Intermodal Traffic Assignment Modeling: Analytical and Simulation-based Approaches

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SHORT SUMMARY

Urban transportation systems are currently undergoing substantial transformations with the introduction of Shared Mobility Services (SMSs). These services provide convenient alternatives to traditional modes of transport and can help reduce the reliance on private cars. However, their impact on the overall transportation system and on users' choices remains not fully explored, particularly when integrated with public transport (PT). To target this research gap, our study introduces a comprehensive dynamic modeling approach for intermodal transportation systems, integrating SMSs with PT. We propose a user equilibrium flow-based time-independent model to simultaneously handle path and mode choice, passenger-driver matching, and intermodality. Additionally, we use a rolling horizon approach with a trip-based macroscopic simulator to consider the system's dynamics. The proposed framework provides a baseline for analyzing commuters' behavior within multi-modal transportation networks. We apply the proposed framework to an analytical multi-modal network and the results validate the ability of the model to represent all interactions between the modes, formulate users' choices at equilibrium, and capture the dynamics of traffic conditions and demand.

Keywords: shared mobility services, intermodality, user equilibrium, macroscopic simulation, rolling horizon.

1 INTRODUCTION

The growing demand for urban transportation has led to the rise of shared mobility services (SMSs), providing a cost-effective and sustainable mobility option. These services, such as carpooling and ridesharing, offer new opportunities to enhance the transportation system's efficiency through the shared use of vehicles. Integrating these SMSs with public transport (PT) has transformed traveling behavior and created a complex multi-modal system (Pi et al., 2019). On the one hand, commuters face mode and route choices, aiming to reduce travel expenses (Ameli, Lebacque, & Leclercq, 2022). On the other hand, the system owners strive to meet the demand while minimizing overall costs. Modeling these interactions and their impact on user's choices can provide insights for service providers and allow the optimization of mobility services and the system's performance. Numerous studies have focused on modeling multi-modal transportation systems, incorporating SMSs. For carpooling, Wang et al. (2021) studied its impacts on traffic congestion. Furthermore, Beojone & Geroliminis (2021) and Alisoltani et al. (2022) analyzed the efficiency of ridesharing in reducing traffic congestion in large-scale cities.

However, a less explored aspect concerns modeling and evaluating intermodality, i.e., combining modes to accomplish one trip. Particularly, intermodality involving PT and SMSs for first- and last-mile transportation is seldom explored. In particular, Zhu et al. (2020) investigated the impacts of ridesharing on PT ridership. In Pi et al. (2019) and Du et al. (2022), the authors proposed frameworks where commuters access metro stations via bus, car, or e-hailing. Ridesharing was not considered, and the passenger-driver matching for carpooling is not addressed in Pi et al. (2019). The lack of integration between SMSs and PT, especially in terms of intermodality, results in inaccurate traffic predictions and inefficiencies in planning (Zhu et al., 2020). Addressing this gap requires the development of comprehensive transportation models considering all travel options. By integrating SMSs and PT into a unified framework and providing accurate information on costs and travel times, the expenses of both commuters and the urban transportation system can be optimized.

In this paper, we propose a comprehensive modeling approach for multi-modal transportation systems, integrating personal transportation means, public transport, and shared mobility services.

We consider intermodality by enabling the combination of any of these travel modes. We provide a flow-based static model and represent the commuters' choices through two traffic assignment principles: User Equilibrium (UE) and System Optimum (SO). In UE, individual costs are minimized, reaching equilibrium when no commuter is willing to change their choices for lower costs. SO is a system-wide equilibrium, minimizing the total costs. Additionally, we provide a trip-based dynamic model using the Macroscopic Fundamental Diagram (MFD) and a rolling horizon technique to allow the modeling of traffic and commuters' behavior evolution over time.

2 Methodology

This section presents the multi-modal urban transportation modeling approach, illustrated by Figure 1. The **mode choice and traffic assignment** module assigns the travel demand upon the available modes and paths while handling intermodality by optimizing the transfers. The **SMSs optimizer** receives the travel requests and handles the passenger-driver matching for carpooling and ridesharing. The **traffic simulator** assesses the outputs of the two previous modules and updates the estimation of travel and waiting times for the new iteration.

