The Fragile Nature of Road Transportation Systems

Linghang Sun*[1], Yifan Zhang[1], Cristian Axenie[2], Margherita Grossi[3], Anastasios Kouvelas[1], and Michail A. Makridis[1]

[1]Institute for Transport Planning and Systems, ETH Zurich, Switzerland
[2]Computer Science Department and Center for Artificial Intelligence, Technische Hochschule Nürnberg, Germany
[3]Intelligent Cloud Technologies Lab, Huawei Munich Research Center, Germany

Short summary

Traffic demand has been increasing continuously and presents a substantial challenge to the efficiency of traffic control strategies. Meanwhile, the operation of transportation systems is widely believed to display fragile properties, i.e., the loss in performance increases exponentially with the linearly increasing magnitude of disruptions. Currently, the risk engineering community is embracing the novel concept of (anti-)fragility, which enables systems to learn from historical disruptions and exhibit improved performance as disruption levels reach unprecedented magnitudes. In this study, we demonstrate the fragile nature of road transportation systems under demand or supply disruptions. First, we conducted a rigorous mathematical analysis to establish the fragile nature theoretically. Then, by considering real-world stochasticity, we implemented a numerical simulation to bridge the gap between the theoretical proof and the real-world operations. This work aims to help researchers better comprehend the necessity to consider antifragile design for the application of future traffic control strategies.
Keywords: (anti-)fragility, road transportation systems, macroscopic fundamental diagram, model stochasticity.

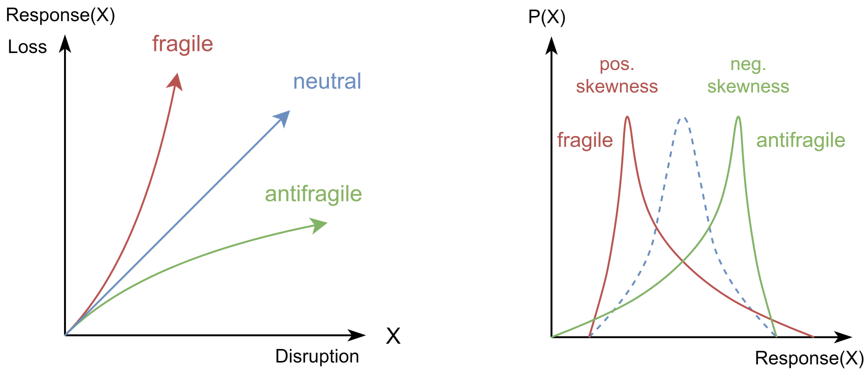
1 Introduction

As reported by the U.S. Department of Transportation, motorized road traffic before the pandemic has experienced an approximate 50% growth over the past few decades. This continuous growth in traffic volume has consequently contributed to a rise in disruptive events, such as severe congestion and more frequent accidents. Meanwhile, there is a common understanding that road transportation networks can exhibit fragile properties. Fragility signifies a system's susceptibility to exponentially worsening performance as disruptions increase in magnitude. One prominent example is the BPR function, which distinctly illustrates that travel time grows exponentially with traffic flow with empirical data at the link level.
Therefore, we introduce the cutting-edge concept of (anti-)fragility to explain this phenomenon. The concept of (anti-)fragility was initially proposed by the famous essayist, and mathematical statistician, Nassim Taleb, in his bestseller *Antifragile: things that gain from disorder* and mathematically elaborated in Taleb & Douady (2013). It serves as a general concept aimed at transforming people's understanding and perception of risk. With antifragility, systems and people can benefit from disruptions and perform better under growing volatility and randomness. Ever since being proposed, antifragility has gained popularity in the risk engineering community across multiple disciplines, such as economy, biology, energy, and robotics.
As demonstrated by the BPR function, previous studies discussing this fragile response have primarily relied on intuitive reasoning and empirical data rather than rigorous mathematical analysis. This paper serves as a proof of concept, aiming to establish the fragile nature of road transportation systems through mathematical analysis. On the microscopic level, we select the two most representative Fundamental Diagrams (FDs), the numerical second-degree polynomial FD introduced in Greenshields et al. (1934) and the analytical one in Daganzo (1994) characterized by two linear functions. On the macroscopic level, we also apply one numerical third-degree Macroscopic Fundamental Diagram (MFD) as introduced in Geroliminis et al. (2013) and the analytical MFD generated through Method of Cuts (MoC) as in Daganzo & Geroliminis (2008). Additionally, as

stochasticity prevails in transportation systems in the real world, we designed a numerical simulation considering real-world stochasticity to study to what extent such realistic uncertainties can influence the fragile characteristics of transportation systems. The overarching objective of this paper is to provide insights to transportation researchers for the future design of transportation systems and control strategies to be not only robust and resilient but also antifragile.

