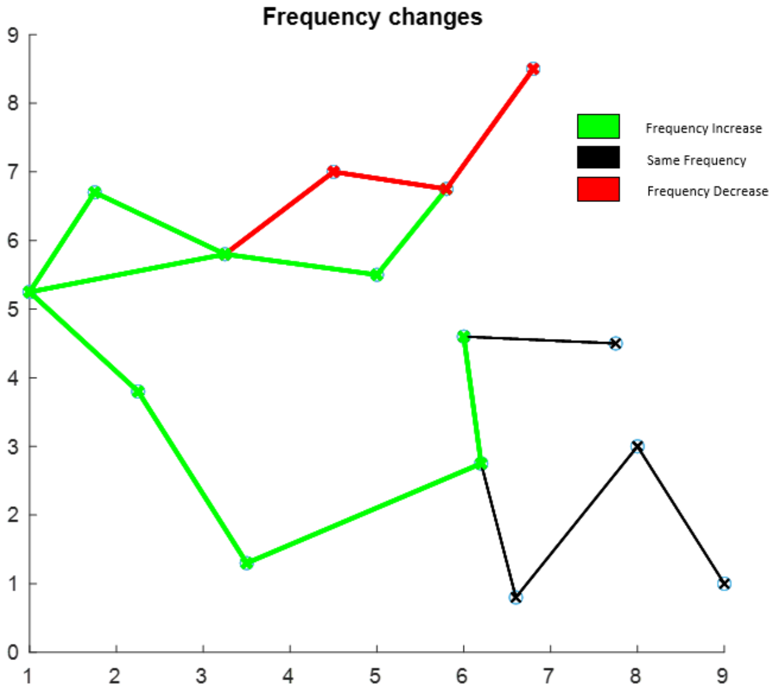


Figure 4 Link Frequency changes

4. CONCLUSIONS AND OUTLOOK

Vulnerability analysis has not been explored within the concept of transit network design problem. This study is a first step to incorporate the impacts of a potential disruption in transit network design problem. After showcasing all candidate network designs accompanied with their critical infrastructure, we identify the solution with the minimum disruption cost and we enhance it further by optimizing the frequency of its transit lines. The final solution has a limited impact on passenger welfare during a disruption and offers a comparable solution to the other candidate solutions that their objective is limited to cost optimization.

Additional results will be presented at the conference, including vulnerability metrics and additional attack strategies. In addition, the application of the methodology to all variations of Mumford network will be presented.

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