

# Multi-hop Control Scheme for Urban Traffic Congestion Mitigation with Non-Compliant Users

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## SHORT SUMMARY

Traffic congestion has negative impacts on both the individual and social aspects of urban life. The multi-hop control scheme (MHCS), which recommends users pass through some intermediate checkpoints (ICs), has shown great potential in alleviating network congestion and enhancing network efficiency. While MHCS enables users to travel through the shortest routes between ICs and thus induces lower unfairness compared to other route control schemes, it cannot prevent non-compliant behaviors of selfish travelers. This study aims to shed light on the impacts of such behavior on the performance of MHCS. We relax the assumption of full compliance by adding a set of constraints to the original MHCS problem that define the non-compliance ratio. The numerical results suggest that, despite the overall negative effect of non-compliance, the ultimate system performance is largely related to the design of ICs. It was also found that enforcing full compliance over all travelers may not be worth the total travel time savings in consideration of the trade-off between fairness and efficiency.

**Keywords:** Multi-hop control scheme, System optimum, Traffic Management, User compliance, User equilibrium.

## 1 INTRODUCTION

Urban traffic congestion has been a critical issue in large metropolitan areas for decades, meanwhile causing huge costs to individuals. A recent report revealed that drivers in the UK wasted an average of 80 hours in congestion over 2022 (INRIX, 2023). To alleviate traffic congestion, various approaches have been developed, ranging from demand management (Afrin & Yodo, 2020), to network capacity expansion (Zhou et al., 2021), to traffic control and route guidance (Jiang et al., 2024; Huo et al., 2023). Compared to other strategies, the route control and/or guidance schemes are relatively cheaper and simpler for implementation (Chellapilla et al., 2023). Although they do not resolve the fundamental imbalance between travel demand and capacity supply, even a few minutes of improvement for every driver can sum up to a great reduction in total travel time and thus achieve a substantial social cost saving (Rocha Filho et al., 2020).

A successful routing scheme must strike a good balance between efficiency and fairness. On the one hand, it should route vehicles to reduce total travel time in the network and ideally, approach the System Optimum (SO) state (Wardrop, 1952). On the other hand, it should address users' preferences and ensure their incentives to participate in the routing scheme. The commonly used benchmark of fairness is the User Equilibrium (UE) state (Sheffi, 1984), where all travelers choose the shortest paths and thus reach zero unfairness.

Since UE and SO are normally incompatible in a congested traffic network, a series of routing schemes are developed to achieve a state in between (e.g., Jahn et al., 2005; Vreeswijk et al., 2015; Zhang & Nie, 2018). In particular, the Multi-hop Control Scheme (MHCS) proposed by Farahani et al. (2021), designates a set of intermediate checkpoints (ICs) in the network and recommends a fraction of travelers to pass through one or several ICs before reaching their destinations. To restrain unfairness, travelers are free to take the shortest routes in each trip segment, referred to as "hop" hereafter. The numerical results demonstrated the ability of MHCS to make a satisfactory balance between efficiency and fairness.

The original MHCS assumes full compliance of travelers, which, however, is unlikely to occur in

real-world scenarios (Kröllner et al., 2021). Even with the increasing share of autonomous vehicles (AVs) that are easier to control compared to human-driven vehicles (Wang et al., 2020), it is still expected that some travelers may never consent to make a detour to ICs for the social good (Ringhand & Vollrath, 2018). Therefore, it is essential to investigate the effectiveness of MHCS without full user compliance, which becomes the main objective of this paper. Previous studies have drawn important insights into the non-compliant behaviors in route guidance, such as the impact of users' familiarity with the network (Bonsall & Parry, 1991) and the clarity of route recommendations (van Essen et al., 2020). Yet, this paper focuses particularly on the sensitivities of MHCS towards non-compliant behaviors.

## 2 METHODOLOGY

### *Preliminary settings*

Consider a transportation network denoted by  $G(N, A)$ , where  $N$  is the set of nodes and  $A$  the set of links. Let  $P \subseteq N$  and  $Q \subseteq N$  denote the set of origins and destinations, respectively, and  $W \subseteq P \times Q$  be the set of OD pairs. We further introduce  $\mathbf{d} = (d_w | w \in W)^T$  to denote the demand vector, where each element  $d_w > 0$  gives the demand flow between OD pair  $w = (p, q) \in W$ . In what follows, we will use  $d_w$  and  $d_{pq}$  interchangeably. For each OD pair  $w \in W$ , we define  $R_w$  as its corresponding route set, thus the set of all routes in the network is given by  $R = \bigcup_{w \in W} R_w$ .