2 Problem formulation

An (anti-)fragile response of a system can be characterized through a nonlinear relationship between the performance loss and the magnitude of the disruption, as shown in Fig. 1(a). Both nonlinear functions can be represented by Jensen's inequality, with either $E[g(X)] \geq g(E[X])$ for a fragile response or $E[g(X)] \leq g(E[X])$ for an antifragile response. This relationship can then be determined through the second derivative, i.e., a positive second derivative featuring a convex function and hence a fragile system.
However, in most real-world scenarios, the mathematical function of the system is unknown, and only discrete measurements of the system's performance are available. In this case, we can calculate the distribution skewness to determine the (anti-)fragile property of the system, and a negative skewness indicates an antifragile response, as shown in Figure 1(b).



(a) Nonlinear relationship (b) Distribution skewness

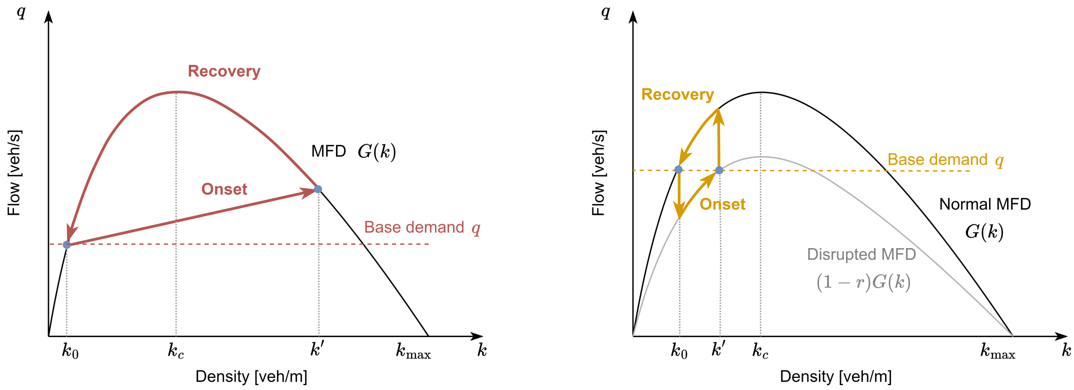
Figure 1: Characteristics and identification of (anti-)fragility

In this paper, we address three sets of opposing concepts for the analysis:

- microscopic / macroscopic
- demand disruption / supply (MFD) disruption
- onset of disruption / recovery from disruption

A demand disruption can be surging traffic due to social events, whereas a supply disruption may indicate a compromised network performance due to adversarial weather or lane closure. Additionally, we consider both the onset of disruptions and the recovery process, as illustrated in Figure 2(a) and 2(b). We denote the FD/MFD profile as $G(k)$ and assume a constant base demand in the network as q. The initial density, critical density, the new density after disruption, and the gridlock density are donated as k_0, k_c, k', and $k_{\max}$. For supply disruption, we introduce a disruption magnitude coefficient r so that the disrupted MFD profile can be represented as $(1-r)G(k)$. On the network level, MFD can also be represented with vehicle accumulation - trip completion instead of traffic flow - density. Hence, with vehicle accumulation denoted as n, $G(k_*)$ can also be replace with $G(n_*)$.
Several assumptions need to be established to define the scope of our study. A critical condition to avoid is the network succumbing to a complete gridlock, where recovery is not possible anymore.

1. For demand disruptions, $k' > k_c$, whereas for supply disruptions, $k' < k_c$.
   A surging demand is considered a disruption only when its presence leads to a reduction in the network's serviceability. For supply disruption with the constant base demand, this assumption aims to avoid gridlock.