For an effective model, addressing all three modules simultaneously is crucial. However, challenges arise for dynamic scenarios and large-scale instances, impacting passenger-driver matching algorithms and mode choice efficiency. These complexities escalate when addressing intermodality. Acknowledging these challenges, we introduce a flow-based, time-independent model to validate the methodology's effectiveness for capturing intermodality by an analytical approach. Additionally, a trip-based, time-dependent framework using a rolling horizon technique extends the static model to capture the dynamics of travel demand and traffic conditions.
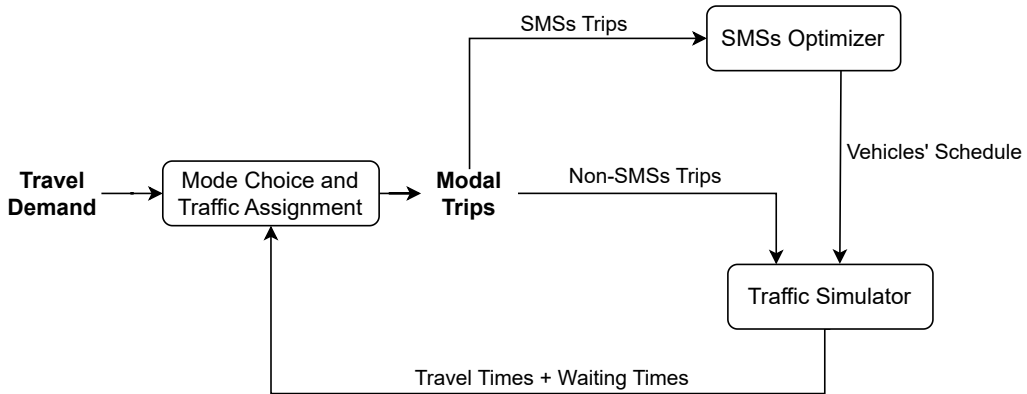

Figure 1: Modeling framework for multi-modal transportation systems

Flow-based Time-independent Model

Let us consider an urban transportation network represented as a directed graph $G(E, A)$, wherein links (A) represent routes, and nodes (E) represent intersections. Let $q^{(i,j)}$ be the travel demand between an Origin-Destination (OD) pair (i, j), $\forall i, j \in E$. We define Ψ as the set of all travel modes illustrated in Figure 2. Commuters can take their car, walk (W), or bike (B) to their destination, be a bus or metro (M) passenger, carpooling driver (CD), or carpooling passenger (CP) as long as the matching takes place, e-hailing passenger (EH) or ridesharing passenger (RS) and thus, be matched with a service vehicle. Additionally, they can choose to combine these travel modes to complement PT.

Commuters traveling between (i, j) simultaneously choose the mode and path to minimize their cost (UE). We define the generalized cost as the sum of travel (t), waiting (WT), service (ST), and monetary (C) costs (Pi et al., 2019). The problem is formulated as a non-convex Mixed-Integer Quadratic Program, and solved through the Beckmann formulation, as described by the program (UE-CMF). All notations are presented in Table 1.

