A Game-theoretical Model of Road Pricing with Endogenized User-equilibrium

Gaurav Malik*[1], Chris Tampère[2]

[1] FWO PhD SB Research Fellow, Department of Mechanical Engineering, KU Leuven, Belgium

[2] Professor, Department of Mechanical Engineering, KU Leuven, Belgium

SHORT SUMMARY

This paper presents an optimization-based game-theoretical model of road pricing. The model incorporates elastic user-demand, multiple user-classes, and an endogenized demand and path-choice user-equilibrium. After providing a generic model formulation, we apply it to a single-player optimization problem to find optimal tolls for a government subjected to user-equilibrium. We propose a post-processing method that avoids unfavorable outcomes caused by the non-uniqueness of user-equilibrium path flows. We use the single-player optimization problem to develop a game-theoretical framework to solve different competition scenarios. The model is applied to two fictional case-studies. The first involves a two-player game-theoretical problem and four competition scenarios are elaborated, and the second involves user-class specific instruments and demonstrates the relevance of the post-processing.

Keywords: pricing and capacity optimization; Traffic, network, and mobility management; transport economics and policy.

INTRODUCTION

Transportation systems consist of several stakeholders who influence each other's decisions. Road pricing is one domain where this is relevant. Decisions regarding tolls are complicated because of interactions between governments at the same level e.g., two national governments, as well as governments at different levels e.g., regional and federal governments. A toll implementation which ignores such interactions carries the risk of being ill-informed. At the same time, the computational effort for analyzing scenarios with due consideration of these interactions can be prohibitively high if detailed traffic models like traditional four step models (Li et al., 2021) or agent-based simulators (Röder et al., 2013) are used. Such models have detailed representations of the underlying network but the optimality of tolls and competition between different payers remain unaddressed. Models from the domain of transport economics have paid more attention to game-theoretical interactions, like those between governments at equal or different levels (De Borger & Proost, 2021). However, these models remain analytical and lack the computational scalability required even for aggregated versions of pseudo-real case studies. Further, they model only a predetermined fixed set of used/active paths and the crucial choice of not using an available path at all is not modelled. This paper makes two crucial contributions to the state of the art of these models:

- A computational framework for game-theoretical problems with an endogenized demand and path-choice user-equilibrium. This endogenizes the crucial conversion of hitherto active paths into inactive paths and vice-versa as a reaction to changes in toll values.
- The underlying User-Equilibrium (U.E.) model incorporates multiple user-classes with different Willingness-To-Pay (WTP) and different Values of Time (VOT). The U.E. model further allows to target different user-classes with different tolls.

(Ohazulike 2014) and (Najmi, Rashidi, and Waller 2023) have also studied game-theoretical problems in transportation but address only Nash-Cournot interactions, herewith disregarding alternative competition scenarios. The model presented in this paper is modular in nature and can be adapted to different competition scenarios like Nash Cournot and Stackelberg games.

Another relevant issue is the non-uniqueness of path flows in user-equilibrium corresponding to a particular value of toll. The objective function of a government is often dependent on path flows. When a government implements a toll that it believes to be optimal, the users may have multiple ways to respond, some of which may not lead to the expected optimal outcome for the government. To the best of the authors' knowledge, this issue has not been covered in any of the works related to road pricing. As a third contribution:

- We propose a post-processing method that identifies access restrictions necessary to avoid unintended outcomes corresponding to the previously identified optimal tolls.

1. METHODOLOGY

In this paper, the term "player" refers to a government which exerts control over the transportation system via some toll instruments and the term "users" refers to the general travelers in the transportation system.

The single-player optimization problem then is:

$$\min_{\boldsymbol{x},\, \boldsymbol{ti_\mu}} Obj_\mu \quad \text{s.t } U.E. \tag{1}$$

Where Obj_μ stands for the Objective function of player μ, $U.E.$ stands user-equilibrium, $\boldsymbol{x}$ denotes the flow pattern of users and $\boldsymbol{ti_\mu}$ denotes the toll instruments of player μ. This problem represents the Single Leader Multi Follower game between the player and the users. As a Stackelberg leader, the government optimizes its objective function while anticipating user-equilibrium.

User-Equilibrium Model

Supply

Physical network $G(N, A, Z)$ where:

N: set of nodes i = 1,2,3….

A: set of physical links (i, j) where from-node $i \in N$ and to-node $j \in N$.

Z: set of centroids i = 1,5,9… $Z \subseteq N$.

Demand

D: set of OD pairs (r, s) where origin $r \in Z$ and destination $s \in Z$.

$d_{(r,s)}$: Demand level for OD pair (r, s).

User-class

User-classes $u \in U$, where U is the set of all modelled user-classes.

$U_{(r,s)}$: set of all user-classes $u \in U$ which are relevant to be modelled for OD pair (r, s) with $U_{(r,s)} \subseteq U \; \forall \; (r, s) \in D$.

$D_{(u,(r,s))}$: set of Origin-Destination-User-class (ODU) triplets $(u, (r, s))$.

$d_{(u,(r,s))}$: Demand level for ODU triplet $(u, (r, s))$.

Demand Elasticity

Each $d_{(u,(r,s))}$ may have a different elasticity, which is modelled by linear WTP curve as follows:

$$WTP_{(u,(r,s))} = A_{(u,(r,s))} - B_{(u,(r,s))} * d_{(u,(r,s))} \; \forall \; (u, (r, s)) \in D_{(u,(r,s))} \tag{2}$$

Paths

A path is a sequence of connected links from origin r to destination s for an ODU triplet $(u, (r, s))$.