(a) Onset and recovery of a demand disruption

(b) Onset and recovery of a supply disruption

Figure 2: Onset and recovery of disruptions

2. For demand disruptions, $q < G(k')$, whereas for supply disruptions, $q < (1-r)G(k_c)$. Likewise, the necessity of this assumption also lies in the avoidance of gridlock for both demand and supply disruptions.
3. The onset of the disruptions can happen instantaneously, while the recovery from disruptions is a gradual process.

3 Mathematical proof for the fragility of road transportation systems

In this section, we conduct mathematical proof to demonstrate the fragile nature of the transportation systems on both the microscopic and the macroscopic levels. In the following study, the indicator to study the instantaneous disruption onset and between different stable states is the Average Time Spent (ATS), as ATS is the same for each vehicle that entered the network. On the other hand, for the study of the disruption recovery, we use Total Time Spent (TTS), to better reflect the overall temporal costs of all the vehicles in this process.
To illustrate the system's fragility to demand disruption, we analyze the derivatives of time spent relative to the initial disruption demand, either represented by disruption density k', or disruption vehicle accumulation n'. For supply disruptions, establishing a positive second derivative of time spent concerning the magnitude of MFD disruption r would demonstrate the fragility. After the supply disruption, the new equilibrium point is $q = (1-r)G(k'(r))$.

Proposition 1. *Road transportation systems are fragile with the onset of demand disruptions on the microscopic level.*

Proof. For the Greenshields FD, the following equations describe traffic in a stable state. The traffic flow, density, and speed are denoted as q, k, and v respectively, while a and b are polynomial coefficients.

$$G(k) = ak^2 + bk \tag{1}$$

$$v(k) = \frac{q}{k} = ak + b \tag{2}$$

With the sudden onset of a demand disruption k', for a link with a given length of L, the ATS and its first and second derivatives over such disruption are:

$$ATS = \frac{L}{v(k')} = \frac{L}{ak' + b} \tag{3}$$

$$\frac{dATS}{dk'} = -aL(ak' + b)^{-2} \tag{4}$$

$$\frac{d^2ATS}{dk'^2} = 2a^2L(ak' + b)^{-3} \tag{5}$$

While a is negative, $ak' + b$ has a physical meaning of the average speed and should always be non-negative, thus, the derivatives are positive, indicating a fragile response.
In the Daganzo two-regime FD, the speed can be formulated as the following Eq. 6.

$$v(k) = \begin{cases} v_f, & 0 \leq k < k_c \\ w + \frac{c}{k}, & k_c \leq k \leq k_{\max} \end{cases} \tag{6}$$

When the disruption density k' is below the critical density k_c, the ATS and its derivatives are:

$$ATS = \frac{L}{v_f} \tag{7}$$

$$\frac{dATS}{dk'} = 0 \tag{8}$$

$$\frac{d^2ATS}{dk'^2} = 0 \tag{9}$$

It indicates the traffic states before k_c are neither fragile nor antifragile. However, as per Assumption 1, the congested area of the MFD is the study focus for demand disruptions, now we calculate the derivatives when k' is over k_c:

$$ATS = \frac{L}{v(k')} = \frac{L}{w + \frac{c}{k'}} \tag{10}$$

$$\frac{dATS}{dk'} = \frac{cL}{(wk' + c)^2} \tag{11}$$

$$\frac{d^2ATS}{dk'^2} = \frac{-2wcL}{(wk' + c)^3} \tag{12}$$

Before k' reaches $k_{\max}$, $wk' + c > 0$ always holds true, and since $w < 0$ as well as $c > 0$, both derivatives are positive.

□

Proposition 2. *Road transportation systems are fragile with the onset of demand disruptions on the macroscopic level.*

Proof. For MFD approximated with a third-degree polynomial, similar to the Greenshields FD in Eq. 1, we have:

$$G(k) = ak^3 + bk^2 + ck \tag{13}$$

$$v(k) = \frac{q}{k} = ak^2 + bk + c \tag{14}$$

Consequently, the ATS and its derivatives are:

$$ATS = \frac{L}{v(k')} = \frac{L}{ak'^2 + bk' + c} \tag{15}$$

$$\frac{dATS}{dk'} = \frac{-(2ak' + b)L}{(ak'^2 + bk' + c)^2} \tag{16}$$

$$\frac{d^2ATS}{dk'^2} = \frac{\frac{3}{2}(2ak' + b)^2 + \frac{1}{2}(b^2 - 4ac)}{(ak'^2 + bk' + c)^3} L \tag{17}$$