$$\min_{x_a, x_{a,m}} Z_{UE} = \sum_{n \in N} \sum_{a \in A_n} \int_0^{x_a} \alpha \cdot t_{a,n}(\omega) \cdot d\omega \quad + \quad \sum_{a \in A} \sum_{m \in \Psi} [\alpha(WT_{m,a} + ST_{m,a}) + C_{m,a}] \cdot x_{a,m} \quad \text{(UE-CMF)}$$
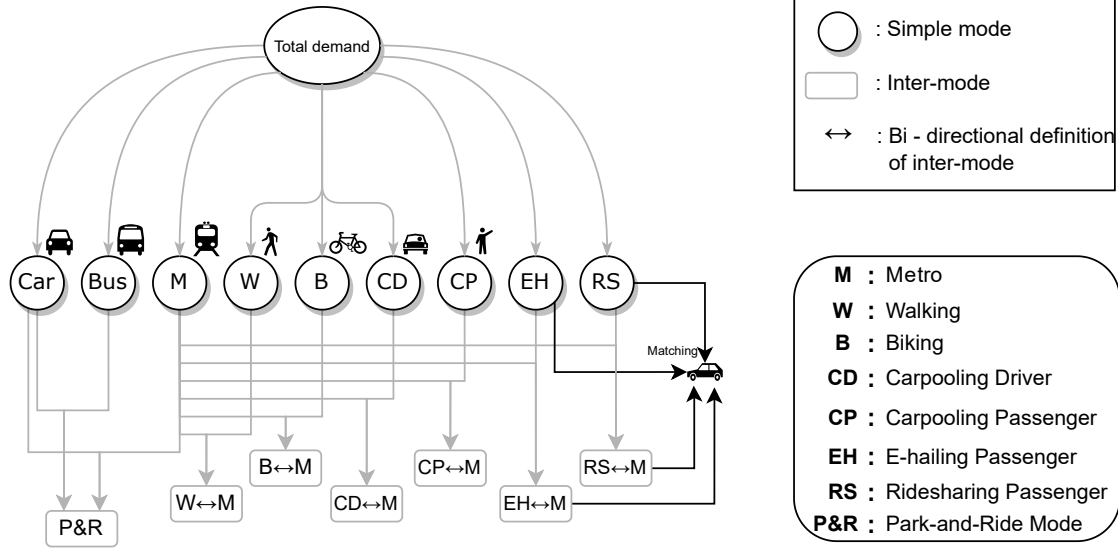
Figure 2: Available travel options for multi-modal networks.

Cost Functions

$$t_{a,n}(x) = \begin{cases} BPR(x) & if\ n = RN \\ \frac{L_a}{Sp_n} & if\ n \in \{MN, WN, BN\} \end{cases} \tag{1}$$

$$WT_{m,a} = \begin{cases} 0 & if\ m \in \{car, CD, W, B\} \\ \frac{1}{2freq_{m,a}} \cdot o_a & if\ m \in \{Bus, M\} \\ \frac{q_m}{R_m} \cdot o_a & if\ m \in \{CP, RS, EH\} \end{cases} \tag{2}$$

$$ST_{m,a} = \begin{cases} 0 & if\ m = W \\ S_m & if\ m \in \{Bus, M\} \\ S_m \cdot (o_a + d_a) & if\ m \in \{CP, RS, EH\} \\ S_m \cdot (o_a + d_a) + P_1 \cdot d_a & if\ m = CD \\ P_m \cdot d_a & if\ m \in \{car, B\} \end{cases} \tag{3}$$

$$C_{m,a} = \begin{cases} \gamma \cdot L_a + PF_m \cdot d_a & if\ m = car \\ TF_{m,a} & if\ m \in \{Bus, M, CP, EH, RS\} \\ \gamma \cdot L_a + PF_m \cdot d_a - TF_{m,a} & if\ m = CD \\ 0 & if\ m \in \{W, B\} \end{cases} \tag{4}$$

Constraints

$$q^{(i,j)} = \sum_{m \in \Psi} \sum_{p \in P_{ij}^m} f_{p,m}^{(i,j)} \qquad\qquad \forall i, j \in E \tag{5}$$

$$f_{p,CD}^{(i,j)} = \sum_{r \in E} f_{p,CD}^{(i,j)\ r} \qquad\qquad \forall i, j \in E;\ \forall p \in P_{ij}^{CD} \tag{6}$$

$$f_{p,CP}^{(i,j)} \leq CAP_{CP} \cdot \Big[\sum_{r,s \in E} \sum_{l \in Q_p^{(r,s)}} f_{l,CD}^{(r,s)\ i,j} \Big] \qquad\qquad \forall i, j \in E\quad;\ \forall p \in P_{ij}^{CP} \tag{7}$$

$$f_{l,CD}^{(i,j)\ r,s} \leq f_{p,CP}^{(r,s)} + \sum_{I(CP) \in \Psi} \sum_{u,v \in E} \sum_{k \in X} f_{k,I(CP)}^{(u,v)} \qquad \forall r, s, i, j \in E;\ \forall l \in P_{ij}^{CD};\ \forall p \in R_l^{(r,s)} \tag{8}$$

$$f_{p,EH}^{(i,j)} = y_{p,EH}^{(i,j)} \qquad\qquad \forall i, j \in E\quad;\ \forall p \in P_{ij}^{EH} \tag{9}$$

$$y_{p,RS}^{(i,j)} = \sum_{r \in E} y_{p,RS}^{(i,j)\ r} \qquad\qquad \forall i, j \in E;\ \forall p \in P_{ij}^{RS} \tag{10}$$