P: set of all modelled paths q over all $(u, (r, s)) \in D_{(u,(r,s))}$.

$P_{(u,(r,s))}$: set of all paths $q \in P$ specific to ODU triplet $(u, (r, s))$ with $P_{u,(r,s)} \subseteq P$.

Pre-processing

Pre-processing enumerates relevant available paths. This results in a Link-Path incidence matrix where:

$$\delta_{(i,j),q} = \begin{cases} 1, if\ path\ q\ traverses\ the\ link\ (i,j) \\ 0, otherwise \end{cases} \tag{3}$$

Path Flows

x_q: Flow of users on path $q \in P$.

Link Flows

$$f_{(i,j)} = \sum_{q \in \mathrm{P}} \delta_{(i,j)q} * x_q \tag{4}$$

Link Travel Time

$$lc_{(i,j)} = a_{(i,j)} + b_{(i,j)} * \left(f_{(i,j)}\right)$$
$$a_{(i,j)} > 0, b_{(i,j)} > 0 \; \forall \; (i,j) \in A \tag{5}$$

Generalized Link Cost

Generalized link costs (glc) are defined for each link (i,j) and for each ODU triplet $(u,(r,s))$:

$$glc_{(i,j)}^{(u,(r,s))} = lc_{(i,j)} * VoT^{(u,(r,s))} + \sum_{t \in T_{(i,j)}} t + \sum_{t \in T_{(i,j)}^{(u,(r,s))}} t$$

$$where$$

$$T_{(i,j)} = T\_fixed_{(i,j)} \cup (\bigcup_{\mu \in PS} T\_control_{(i,j),\mu})$$

$$T_{(i,j)}^{(u,(r,s))} = T\_fixed_{(i,j)}^{(u,(r,s))} \cup (\bigcup_{\mu \in \mathrm{PS}} T\_control_{(i,j),\mu}^{(u,(r,s))}) \tag{6}$$

$T_{(i,j)}$ and $T_{(i,j)}^{(u,(r,s))}$ are the sets entry-based monetary costs on link (i,j) including fixed costs and control instruments implemented by players. $T_{(i,j)}$ is for all users while $T_{(i,j)}^{(u,(r,s))}$ targets specifically ODU triplet $(u,(r,s))$.

PS: set of players.

$VoT^{(u,(r,s))}$: VoT of ODU triplet $(u,(r,s))$.

Generalized Path Cost (gpc)

$$gpc_q = \sum_{(i,j) \in A} \delta'_{(i,j)q} * glc_{(i,j)}$$

$$\forall q \in P_{u,(r,s)}, \qquad \forall (u,r,s) \in D_{u,r,s}$$

Active Path

All paths $q \in P_{(u,(r,s))}$ with $WTP_{(u,(r,s))} - gpc_q = 0$ at user-equilibrium are considered active whereas, those with $WTP_{(u,(r,s))} - gpc_q < 0$ are considered inactive.

Mixed Integer User-Equilibrium (MIUE)

Binary variable $i_q = 0$ if path q is inactive, and $i_q = 1$ if it is active. Using the big-M concept, we express the U.E. as the following mixed-integer user-equilibrium conditions:

$$\sum_{q \in P_{(u,(r,s))}} x_q = d_{(u,(r,s))} \ \forall (u,(r,s)) \in D_{(u,(r,s))}$$

$$x_q \geq 0 \ \forall q \in P$$

$$d_{(u,(r,s))} \leq d\_max_{(u,(r,s))} \ \forall (u,(r,s)) \in D_{u,(r,s)}$$

$$x_q - M * i_q \leq 0 \ \forall q \in P$$

$$M * \left(WTP_{(u,(r,s))} - gpc_q\right) + \left(1 - i_q\right) \leq 0 \ \forall q \in P_{(u,(r,s))}, \qquad \forall (u,(r,s)) \in D_{(u,(r,s))}$$

$$-(WTP_{(u,(r,s))} - gpc_q) + M * i_q - M \leq 0 \ \forall q \in P_{(u,(r,s))}, \qquad \forall (u,(r,s)) \in D_{u,(r,s)} \tag{7}$$

Existence and Uniqueness of User-Equilibrium

If all $glc_{(i,j)}$, gpc_q, and $WTP_{(u,(r,s))}$ are continuous in $f_{(i,j)}$, x_q and $d_{(u,(r,s))}$ respectively and all $d_{(u,(r,s))}$ are bounded above with 1) $WTP_{(u,(r,s))}$ at $d\_max_{(u,(r,s))}$ being lower and 2)

$WTP_{(u,(r,s))}$ at zero demand being higher than gpc of the cheapest path for that $(u,(r,s))$, then there is at least one feasible path flow vector satisfying the user-equilibrium conditions for given sets of $T_{(i,j)}$ and $T_{(i,j)}^{(u,(r,s))}$. This can be proven with Brouwer's and Kakutani's theorems but for the sake of brevity, the proof is not included.

For the general case, uniqueness of equilibrium path flows is not guaranteed. This is crucial when players' objective functions depend on path flows.