As Eq. 14 should have real roots, indicating the speed to be a real number, so $b^2 - 4ac > 0$ should also hold. Therefore, the derivatives are positive.
In MoC, the MFD can be approximated by a series of linear functions. Likewise to the Daganzo two-regime linear FD as in Eq. 12, for any linear function, the second derivative is:

$$\frac{d^2ATS}{dk'^2} = \frac{-2u_i c_i L}{(u_i k' + c_i)^3} \tag{18}$$

Conforming to Assumption 1, we focus on the cuts with intercepts larger than the critical density k_c ($u_i < 0$). The second derivative for these cuts is positive indicating a fragile property.

□

Proposition 3. *Road transportation systems are fragile when going through the recovery process from demand disruptions.*

Proof. According to Assumption 3, we consider this demand as a disruption that happens instantly in the network, which is denoted as n' at time $t' = 0$. The complete MoC can be represented into multiple sets of consecutive duo linear functions as Figure 3 shows. The four constants a_1, a_2, b_1, and b_2 are the slope and y-intercept on the coordinates, with $a_2 > a_1$ and $b_1 > b_2 > 0$. After a certain period t_c, the number of vehicles in the network reaches this critical accumulation n_c of these two cuts. After any period $t > t_c$, the vehicle accumulation becomes n. We also denote the initial trip completion and critical trip completion as $m_0 = a_1 n' + b_1$ and $m_c = a_1 n_c + b_1 = a_2 n_c + b_2$ respectively.

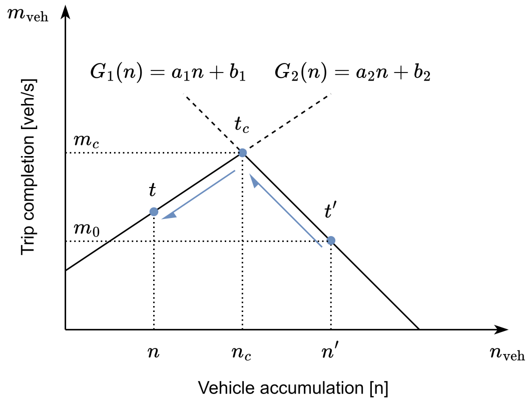


Figure 3: Simplification of MoC

Any two consecutive cuts of the MFD can be formulated into the following Eq. 19:

$$G(n) = \begin{cases} a_1 n + b_1, & n_c \leq n < n_{\max} \\ a_2 n + b_2, & 0 \leq n < n_c \end{cases} \tag{19}$$

The system dynamics can be summarized as:

$$\frac{dn}{dt} = -G(n) + q = -a_i n - b_i + q \tag{20}$$

When the traffic states stay only on a single branch, and given any initial vehicle accumulation n_1 at the beginning of a period from t_1 to t_2, the number of vehicles n_2 at the end of this period can be determined as:

$$\int_{t_1}^{t_2} dt = -\int_{n_1}^{n_2} \frac{1}{a_i n + b_i - q} dn \tag{21}$$

$$t_2 - t_1 = -\frac{1}{a_i} \ln \left(\frac{a_i n_2 + b_i - q}{a_i n_1 + b_i - q} \right) \tag{22}$$

$$n_2 = \frac{e^{-a_i (t_2 - t_1)} (a_i n_1 + b_i - q)}{a_i} - \frac{b_i - q}{a_i} \tag{23}$$

With the disruption accumulation n', and the traffic states are on the same branch. After any time t, the vehicle accumulation n would be:

$$n = \frac{a_1 n' + b_1 - q}{a_1} e^{-a_1 t} - \frac{b_1 - q}{a_1} \tag{24}$$

The TTS can be calculated as:

$$TTS = \int_0^t n dt = \int_0^t \left(\frac{a_1 n' + b_1 - q}{a_1} e^{-a_1 t} - \frac{b_1 - q}{a_1} \right) dt \tag{25}$$

$$= -\frac{a_1 n' + b_1 - q}{a_1^2} e^{-a_1 t} - \frac{b_1 - q}{a_1} t + \frac{a_1 n' + b_1 - q}{a_1^2} \tag{26}$$

Now we calculate the derivatives of TTS considering t as any constant.