$$f_{p,RS}^{(i,j)} \leq CAP_{RS} \cdot \Big[\sum_{r,s \in E} \sum_{l \in Q_p^{(r,s)}} (y_{l,RS}^{r,i,j,s} + y_{l,RS}^{r,i,s,j}) \Big] \qquad\qquad \forall i, j \in E;\ \forall p \in P_{ij}^{RS} \tag{11}$$

$$f_{p,I(RS)}^{(i,j)} \leq \sum_{r,s \in E} \sum_{k \in Q_p^{(r,s)}} (y_{k,RS}^{r,i,TR(p),s} + y_{k,RS}^{r,TR(p),j,s}) \qquad \forall i,j \in E \quad ; \forall p \in P_{ij}^{I(RS)} \quad (12)$$

$$y_{l,RS}^{i,r,s,j} + y_{l,RS}^{i,r,j,s} \leq f_{p,RS}^{(r,s)} \qquad \forall r,s,i,j \in E; \forall l \in P_{ij}^{RS}; \forall p \in R_p^{(r,s)} \quad (13)$$

$$f_{p,RS}^{(i,j)} \geq 2 \cdot y_{p,RS}^{i,i,j,j} + \sum_{r,s \in E} \sum_{l \in Q_p^{(r,s)}} (y_{l,RS}^{r,i,j,s} + y_{l,RS}^{r,i,s,j}) \qquad \forall i,j \in E; \forall p \in P_{ij}^{RS} \quad (14)$$

$$q_o^{(i,j)\ r} = \sum_{p \in P_{ij}^{RS}} y_{p,RS}^{i,r,s,j} + y_{p,RS}^{i,s,r,j} \qquad \forall i,j,r \in E \quad (15)$$

$$q_o^{(i,j)} = \sum_{r \in E} q_o^{(i,j)\ r} + \sum_{p \in P_{ij}^{EH}} y_{p,EH}^{(i,j)} \qquad \forall i,j \in E \quad (16)$$

$$q_e^{(i,j)} = \sum_{p \in P_{ij}^e} y_{p,e}^{(i,j)} \qquad \forall i,j \in E \quad (17)$$

$$\sum_{i,j \in E} q_o^{(i,j)\ s} + \sum_{j \in E} q_e^{(j,s)} = \sum_{j \in E} q_o^{(s,j)} + \sum_{j \in E} q_e^{(s,j)} \qquad \forall s \in D \quad (18)$$

$$\sum_{i \in E} q_e^{(i,j)} \geq \sum_{s \in E} q_o^{(j,s)} \qquad \forall j \in O \quad (19)$$

$$\sum_{i,j \in E} q_o^{(i,j)} + q_e^{(i,j)} = |V| \qquad (20)$$

$$\sum_{i,j \in E} \sum_{p \in PI((i,j),s)} f_{p,I_{car,M}}^{(i,j)} + f_{p,I_{CD,M}}^{(i,j)} \leq Pk\_cap_s \qquad \forall s \in TR \quad (21)$$

$$x_{a,m} = \sum_{i,j \in E} \left(\sum_{p \in P_{ij}^m} f_{p,m}^{(i,j)} + \sum_{I(m) \in \Psi} \sum_{k \in P_{ij}^{I(m)}} f_{k,I(m)}^{(i,j)} \right) \cdot \delta_{a,p,m}^{(i,j)} \qquad \forall a \in A; \forall m \in \Psi \quad (22)$$

$$x_a = \sum_{m \in \Psi'} x_{a,m} + \sum_{i,j \in E} \sum_{p \in P_{ij}^{RS}} \sum_{t \in T} y_{p,t}^{(i,j)} \cdot \delta_{a,p,t}^{(i,j)} \qquad \forall a \in A; \ T = \{RS, EH, e\} \quad (23)$$

$$f_{p,m}^{(i,j)}, f_{p,m}^{(i,j)r,s} \geq 0 \qquad \forall p \in P_{ij}^m; \forall i,j,r,s \in E; \forall m \in \Psi \quad (24)$$

$$y_{p,m}^{(i,j)} \geq 0 \qquad \forall p \in P_{ij}^m; \forall i,j \in E; \forall m \in \{EH, RS, e\} \quad (25)$$

$$y_{p,RS}^{i,r,s,j} \geq 0 \qquad \forall p \in P_{ij}^{RS}; \forall i,j,r,s \in E \quad (26)$$