Objective Function Formulation

The objective function of government-type players is modelled as:

$$Obj_{\mu} = -\left(SWlf_{\mu} - STC_{\mu} + TR_{\mu}\right) \forall \mu \in PS \tag{8}$$

$$where$$

$$SWlf_{\mu} = \sum_{(u,(r,s)) \in D_{(u,(r,s)),\mu}} \int_0^{d_{(u,(r,s))}} \left(WTP_{(u,(r,s))}\right) \mathrm{d}d_{(u,(r,s))}$$
$$= \sum_{(u,(r,s)) \in D_{(u,(r,s)),\mu}} \left(A_{(u,(r,s))} * d_{(u,(r,s))} - \frac{B_{(u,(r,s))} * d_{(u,(r,s))}^2}{2}\right) \tag{9}$$

$$STC_{\mu} = TTC_{\mu} + SEC_{\mu}$$

$$TTC_{\mu} = \sum_{(u,(r,s)) \in D_{(u,(r,s)),\mu}} \left(\sum_{q \in P_{u,(r,s)}} x_q * \left(gpc_q\right)\right)$$

$$SEC_{\mu} = \sum_{(i,j) \in A^{\mu}} \left\{\lambda_{(i,j)} * \left(lc_{(i,j)}\right)\right\} + \sum_{q \in P} \left(\sigma_q^{\mu} * x_q\right) \tag{10}$$

$D_{(u,(r,s)),\mu}$: set of ODU triplets under the electorate of government-type player μ.

$SWlf_{\mu}$ and TTC_{μ}: <u>social welfare</u> and <u>total travel costs</u> of the society under the jurisdiction of player μ respectively. $SWlf_{\mu}$ represents the total benefit that the society makes by travelling.

SEC_{μ}: <u>social external costs</u> of the society under jurisdiction of player μ. $\lambda_{(i,j)}$ and σ_q are pre-decided scalar factors.

TR_{μ}: revenue collected by player μ.

$$TR_{\mu} = TR_{\mu,link} + TR_{\mu,link-(u,(r,s))}$$

$$TR_{\mu,link} = \sum_{(i,j) \in A^{\mu}} \left[\left\{\sum_{t \in T\_control_{(i,j),\mu}} t\right\} * f_{(i,j)}\right]$$

$$TR_{\mu,link-(u,(r,s))} = \sum_{(u,(r,s)) \in D_{(u,(r,s))}} \left[\sum_{q \in P_{(u,(r,s))}} \left[\left[\sum_{(i,j) \in A^{\mu}} \left\{ \delta'_{(i,j)q} * \sum_{t \in T_{control}{}^{(u,(r,s))}_{(i,j),\mu}} t \right\} \right] * x_q \right] \right] \tag{11}$$

Obj_{μ} depends on path flows and can't be fully evaluated with just link flows.

Single-Player Optimization: Mixed-Integer Quadratic Programming (MIQP)

The optimization problem of a single government-type player, initially presented in **Equation (1)**, is formulated as an MIQP employing the MIUE of **Equation (7)** as a constraint and can be solved using MIP solvers like Gurobi and CPLEX.

$$\min_{\boldsymbol{i,x,d,\tau_{\mu}}} Obj_{\mu} \ \forall \ \mu \ \in \mathrm{PS}$$
$$\text{s.t} \quad MIUE \tag{12}$$

Post-processing:

Once the optimal value of instruments is determined, the uniqueness of user-equilibrium path flows corresponding to those values can be determined by post-processing. There may exist multiple feasible user-equilibrium path flow vectors for the optimal values of instruments. Some of these vectors may have an objective function value worse than the intended optimal point. So, it is important for the player to verify whether the optimal instrument values lead to a unique user-equilibrium and hence, to the intended objective function value without the possibility of an unfavorable response by users. We propose the following post-processing method to address this:

1. After solving the optimization problem, set/fix all instruments to solution values.
2. Solve the optimization problem again with fixed instruments but instead of minimizing the objective function, maximize it.
3. This represents the most unfavorable response (MUR) of users to the previously determined optimal instrument values. If the original solution and the MUR case lead to same objective function values, the player can be assured of the outcome.
4. If, however, the difference is significant, compare path flows in the two solutions. Now, depending on the ease of implementation of available access restrictions e.g., metered access etc., the player should start introducing explicit constraints on the corresponding path and link flow variables in the optimization problem and solve the MUR case again.
5. This can be repeated until the difference between the objective functions in the original and the MUR solution is acceptable. In this way, the player can iteratively determine the minimum explicit constraints or access restrictions required to prevent the unfavorable responses of users.

Multi-Player Games: Nash-Cournot (NC) Game

When there are several governments taking decisions simultaneously, the resulting game is a Nash-Cournot game. It can be formulated as a fixed-point problem in which the players sequentially solve their single-player optimization problems while considering fixed instruments of other players fixed. Each player, individually, still acts a leader over the users. This process is shown in **Figure 1**.

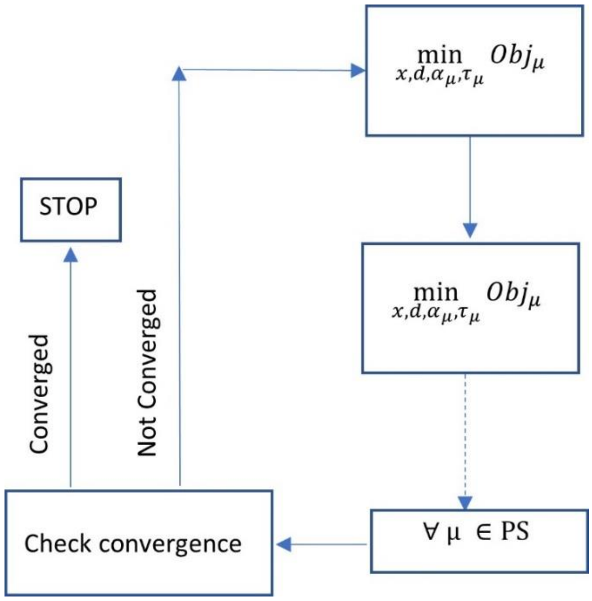

Figure 1: NC game solution process

Existence

Existence conditions for Nash equilibrium of a general n-player game are given by the Debreu-Glicksberg-Fan Theorem. This model does not follow these conditions. (Ohazulike, 2014) also confirms that Nash Equilibrium for Nash-Cournot game in road pricing may not exist in general.