Following the literature, we denote  $\mathbf{f} = (f_{rw} | r \in R_w, w \in W)^T$  as the path flow vector and  $\mathbf{v} = (v_a | a \in A)^T$  as the link flow vector. Besides, we use  $\Gamma \in \{0, 1\}^{|W| \times |R|}$  as the OD-path incidence matrix (where  $\gamma_{wr}$  equals 1 if path  $r$  connects OD pair  $w$  and zero otherwise) and  $\Delta \in \{0, 1\}^{|A| \times |R|}$  as the link-route incidence matrix (where  $\delta_{ar}$  equals 1 if the route  $r$  uses link  $a$  and zero otherwise). Accordingly, the relationship among link flows, path flows, and OD demands can be stated as  $\mathbf{d} = \Gamma \mathbf{f}$  and  $\mathbf{v} = \Delta \mathbf{f}$ , which yields the sets of feasible path and link flows as follows:

$$\Omega_{\mathbf{f}} = \{\mathbf{f} | \mathbf{d} = \Gamma \mathbf{f}, \mathbf{f} \geq 0\} \quad (1)$$

$$\Omega_{\mathbf{v}} = \{\mathbf{v} | \mathbf{v} = \Delta \mathbf{f}, \mathbf{d} = \Gamma \mathbf{f}, \mathbf{f} \geq 0\} \quad (2)$$

We assume that travel cost functions are separable (i.e., the travel time on a link is the function of the flow on that link only) and that the travel time on a route is the sum of travel times on all links forming the route. Hence,  $\mathbf{c} = \Delta^T \mathbf{t}$ . Recalling  $\mathbf{v} = \Delta \mathbf{f}$ , we may write the path cost vector as  $\mathbf{c} = \mathbf{c}(\mathbf{f})$ . Consider  $\mu_w$  as the least path cost for OD pair  $w$  and  $\boldsymbol{\mu} = (\mu_w | w \in W)^T$  where

$$\mu_w = \min\{c_{rw} | r \in R_w\} \quad \forall w \in W. \quad (3)$$

Therefore, the route flow vector  $\mathbf{f}^*$  satisfies the Wardrop's user equilibrium (UE) condition if and only if:

$$f_{rw}^* > 0 \Rightarrow c_{rw} = \mu_w \quad (4a)$$

$$f_{rw}^* = 0 \Rightarrow c_{rw} \geq \mu_w. \quad (4b)$$

The above condition can be stated as an equivalent variational inequality (UE-VI) problem using path flows: Define a path flow vector  $\mathbf{f}^* \in \Omega_{\mathbf{f}}$  such that

$$\mathbf{c}(\mathbf{f}^*)^T (\mathbf{f} - \mathbf{f}^*) \geq 0 \quad \forall \mathbf{f} \in \Omega_{\mathbf{f}}. \quad (5)$$

The above VI problem can also be stated in terms of link flows: Find a link flow vector  $\mathbf{v}^* \in \Omega_{\mathbf{v}}$  such that

$$\mathbf{t}(\mathbf{v}^*)^T (\mathbf{v} - \mathbf{v}^*) \geq 0 \quad \forall \mathbf{v} \in \Omega_{\mathbf{v}}. \quad (6)$$

To refer to the system optimal (SO) solution, let the marginal link cost function be represented by  $s_a = t_a(v_a) + v_a \frac{dt_a(v_a)}{dv_a}$  for all  $a \in A$ . The vector of all link marginal travel times is then  $\mathbf{s} = (s_a(v_a) | a \in A)^T$ . We define the vector of route marginal costs as  $\mathbf{u} = \Delta^T \mathbf{s}$  including the elements  $u_{rw}$  (the marginal cost of route  $r$  connecting OD pair  $w$ ). The corresponding variational inequality formulation of the SO problem (SO-VI) reads: Define a path flow vector  $\mathbf{f}^* \in \Omega_{\mathbf{f}}$  such that

$$\mathbf{u}(\mathbf{f}^*)^T (\mathbf{f} - \mathbf{f}^*) \geq 0 \quad \forall \mathbf{f} \in \Omega_{\mathbf{f}}. \quad (7)$$