In (UE-CMF), Equation (1) denotes the travel time on link a in network n. For the road network, we use the Bureau of Public Roads (BPR) function in the static configuration. Non-vehicular travel time relies on link length and average speed of walking, biking, or metro. Equation (2) defines the waiting time of mode m on link a. Commuters experience waiting times at the trip's start. For PT, the waiting time is assumed as half of the headway, while for SMSs, it depends on the passengers count and a meeting rate (Nourinejad & Ramezani, 2020). Equation (3) defines the service time of mode m on link a. For PT, it denotes the average time at each stop. For car or bike users, it represents the parking time. For SMSs, it includes matching time with a driver/passenger and the boarding and drop-off times. Equation (4) denotes the monetary cost of mode m on link a. Car drivers' cost comprises fuel and parking fare, while other passengers' cost reflects the trip fare to be paid.

Constraint (5) represents the demand conservation. Constraints (6)-(8) ensure the correspondence between the number of drivers and passengers per OD pair and path p. Similarly, Constraints (9)-(13) are for the e-hailing and ridesharing services. Constraint (14) ensures that a ridesharing trip is shared (not an e-hailing service). Constraints (15) and (16) express occupied fleet cars as the sum of those in e-hailing and ridesharing services. Constraint (17) defines the number of empty vehicles circulating between (i,j). Constraint (18) ensures, for destination nodes, that the number of entering vehicles equals the number of exiting vehicles. Constraint (19) ensures enough empty cars for the e-hailing and ridesharing passengers at the origin nodes. Constraint (20) ensures the fleet size ($|V|$) conservation. Constraint (21) limits car commuters in inter-mode transfers at node s to the parking capacity. Constraints (22) and (23) present the link-path flow conversion, and (24)-(26) are integrality conditions.

The model simultaneously addresses mode choice, traffic assignment, and passenger-driver matching per OD pair. It includes choosing intermodality transfer location in path selection, creating a unified framework for complete travel patterns. This model is valid for analyzing scenarios within a specific time period, during which travel demand can be given and remains stable. We prove that this analytical model can be solved with the Beckmann transformation to obtain an exact equilibrium solution at the link level. We refer the reader to Kadem et al. (2024a) for a more detailed description of the model and discussion of its theoretical aspects.

Table 1: **List of notations.**

Network Structure

E	Set of nodes.
O	Set of origin nodes.
D	Set of destination nodes.
TR	Set of transfer nodes.
N	Set of networks ; $N = \{Road(RN),\ Metro(MN),\ Bike(BN),\ Walk(WN)\}$.
A_n	Set of links in network n.
A	Set of links: $A = \cup_{n \in N} A_n$.
Ψ	Set of all travel modes ; $\Psi = \{$ car, bus, M, CP, CD, EH, RS, W, B, $I_{m1,m2}$ $\}$ where $I_{m1,m2}$ is a combination of mode $m1$ and $m2$.
Ψ'	Set of travel modes that contribute to traffic congestion; $\Psi' = \{$ car, bus, CD $\}$
V	Set of fleet vehicles for ridesharing and e-hailing services.
P_{ij}^m	Set of paths for origin-destination (OD) pair (i,j) with mode m.
o_a	Binary coefficient equal to 1 if link a is an origin link ; 0 Otherwise.
d_a	Binary coefficient equal to 1 if link a is a destination link ; 0 Otherwise.
$\delta_{a,p,m}^{(i,j)}$	Binary coefficient equal to 1 if path p of mode m traverse link a between OD pair (i,j) ; 0 Otherwise.
$q^{(i,j)}$	Travel demand for OD pair (i,j).
Q	Total demand for the network.
$Q_p^{(r,s)}$	Set of paths l between (r,s), where the path p is a sub-path of l.
$R_p^{(r,s)}$	Set of paths l (r,s), where l is a sub-path of p.
$I(m)$	Intermode using mode m as a first-mile or last-mile option.
$TR(p)$	Transfer node in the intermodal path p.