Multi-Player Games: Stackelberg Games

In Stackelberg games, the leader optimizes its objective function subject to the optimum of the follower or the Nash-Cournot game equilibrium (NCE) of followers as represented in **Figure 2**.

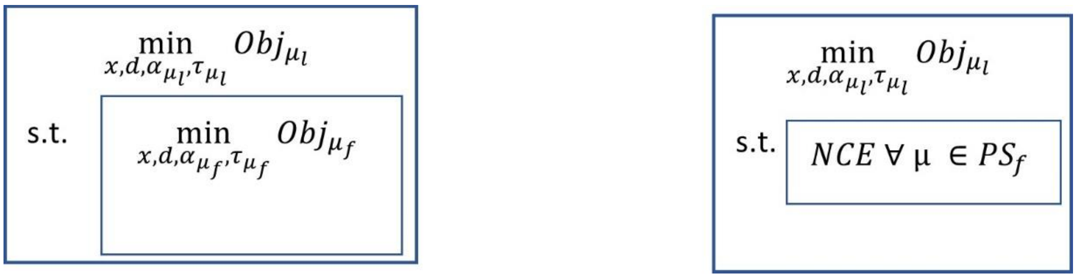

Figure 2: Stackelberg game as an optimization problem of the leader with single follower (left) and multiple followers in NC game (right)

Even with a single follower player, this problem is an optimization problem which is subject to another optimization problem which itself is subject to the U.E. of users. It is highly non-convex and is not suitable to be solved by exact methods. We solve this problem with the use of meta-model-based Blackbox optimization methods like Bayesian Optimization, SHERPA (*SHERPA White Paper*, n.d.) etc.

Existence

If the follower(s) has/have a unique response (instruments' values) for each feasible value of the leader's instruments, and the leader's instruments are bounded on both sides, then Stackelberg equilibrium should exist. For the case of a single follower player, the former condition and hence the existence is guaranteed if the objective function is quasi-convex. For the case of Nash-Cournot game between followers, as mentioned before, NCE may not exist. Consequently, the overall Stackelberg equilibrium may also not exist.

2. RESULTS AND DISCUSSION

We present two case studies. The parameters used in these case studies are not calibrated to any real scenario and should only be interpreted in relative sense.

Case Study 1: Multi-player games and multi-user-classes

The city government (player 1) charges the toll t_{rad} for radial roads and the rural government (player 2) charges the toll t_m for the neighbouring rural roads. The primary aim of this case study is to demonstrate an application of the multi-player framework.

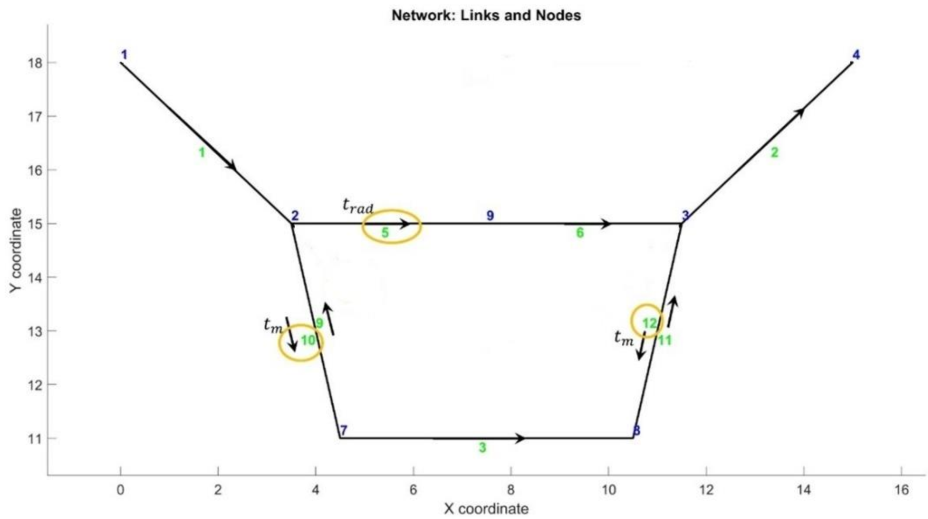

Figure 3: Network-Case Study 1: Node ID (blue), link indices (green) and circled link indices of tolled links

Table 1: Problem Specification-Case Study 1

Part	Specifications	Comment
Supply	• Network: $G_1(N, A, Z)$ • $N = \{1,2,3,4,7,8,9\}$ = set of 7 nodes. • $A = \{(1,2), (3,4), (7,8) \ldots\}$ = set of 9 links. • $Z = \{1,4,7,8,9\}$ = set of 5 centroids.	• See **Figure 3** for network. • Parameters for link travel time functions in **Table 5**

Demand	• $D = \begin{Bmatrix} (1,4), (1,7), (1,8), (7,4), (8,4), \\ (1,9), (9,4), (7,8), (7,9), (9,8) \end{Bmatrix}$ =set of 10 OD pairs.
	• $U = \begin{Bmatrix} 1{:}\, normal\ users \\ 2{:}\, cost\ insensitive\ travelers \end{Bmatrix}$ =set of user classes.
	    ○ $U_{(r,s)}= \begin{Bmatrix} 1{:}\, normal\ users \\ 2{:}\, cost\ insensitive\ travelers \end{Bmatrix}$ for all $(r, s) \in D$
	    ○ $n(D_{(u,(r,s))}) = 20,\ n(D) = 10.$
		The parameters for WTP functions are mentioned in **Table 2** and **Table 3**.