$$\frac{dTTS}{dn'} = \frac{1}{a_1} - \frac{e^{-a_1 t}}{a_1} \tag{27}$$

$$\frac{d^2TTS}{dn'^2} = 0 \tag{28}$$

The second derivative of TTS is 0, indicating that when the traffic states move only along a single branch, it shows neither fragility nor antifragility.
When the traffic state goes over the critical vehicle accumulation n_c, we calculate the TTS separately on both the more and the less congested branches, denoted as TTS_1 and TTS_2. Since the critical time t_c is still unknown, we need to determine t_c first, similar to Eq. 22.

$$t_c = -\frac{1}{a_1} \ln\left(\frac{a_1 n_c + b_1 - q}{a_1 n' + b_1 - q}\right) \tag{29}$$

$$t_c = -\frac{1}{a_1} \ln\left(\frac{m_c - q}{a_1 n' + b_1 - q}\right) \tag{30}$$

Likewise to Eq. 26, the TTS_1 for the more congested branch is:

$$TTS_1 = -\frac{a_1 n' + b_1 - q}{a_1^2} e^{-a_1 t_c} - \frac{b_1 - q}{a_1} t_c + \frac{a_1 n' + b_1 - q}{a_1^2} \tag{31}$$

$$= -\frac{m_c - q}{a_1^2} + \frac{b_1 - q}{a_1^2} \ln\left(\frac{m_c - q}{a_1 n' + b_1 - q}\right) + \frac{a_1 n' + b_1 - q}{a_1^2} \tag{32}$$

Since TTS is the sum of both TTS_1 and TTS_2, the second derivative of TTS would also be the sum of the derivatives. The derivatives for TTS_1 are:

$$\frac{dTTS_1}{dn'} = -\frac{b_1 - q}{a_1}(a_1 n' + b_1 - q)^{-1} + \frac{1}{a_1} \tag{33}$$

$$\frac{d^2TTS_1}{dn'^2} = (b_1 - q)(a_1 n' + b_1 - q)^{-2} \tag{34}$$

According to Eq. 23, the vehicle accumulation on the less congested branch from t_c to t would be:

$$n = \frac{e^{-a_2(t - t_c)}(a_2 n_c + b_2 - q)}{a_2} - \frac{b_2 - q}{a_2} \tag{35}$$

$$= \frac{e^{a_2 t_c}(m_c - q)}{a_2} e^{-a_2 t} - \frac{b_2 - q}{a_2} \tag{36}$$

The TTS_2 for the less congested branch would be:

$$TTS_2 = \int_{t_c}^{t} n dt = \int_{t_c}^{t} \left(\frac{e^{a_2 t_c}(m_c - q)}{a_2} e^{-a_2 t} - \frac{b_2 - q}{a_2}\right) dt \tag{37}$$

$$= -\frac{e^{-a_2 t}(m_c - q)}{a_2^2} e^{a_2 t_c} + \frac{b_2 - q}{a_2} t_c + \frac{m_c - q}{a_2^2} - \frac{b_2 - q}{a_2} t \tag{38}$$

The derivatives on the less congested branch are:

$$\frac{dTTS_2}{dn'} = -\frac{(m_c - q)^{1 - \frac{a_2}{a_1}} e^{-a_2 t}}{a_2} (a_1 n' + b_1 - q)^{\frac{a_2}{a_1} - 1} + \frac{b_2 - q}{a_2}(a_1 n' + b_1 - q)^{-1} \tag{39}$$

$$\frac{d^2TTS_2}{dn'^2} = -\left(\frac{(a_2 - a_1)(m_c - q)^{1 - \frac{a_2}{a_1}} e^{-a_2 t}}{a_2} (a_1 n' + b_1 - q)^{\frac{a_2}{a_1}} + \frac{a_1(b_2 - q)}{a_2}\right)(a_1 n' + b_1 - q)^{-2} \tag{40}$$

As per Assumption 2, $m_0 - q > 0$ holds. The second derivative of the whole process would be:

$$\frac{d^2TTS}{dn'^2} = \frac{d^2TTS_1}{dn'^2} + \frac{d^2TTS_2}{dn'^2} \tag{41}$$

$$= \left(b_1 - q - \frac{e^{-a_2 t}}{a_2}(a_2 - a_1)(m_c - q)^{1-\frac{a_2}{a_1}}(m_0 - q)^{\frac{a_2}{a_1}} - \frac{a_1(b_2 - q)}{a_2}\right)(m_0 - q)^{-2} \tag{42}$$