Indices

i,j,r,s	Index of node, $i,j,r,s \in E$.
a	Index of link (edge), $a \in A$.
m	Index of mode, $m \in \Psi$.
p	Index of path, $p \in P_{ij}^m$.
n	Index of network, $n \in N$.

Input Parameters

L_a	Length of link a.
Sp_n	Mean speed in network n.
$freq_{m,a}$	Frequency of travel mode m on link a (vehicles/time unit).
R_m	Meeting rate of travel mode m (service/time unit).
S_m	Average service time for mode m.
P_m	Average parking time for mode m.
$TF_{m,a}$	Trip fare of travel mode m on link a.
PF_m	Parking fare for travel mode m.
CAP_m	Maximum passenger capacity for the travel mode m.
Pk_cap_s	Parking capacity for transfer node s.
$|V|$	Fleet size for the ridesharing and e-hailing services.
α	Monetary cost per unit of time (monetary unit/time unit).
γ	Monetary cost per unit of distance (monetary unit/distance unit).

Variables

$t_{a,n}$	Travel time on link a belonging to network n.
$WT_{m,a}$	Waiting time for travel mode m on link a .
$ST_{m,a}$	Service time for travel mode m on link a.
$C_{m,a}$	Monetary cost of using travel mode m on link a.
x_a	Aggregated traffic flow on link a.
$x_{a,m}$	Aggregated traffic flow on link a with travel mode m.
q_m	Travel demand for mode m.
$q_m^{(i,j)}$	Travel demand for mode m and OD pair (i,j).
$q_t^{(i,j)}$	Number of empty or occupied service vehicles between OD pair (i,j) ; $t \in \{e,o\}$.
$q_o^{(i,j)\ r}$	Number of occupied service vehicles between OD pair (i,j) stopping at node r.
$f_{p,m}^{(i,j)}$	Traffic flow of mode m on path p between OD pair (i,j).
$f_{p,CD}^{(i,j)r,s}$	Traffic flow of carpooling drivers on path p between (i,j) stopping at nodes r and s.
$y_{p,t}^{(i,j)}$	Flow of empty or occupied service vehicles on path p between OD pair (i,j) ; $t \in \{e,EH,RS\}$.
$y_{p,RS}^{i,r,s,j}$	Flow of ridesharing occupied vehicles on path p stopping at nodes i,r,s and j, in this order.
$c_{p,m}^{(i,j)}$	Generalized cost of path p with travel mode m between OD pair (i,j).

Trip-based Macroscopic Approach

This section proposes a dynamic approach to represent multi-modal transportation systems. The **mode choice and traffic assignment** and the **SMSs Optimizer** modules of Figure 1 are handled by the flow-based model, replacing the BPR with an estimated mean speed. We use the trip-based MFD for the **traffic simulation**. The two models are integrated through a rolling horizon technique (Alisoltani et al., 2021), in which one iteration refers to the feedback phase between the **traffic simulation** and the **mode choice and traffic assignment**. Thus, the assignment model is solved in every iteration using estimated travel times for the prediction period (T_p time unit). The traffic simulation is carried out to evaluate the assignment solution and update the prediction. A re-optimization is performed every T_s time unit to consider the dynamics of demand and traffic conditions. As an initial step and to validate the methodology, we keep T_p as the full time horizon for every iteration wherein we optimize mode choice and traffic assignment. In the trip-based MFD, each trip has a remaining length, updated based on the speed with respect to the vehicle accumulation at time t (Daganzo & Geroliminis, 2008). The trips with a passenger role are not considered in the accumulation (Ameli, Faradonbeh, et al., 2022). We represent PT with a frequency-based service in which buses use the same road infrastructure as cars. The waiting time for SMSs passengers is the time between their departure time and the actual pickup. Intermodal trips are a sequence of modal trips. Thus, the commuter's role is updated dynamically. Consider a commuter departing at time t, using carpooling for distance l_1, then the metro for l_2. The trip begins effectively at t_1^s when picked up. l_1 is updated by the MFD, reaching zero at time t_1^e (i.e. the transfer location). The commuter shifts from carpooling to the metro at t_2^s, updating l_2 based on metro speed until reaching the destination at t_2^e.