Paths	$P = \{1,2,3 \ldots .14, 15, 16, \ldots .28\}$ i.e., 28 paths where the last 14 paths are duplicates of the first 14 paths in terms of link-path incidence, but they are for the second user class.	$[\delta]_{9\,X\,14}$ = links-paths incidence matrix in **Table 4**
Set of Players	$PS = \{1{:}\, city\ government$ $2{:}\, rural\ government\}$	
Link-based costs and instruments	• There are no fixed entry costs on any link. • $T_{(2,9)} = T\_control^1_{(2,9)} = \{t_{rad}\}, T_{(2,7)} = T\_control^2_{(2,7)} = \{t_m\}, T_{(3,8)} = T\_control^2_{(3,8)} = \{t_m\}$ • $T_{(i,j)} = \emptyset$ for remaining $(i, j) \in A$ i.e., there are no entry instruments on the remaining links.	
Link-ODU triplet-based costs and instruments	There are no such fixed costs or instruments	
Objective Function	• $T\_control^1_{(2,9)} = \{t_{rad}\}$, $T\_control^2_{(2,7)} = \{t_m\}, T\_control^2_{(3,8)} = \{t_m\}$ • $T\_control^{\mu}_{(i,j)} = \emptyset$ for remaining $(i, j) \in A$ for all $\mu \in PS$ • $D_{(u,(r,s)),1}= \begin{Bmatrix} (1,(1{,}9)), (1,(9{,}4)), (1,(7{,}9)), (1,(9{,}8)), \\ (2,(1{,}9)), (2,(9{,}4)), (2,(7{,}9)), (2,(9{,}8)) \end{Bmatrix}$ • $D_{(u,(r,s)),2}= \begin{Bmatrix} (1,(1{,}7)), (1,(1{,}8)), (1,(7{,}4)), (1,(8{,}4)), (1,(7{,}8)), (1,(7{,}9)), (1,(9{,}8)), \\ (2,(1{,}7)), (2,(1{,}8)), (2,(7{,}4)), (2,(8{,}4)), (2,(7{,}8)), (2,(7{,}9)), (2,(9{,}8)) \end{Bmatrix}$ • $A^1 = \{(7,8),\ (7,2), (2,7), (8,3), (3,8)\}$ • $A^2 = \{(2,9),\ (9,3)\}$ • $TR_{1,link-(u,(r,s))} = TR_{2,link-(u,(r,s))} = 0$ as $T\_control^{(u,(r,s)),\mu}_{(i,j)} = \emptyset$ for both $\mu = 1$ and 2. • Remaining terms i.e., STC_{μ} and $TR_{\mu,link}$ can be calculated using $D_{(u,(r,s)),\mu}$, A^{μ}, $\sigma^1_q = \sigma^2_q = 0$ for all $q \in P$ and link parameters in **Table 5**. • $SWlf_{\mu}$: the welfare of the 10 cost-insensitive $(u, (r, s))$ is not added but all 20 $(u, (r, s))$ are considered for the STC_{μ}.	

Table 2: WTP Parameters- normal users

S. No.	$(u,(r,s))$	A	B
1	(1, (1,4))	15	0.015/2
2	(1, (1,7))	10	0.02
3	(1, (1,8))	12	12/1000
4	(1, (7,4))	12	12/1000
5	(1, (8,4))	10	10/1000
6	(1, (1,9))	10	0.01
7	(1, (9,4))	10	0.01
8	(1, (7,8))	8	8/500
9	(1, (7,9))	8	8/500
10	(1, (9,8))	8	8/500

Table 3: WTP Parameters- cost-insensitive users

S. No.	$(u,(r,s))$	A	B
11	2, (1,4)	10^5	10^5/50
12	2, (1,7)	10^5	10^5/0.001
13	2, (1,8)	10^5	10^5/25
14	2, (7,4)	10^5	10^5/25
15	2, (8,4)	10^5	10^5/0.001
16	2, (1,9)	10^5	10^5/0.001
17	2, (9,4)	10^5	10^5/0.001
18	2, (7,8)	10^5	10^5/20
19	2,(7,9)	10^5	10^5/0.001
20	2, (9,8)	10^5	10^5/0.001

Table 4: Links-paths incidence matrix

Links\Paths	1	2	3	4	5	6	7	8	9	10	11	12	13	14
(1,2)	1	1	1	1	1	0	0	0	1	0	0	0	0	0
(3,4)	1	1	0	0	0	1	1	1	0	1	0	0	0	0
(7,8)	0	1	0	0	1	0	1	0	0	0	0	1	0	0
(2,9)	1	0	0	1	0	1	0	0	1	0	1	0	1	0
(9,3)	1	0	0	1	0	1	0	0	0	1	1	0	0	1
(7,2)	0	0	0	0	0	1	0	0	0	0	1	0	1	0
(2,7)	0	1	1	0	1	0	0	0	0	0	0	0	0	0
(8,3)	0	1	0	0	0	0	1	1	0	0	0	0	0	0
(3,8)	0	0	0	1	0	0	0	0	0	0	1	0	0	1