Since $m_0 - q > 0$, and if a transportation system is to be fragile, $\frac{d^2TTS}{dn'^2}$ should also be positive, and the following equation has to be true:

$$b_1 - q - \frac{e^{-a_2 t}}{a_2}(a_2 - a_1)(m_c - q)^{1-\frac{a_2}{a_1}}(m_0 - q)^{\frac{a_2}{a_1}} - \frac{a_1(b_2 - q)}{a_2} > 0 \tag{43}$$

As $t > t_c$ and $a_2 > a_1$, regardless of whether a_2 is positive or negative, the following relationship always holds:

$$-\frac{e^{-a_2 t}}{a_2} > -\frac{e^{-a_2 t_c}}{a_2} \tag{44}$$

As the following three terms, $a_2 - a_1$, $(m_c - q)^{1-\frac{a_2}{a_1}}$, and $(m_0 - q)^{\frac{a_2}{a_1}}$ are all positive, the following relationship is true:

$$b_1 - q - \frac{e^{-a_2 t}}{a_2}(a_2 - a_1)(m_c - q)^{1-\frac{a_2}{a_1}}(m_0 - q)^{\frac{a_2}{a_1}} - \frac{a_1(b_2 - q)}{a_2} > \tag{45}$$

$$b_1 - q - \frac{e^{-a_2 t_c}}{a_2}(a_2 - a_1)(m_c - q)^{1-\frac{a_2}{a_1}}(m_0 - q)^{\frac{a_2}{a_1}} - \frac{a_1(b_2 - q)}{a_2} \tag{46}$$

Here we substitute the t_c in Eq. 46 with Eq. 30 and get:

$$b_1 - q - \frac{(a_2 - a_1)(m_c - q)}{a_2} - \frac{a_1(b_2 - q)}{a_2} = \tag{47}$$

$$a_1\left(\frac{b_1 - m_c}{a_1} - \frac{b_2 - m_c}{a_2}\right) = a_1(n_c - n_c) = 0 \tag{48}$$

Hence, Eq. 43 is true and the second derivative of TTS is positive. □

Proposition 4. *Road transportation systems are fragile with the onset of supply disruptions on the microscopic level.*

Proof. For the Greenshields FD, as the base demand q is constant, we have:

$$q = G(k_0) = (1 - r)G(k'(r)) = (1 - r)(ak'(r)^2 + bk'(r)) \tag{49}$$

So, the traffic density of the new equilibrium point after the MFD disruption would be:

$$k'(r) = \frac{\sqrt{b^2 + \frac{4aq}{1-r}} - b}{2a} \tag{50}$$

The ATS and its derivatives are:

$$ATS = \frac{L}{v(k')} = \frac{Lk'(r)}{q} = \frac{L}{2aq}((b^2 + \frac{4aq}{1-r})^{\frac{1}{2}} - b) \tag{51}$$

$$\frac{dATS}{dr} = L(b^2(1 - r) + 4aq)^{-\frac{1}{2}}(1 - r)^{-\frac{3}{2}} \tag{52}$$

$$\frac{d^2ATS}{dr^2} = \frac{b^2L}{2}(b^2(1 - r) + 4aq)^{-\frac{3}{2}}(1 - r)^{-\frac{3}{2}} + \frac{3L}{2}(b^2(1 - r) + 4aq)^{-\frac{1}{2}}(1 - r)^{-\frac{5}{2}} \tag{53}$$

As k' has a physical meaning of disruption density, hence, it should have real roots with $b^2 + \frac{4aq}{1-r}$ being positive. And because $1 - r$ needs to be positive as well, therefore, $\frac{d^2ATS}{dr^2}$ is always positive and indicates the fragility. Likewise to the proof of demand disruption, the Daganzo FD is a special case of the MoC, so we directly prove the fragility on the macroscopic level with MoC.