3 RESULTS AND DISCUSSION

In this section, we first solve the static model and analyze the mode choices under UE and SO to offer valuable insights into commuters' decision-making process and the influencing factors. Then, we show the applicability of our model in a dynamic context. For validation, we consider the network illustrated by Figure 3.
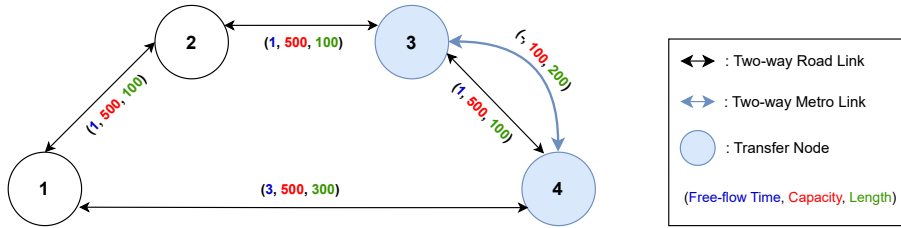

Figure 3: A multi-modal synthetic network with twelve OD pairs.

Analysis of mode choice

We consider a given OD demand and calculate the UE (UE-CMF) with the Gurobi Solver. Additionally, we formulate the SO by replacing the objective function with the Equation (27) and compare the use of modes under these two principles. All parameters are similar to our previous study in Kadem et al. (2024b).

$$\min_{x_a, x_{a,m}} Z_{SO} = \sum_{n \in N} \sum_{a \in A_n} \alpha \cdot t_{a,n}(x_a) \cdot x_a \quad + \quad \sum_{a \in A} \sum_{m \in \Psi} [\alpha(WT_{m,a} + ST_{m,a}) + C_{m,a}] \cdot x_{a,m} \quad (27)$$

Figure 4 presents the use of modes. With UE traffic assignment, 36.6% of commuters use PT due to its low cost. For OD pairs without direct PT connections or with saturated PT links, commuters use SMSs. 10.8% of carpooling passengers have door-to-door trips, and 10% participate in ridesharing, while 15.7% use CD mode. Drivers face high costs with their cars but can benefit from having a passenger onboard to decrease expenses. Sharing can occur with a drop-off at a transfer location if intermodality is used. Particularly, intermodality with SMSs represents 26.6% of trips. Additionally, ridesharing is mainly used for short trips (door-to-door or combined with metro

for short distances), while carpooling is chosen for long-distance trips. This is due to ridesharing's high fares, which increase with trip length.

When traffic is assigned under SO, ridesharing is not used. Our SO objective function excludes the service provider's profits. Thus, the system aims to reduce both passengers' and drivers' costs. Increasing the number of CP will increase the number of CD accordingly, and attract the ridesharing demand. These observations suggest that encouraging more carpooling drivers, especially through intermodality, allows the shift from UE towards the SO state. The experiment demonstrates the static model's ability to calculate UE in multi-modal systems and its applicability to the assignment process with a given OD demand.
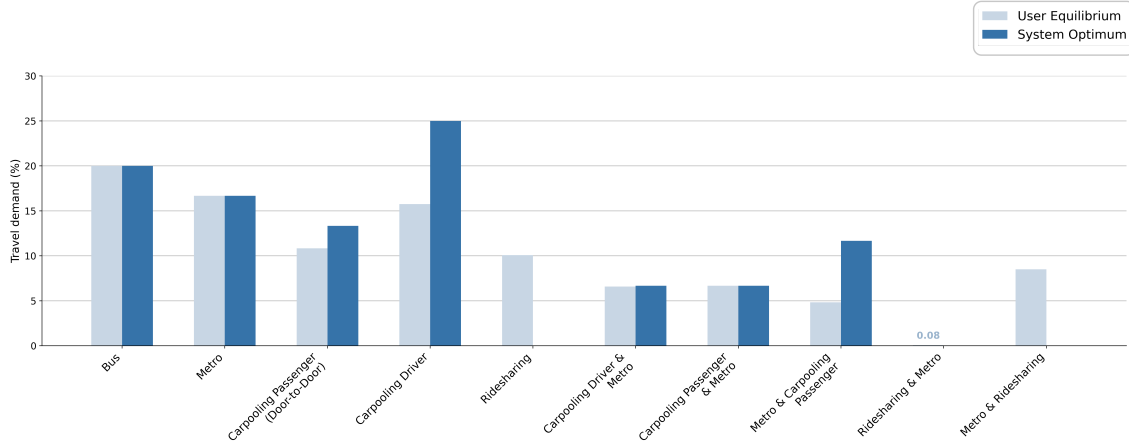

Figure 4: Modes usage in the synthetic multi-modal network.