Table 5: Link travel time and external cost parameters

Link Index	(i,j)	a	b	λ
1	(1,2)	4	10*0.01/36	0
2	(3,4)	4	10*0.01/36	0
3	(7,8)	0.5	10*0.05/36	0.12
5	(2,9)	0.35	10*0.25/36	0.2
6	(9,3)	0.35	10*0.25/36	0.2
9	(7,2)	0.5	10*0.05/36	0.12
10	(2,7)	0.5	10*0.05/36	0.12
11	(8,3)	0.5	10*0.05/36	0.12
12	(3,8)	0.5	10*0.05/36	0.12
13	(1,4)	14	2.5*0.01/36	0

Central optimization:

Central optimization problem represents the case when both governments act jointly as a single player. The central objective function is modelled by considering:

1. $T\_control^{c}_{(i,j)} = T\_control^{1}_{(i,j)} + T\_control^{2}_{(i,j)}$ for all $(i,j) \in A$.
2. $D_{(u,(r,s)),c} = D_{(u,(r,s)),1} \cup D_{(u,(r,s)),2}$
3. $A^{c} = A^{1} \cup A^{2}$

The results are summarized in **Table 6**.

Nash-Cournot game equilibrium:

The game is initialized with all instruments set to zero. The evolution of toll values and the three objective functions is shown in **Figure 4** and **Figure 5** respectively.

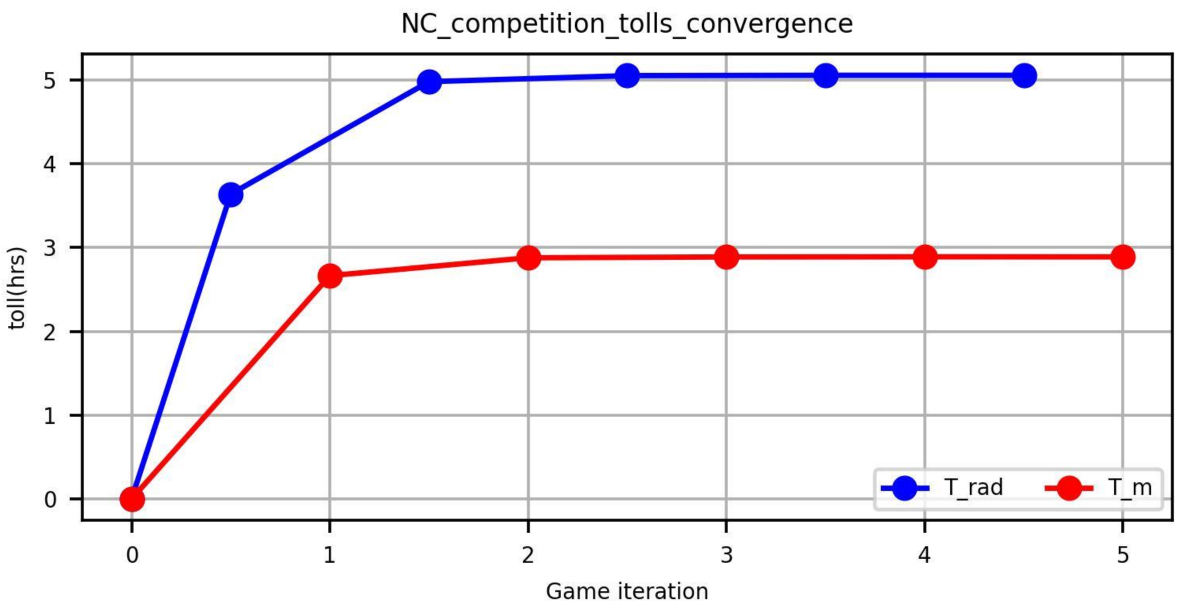

Figure 4: Evolution of the two tolls over NC game iterations

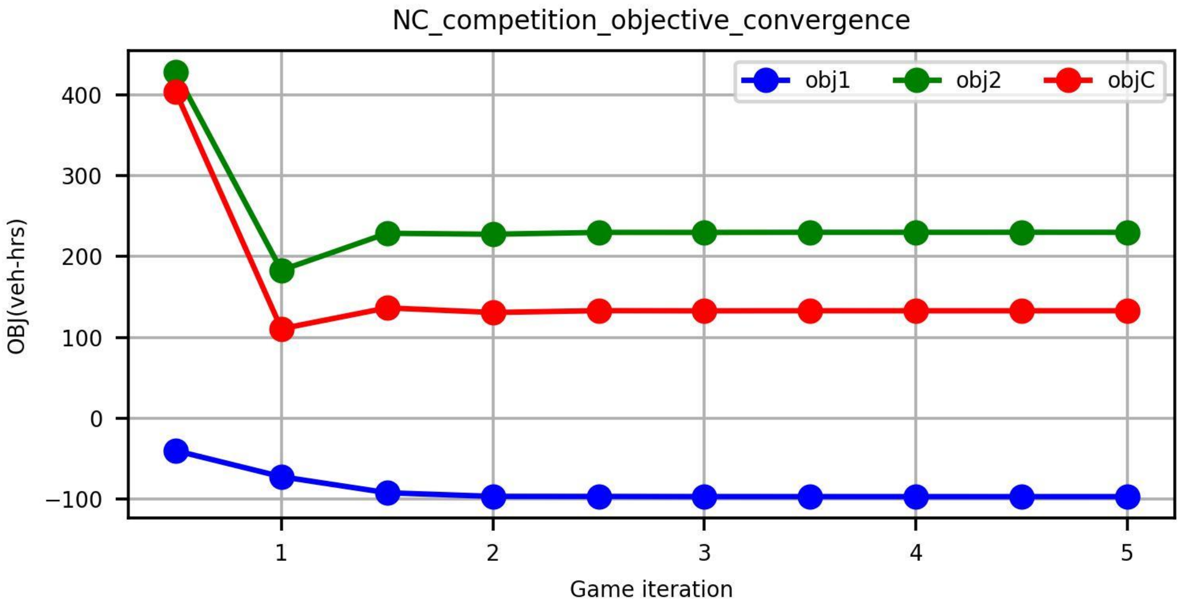

Figure 5: Evolution of the central and the two player's objective functions over NC game iterations

Reaction functions of the two players are plotted in **Figure 6**. The Nash-Cournot game equilibrium is marked by a black circle. The results are summarized in **Table 6**.