□

Proposition 5. *Road transportation systems are fragile with the onset of supply disruptions on the macroscopic level.*

Proof. Since the proof of fragility under supply disruption with Geroliminis MFD involves calculating the roots for cubic equations. For simplicity reasons, we prove only With Daganzo MoC:

$$q = (1-r)(uk'(r) + c) \tag{54}$$

$$k'(r) = \frac{\frac{q}{1-r} - c}{u} \tag{55}$$

The ATS and its derivatives are:

$$ATS = \frac{Lk'(r)}{q} = \frac{L}{qu}(\frac{q}{1-r} - c) \tag{56}$$

$$\frac{dATS}{dr} = \frac{L}{u}(1-r)^{-2} \tag{57}$$

$$\frac{d^2ATS}{dr^2} = \frac{2L}{u}(1-r)^{-3} \tag{58}$$

As per Assumption 1, when studying supply disruptions, we focus on the uncongested zone of the MFD, meaning the slope of these relevant cuts is positive so that both derivatives are positive.

□

Proposition 6. *Road transportation systems are fragile when going through the recovery process from supply disruptions.*

Proof. To study the possible fragile properties of road transportation networks regarding the recovery process from supply disruptions, we need to combine the conclusions from Proposition 3 and Proposition 5. In Proposition 3, we've proven $\frac{d^2TTS}{dn'^2} \geq 0$. Similarly, with $a_1 > 0$ as the branch is below the critical density for supply disruptions following Assumption 1, we can easily prove the first derivative $\frac{dTTS}{dn'}$ to be non-negative as well. Likewise to Proposition 5, replacing q with $un_0 + c$, we have:

$$q = un_0 + c = (1-r)(un' + c) \tag{59}$$

$$n'(r) = \frac{un_0 + c}{u(1-r)} - c/u \tag{60}$$

The derivatives of n' over the coefficient r are:

$$\frac{dn'}{dr} = \frac{un_0 + c}{u}(1-r)^{-2} \tag{61}$$

$$\frac{d^2n'}{dr^2} = \frac{2(un_0 + c)}{u}(1-r)^{-3} \tag{62}$$

Since u and $un_0 + c$ are both positive, the derivatives of n' over r are positive as well. Additionally, it can be easily proven that when considering the transition from a more congested to a less congested branch, the same conclusion also holds.

As TTS is a function of n' and n' is again a function of r, by applying the chain rule, we can get the second derivative of TTS over r:

$$\frac{d^2TTS}{dr^2} = \frac{d}{dr}\left(\frac{dTTS}{dn'} \cdot \frac{dn'}{dr}\right) \tag{63}$$

$$= \frac{d^2TTS}{dn_0'^2} \cdot \left(\frac{dn'}{dr}\right)^2 + \frac{dTTS}{dn'} \cdot \frac{d^2n'}{dr^2} \tag{64}$$

Since all the four components of the Eq. 64 have been proven to be non-negative. Hence $\frac{d^2TTS}{dr^2}$ is also non-negative and we've proven the fragile nature of road transportation systems regarding the recovery process of supply disruptions.

□

4 Numerical simulation

In the real world, road transportation systems and MFDs are subject to stochasticity and these uncertainties cannot be reflected in the above mathematical analysis. Therefore, we show the impact of realistic stochasticity on the network performance with a numerical simulation of a congestion recovery process. The MFD of the studied region is created by applying Daganzo MoC with realistic parameters in the city center of Zurich, for example, free-flow speed, back-propagation speed, maximal density, and capacity are provided in Ambühl et al. (2020) with queried routes in Google API. The total and average lane length for District 1 is queried through SUMO. By assuming the studied region to be homogenous with traffic and signal settings, we introduce stochasticity in this region with real traffic light data, as the signalization in Zurich is actuated and does not strictly follow a fixed-time signal cycle. The parameters are summarized in Table. 1.

Table 1: Estimated parameters for the city center of Zurich

Parameters	Notation	Unit	Value
Free-flow speed	u_0	m/s	12.5
Back-propagation speed	w_0	m/s	6.0
Maximal density	κ	veh/m	0.145
Capacity	s	veh/s	0.51
Total lane length	D	m	68631
Average lane length	l	m	167
Average trip length	L	m	7110
Signal cycle time	C	s	50
Signal green time (mean)	μ_G	s	14.8
Signal green time (std.)	σ_G	s	2.5
Offset	δ	s	0

As the average green time is 14.8 s and the standard deviation is 2.5 s, following Daganzo MoC, we produce three groups of cuts with green time being $\mu_G - \sigma_G$, μ_G, or $\mu_G + \sigma_G$, as Fig. 4 shows.