Validation in the dynamic context

To consider traffic dynamics, we use the rolling horizon technique (Ameli, 2019) and assume convergence when speed estimation remains consistent across the iterations (section 2). We consider the first two iterations for the same time slot, and presents the changes in modes in Figure 5. In the first iteration, the road speed was over-estimated. Thus, 32% of commuters use their cars and participate as a carpooling driver (CD). The simulation phase then adjusts the speed estimation, and commuters adapt their mode in the second iteration. We notice an increase in the metro mode and a decrease in the CD mode. The explanation is that most drivers notice that using the road network, especially for a long distance, is time-consuming. Thus, they use PT and/or SMSs as a first-mile option instead of a door-to-door service (CD&M increases by 8%). The experiment shows that dynamic traffic patterns in each time slot align with the static configuration regarding mode use and equilibrium state. The rolling horizon can thus consider the traffic dynamics and adapt the mode and path choices accordingly. However, an extended analysis is required for the effects of traffic congestion on mode choices to address large-scale scenarios.

4 Conclusion

This research proposed a comprehensive modeling framework for multi-modal urban transportation networks, including shared mobility services (SMSs) and public transport (PT). We integrate a flow-based time-independent model with a trip-based macroscopic simulator through a rolling horizon technique to represent the evolution of traffic and commuters' choices over time. Addressing the impacts of SMSs on transportation systems and travelers' choices when integrated with PT, our model fills existing gaps by providing an integrated framework for evaluating, combining, and optimizing travel options.

Preliminary results reveal that carpooling is preferred over ridesharing when PT is not accessible. However, ridesharing becomes more attractive in high-demand scenarios due to increased waiting times for the carpooling service. Additionally, carpooling attracts more commuters for long-distance trips compared to ridesharing. Future research aims to apply the approach to realistic cities, like Paris or Lyon in France, providing insights into commuters' choices and system
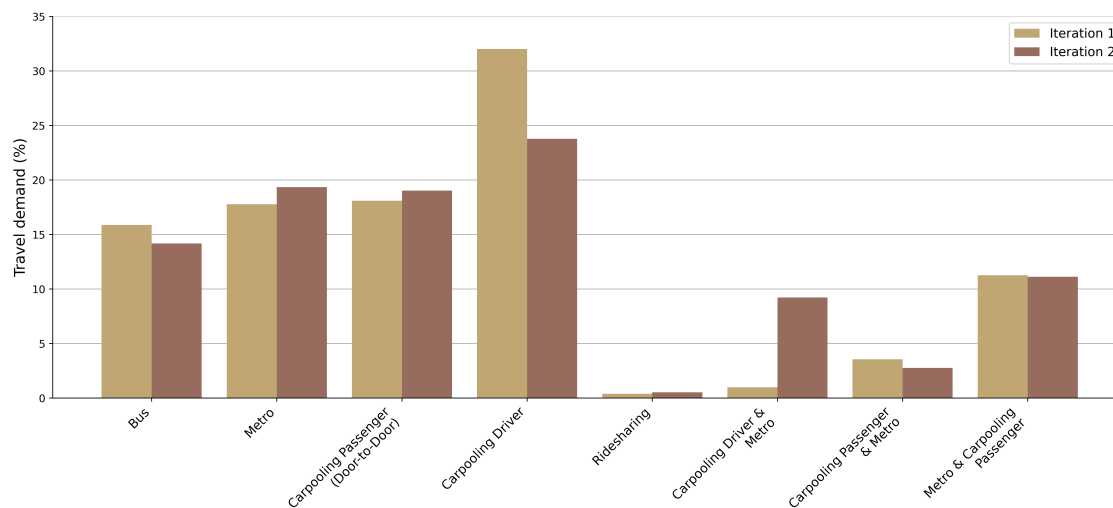
Figure 5: Evolution of the use of modes in a dynamic context.

evolution. Considering stochastic user equilibrium in the formulation is also interesting to explore.

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