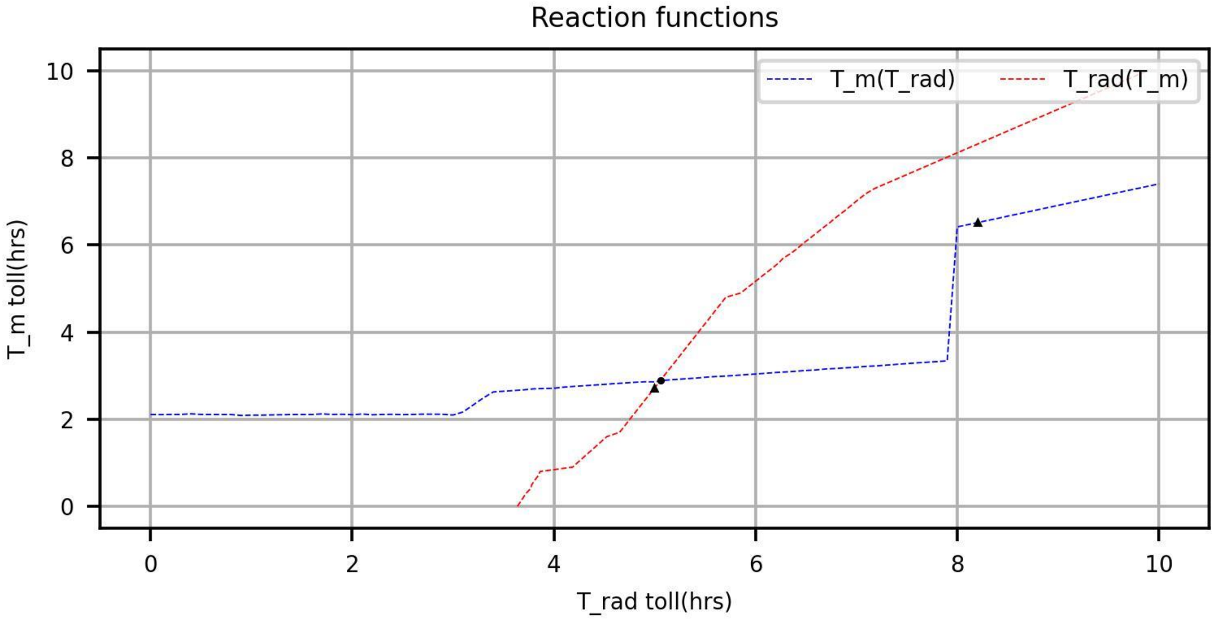

Figure 6: Reaction functions: Player 1 (red curve; setting T_rad) and Player 2 (blue curve, setting T_m)

<u>Stackelberg game equilibrium:</u>

The Nash Stackelberg equilibrium points when player 1 and when player 2 act as leaders are visualized in **Figure 6** as black triangles on the blue and the red response curves respectively. The corresponding values are summarized in **Table 6**. As may be intuitive from the power that Stackelberg leadership grants, both players gain over NC-interaction when given opportunity to be a leader (although Player 2 only marginally). When collaborating (central case), Player 2 sacrifices optimality for the sake of higher Player 1 gain.

Table 6: Results for Case Study 1

Metric/Scenario	**Central case**	**Nash-Cournot game**	**Player 1 as leader**	**Player 2 as leader**
Total Time (s)	0.61	6.59	62	64
Equilibrium Obj. Values (Obj_c, Obj_1, Obj_2)	(-52.39, -384.55, 332.19)	(132.84, -97.06, 229.91)	(113.39, -192.82, 306.24)	(136.57, -92.13, 228.72)
Equilibrium instrument values (t_{rad}, t_m)	(10, 10)	(5.055, 2.890)	(8.28, 6.55)	(4.98, 2.66)

Case Study 3: Non-uniqueness issues and user-class-based instruments

The primary aim of this case study is to demonstrate a case where multiple user-equilibria might exist for the same value of tolls and when user-class differentiated tolls are applied by the governments. The adaptations from first case study are mentioned in **Table 7**.