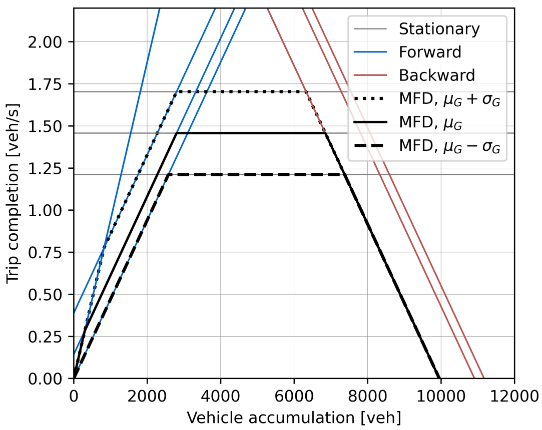
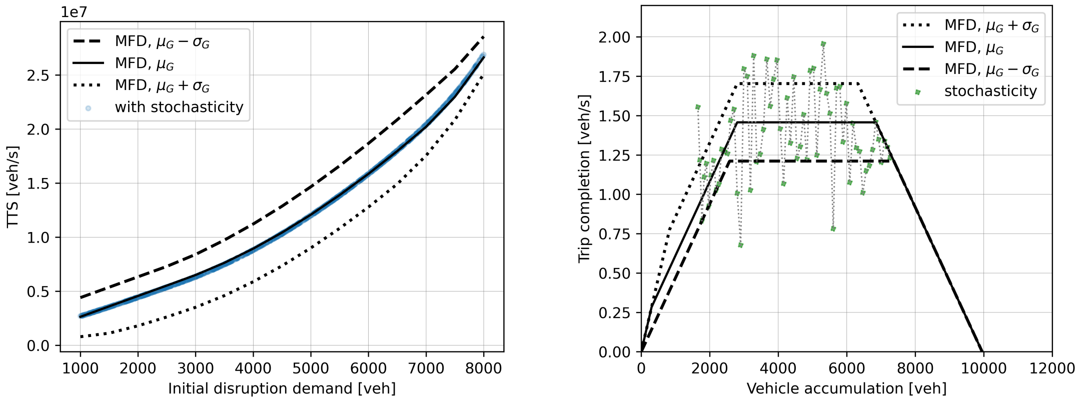


Figure 4: The MFD of the city center of Zurich through MoC

We simulate with different initial disruption demands n' from 1000 to 8000 vehicles with a constant simulation time of 5000 seconds. Fig. 5(a) demonstrates that TTS grows exponentially with linearly increasing n'. Other than these three curves, there are also 500 scattering points with random n' forming the blue curve. Each scatter point is composed of a full recovery process. At each time step, a stochastic signal green time is chosen following normal distribution $G \sim N(14.8, 2.5)$, leading to an uncertain MFD profile as Fig. 5(b) shows as an example.

Since the scattering points closely align with the solid curve, the influence of realistic stochasticity is mostly negligible. However, when the demand is low, the blue curve dips slightly below the solid curve, whereas it appears to rise above with higher demand. To quantify the fragile response, we calculate the skewness of distribution, and the skewness is 0.50 when there is no stochasticity while being 0.53 for the blue curve. As a greater skewness indicates a more fragile system, this means that by introducing realistic stochasticity, the system becomes even more fragile.



(a) Demand disruption in the real world

(b) Stochasticity on the MFD

Figure 5: Numerical simulation with stochasticity

5 Conclusion

This research systematically demonstrated the fragile nature of road transportation systems with rigorous mathematical analysis and numerical simulation under realistic stochasticity. The mathematical proof comprehends the study of fragility under 1) microscopic - macroscopic, 2) demand disruption - supply disruption, and 3) onset of disruption - recovery from disruption. With essential assumptions regarding the disruption density in comparison to the critical density as well as the base traffic demand, we've validated the fragility of road transportation systems from various perspectives. Furthermore, through a numerical simulation with realistic data, we concluded that real-world stochasticity has a limited impact on the fragile characteristics of the system but contributes to rendering the system even more fragile. The fragility observed in urban road networks may be extended to various transportation systems. This study aims to offer insights to researchers, emphasizing the fragile characteristics of transportation systems and encouraging the design of antifragile traffic control strategies in the future.

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