Table 7: Adaptations from Case Study 1

Part Adapted	Adaptation	Comment
Network	2 radial links replaced by 4 radial links	See **Figure 7**
	Each radial path through the city replaced by 4 paths.	See **Figure 8**
Objective Function	$\sigma_q^c = 0.25$ for the radial paths with up-down transfer	Higher external costs for transfers
	$\sigma_q^c = 0.20$ for radial paths with down-up transfer	Higher external costs for transfers
	$\lambda_{(i,j)}$ for links (7,2), (2,7), (8,3) and (3,8) is increased from 0.12 to 0.45.	Higher external costs for rural links

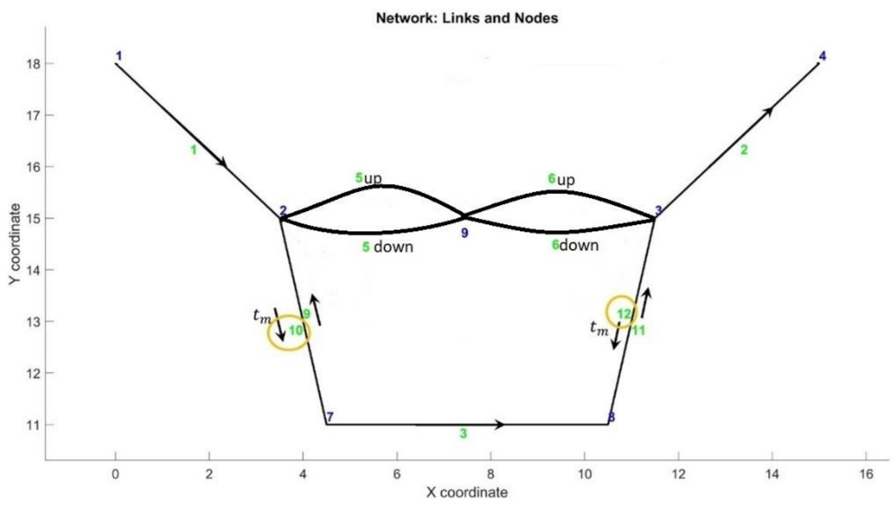

Figure 7: Network- Case Study 2

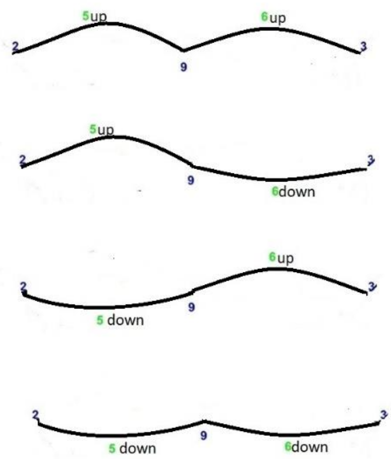

Figure 8: The four radial paths

The solution to the central optimization problem is shown in **Table 8**. The MUR solution, also shown in **Table 8**, is found by fixing the tolls to their optimal values and maximizing Obj_c. There is an appreciable difference between Obj_c values in the two cases. This is primarily due to the non-uniqueness in the path-choice of the fixed demand external ODU triplet (2,(1,4)). In the original solution, the radial paths without up-down or down-up transfers are chosen whereas, in the MUR solution, the paths with transfers are chosen. The latter paths lead to higher external costs thereby worsening Obj_c. Two levels of access restrictions targeting these paths are tested. First, we implement a flow metering measure in which the flow on the transfer paths is limited to 10 units each. This significantly limits the worsening of Obj_c in MUR case. However, to completely avoid any less favorable outcome, these paths need to be completely blocked.

Table 8: Results for Case Study 2

Metric\Case	Original Solution	MUR Solution	MUR Solution with metering	MUR Solution with blocking
Obj. Values (Obj_c, Obj_1, Obj_2)	(556.32, -333.59, 854.54)	(567.55, -322.36, 854.54)	(560.82, -329.09, 854.54)	(556.32, -333.59, 854.54)
Instrument values (t_{rad}, t_m)	(3.51, 2.33)	(3.51, 2.33)	(3.51, 2.33)	(3.51, 2.33)
No. of extra measures	-	-	2	2

Refinement of Case Study 3: User-class-based instruments

The two link-based instruments i.e., t_{rad} and t_m are replaced by four link-ODU triplet-based instruments i.e., $t_{rad}^{1,(r,s)}$, $t_{rad}^{2,(r,s)}$, $t_m^{1,(r,s)}$ and $t_m^{2,(r,s)}$ for all $(r, s) \in D$. The solution of central optimization is shown in **Table 9**. Obj_c is remarkably better than the previous case. Thus, in presence of different user-classes in the transportation system, instruments that discriminate between them can lead to significant gains for the governments.

Table 9: Results for Case Study 2: User-class differentiated tolls.

Metric\Case	Original Solution	MUR Solution	MUR Solution with metering	MUR Solution with blocking
Obj. Values (Obj_c, Obj_1, Obj_2)	(231.43, -660.54, 854.85)	(242.74, -649.23, 854.85)	(235.93, -656.04, 854.85)	(231.43, -660.54, 854.85)
Instrument values ($t_{rad}^{1,(r,s)}$, $t_m^{1,(r,s)}$, $t_{rad}^{2,(r,s)}$, $t_m^{2,(r,s)}$)	(3.33, 2.32, 10, 10)	(3.33, 2.32, 10, 10)	(3.33, 2.32, 10, 10)	(3.33, 2.32, 10, 10)
No. of extra measures	-	-	2	2

3. CONCLUSIONS

We presented an optimization-based game-theoretical model of road pricing, which incorporates elastic user-demand, multiple user-classes, and an endogenized combined demand and path-choice user-equilibrium. Single-player optimization problem and the frameworks for Nash-Cournot and Stackelberg games are elaborated. We also presented a post-processing method to address the issue of non-unique path flows in user-equilibrium.

The demonstration via the case studies serves as a motivation to apply this model to pseudo-real case studies. Moreover, owing to the endogenized demand and path-choice user equilibrium, such a model serves as a candidate for a metamodel like the one mentioned in (Malik & Tampère, 2023), where it is used to guide search in a full network model.

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