### Intermediate checkpoint and demand segmentation

The key novelty of MHCS is the introduction of a set of intermediate checkpoints  $IC \subseteq N$  that split a single trip into multiple segments. From the modeling perspective, this essentially performs a demand segmentation, which transforms the origin demand vector to a virtual demand with additional OD pairs induced by ICs. In what follows, we use the prime symbol ( $'$ ) to identify the virtual demand and all other variables induced by the virtual demand. Therefore,  $\mathbf{d}'$  denotes the set of virtual demand defined on the set of virtual OD pairs  $W' \subseteq \{P \cup IC\} \times \{Q \cup IC\}$ , and the corresponding virtual link and path flow vectors are represented by  $\mathbf{v}'$  and  $\mathbf{f}'$ , respectively, whose feasible sets  $\Omega'_v$  and  $\Omega'_f$  are defined as per Eqs. (2) and (1) except for replacing  $\mathbf{d}$  with  $\mathbf{d}'$ .

Below we provide an example to better illustrate the conception of demand segmentation. Consider the Nguyen and Dupuis network (Nguyen & Dupuis, 1984) in Fig.1 with a total of  $d_{13}$  travelers between OD pair (1,3). Suppose a fraction of travelers are recommended to pass through Node 9 before reaching their destinations. This will create two virtual demand  $d'_{19}$  and  $d'_{93}$  under the demand segmentation. To maintain the flow conservation, the virtual demand must also ensure  $d'_{13} + d'_{19} = d'_{93} + d'_{13} = d_{13}$ . Now define the matrix of *hopping ratios* as  $\boldsymbol{\lambda} \in [0, 1]^{|W'| \times |W|}$ , where each element  $\lambda_{w'w}$  (or  $\lambda_{ij,pq}$ ) gives the share of virtual demand  $d'_{w'}$  in the original demand  $d_w$ . Hence, a compact form of the flow conservation is given by  $\boldsymbol{\lambda}\mathbf{d} = \mathbf{d}'$ .

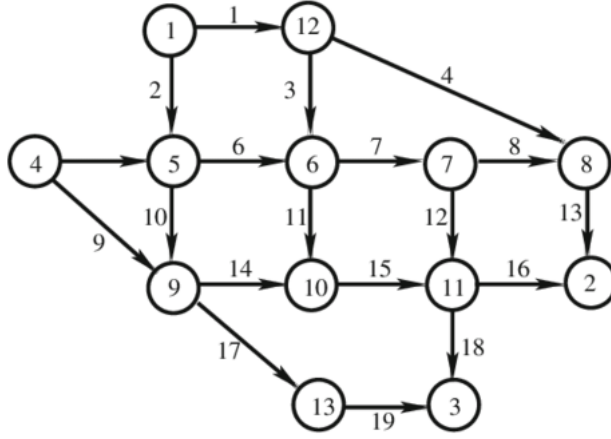


Figure 1: experimental network of Nguyen and Dupuis

### MHCS with non-compliant users

Recall that the ultimate goal of MHCS is to mitigate traffic congestion by recommending ICs to travelers. In other words, the central planner's problem is to determine the ICs and hopping ratios that minimize the total travel time. The selection of ICs, however, is rather complicated as it involves combinatorial optimization. Hence, in this study, we focus on the optimization of hopping ratios given predefined sets of ICs. The problem is then formulated as a bi-level program: at the upper level, the central planner determines the hopping ratios  $\lambda$  in anticipating travelers' route choice, while at the lower level, travelers with augmented ICs make route choices to minimize their own travel time. Since travelers can freely choose any path for each trip segment, the traffic network will consequently reach the UE state under the segmented virtual demand. To account for the non-compliance, we introduce an additional constraint in the upper-level problem that ensures the fraction of travelers without an IC is beyond a certain threshold  $\rho \in [0, 1]$ .

The relaxed MHCS is formulated as follows:

$$\min_{\boldsymbol{\lambda}} Z(\boldsymbol{\lambda}) = \mathbf{t}(\mathbf{v}^*(\boldsymbol{\lambda}))^T \mathbf{v}^*(\boldsymbol{\lambda}) \quad (8a)$$

$$\text{s.t. } \mathbf{d}' = \boldsymbol{\lambda} \mathbf{d}, \quad (8b)$$

$$\sum_{m \in IC \cup \{q\}} \lambda_{pm,pq} = 1, \quad \forall (p, q) \in W, \quad (8c)$$

$$\sum_{i \in IC \cup \{p\}} \lambda_{im,pq} = \sum_{j \in IC \cup \{q\}} \lambda_{mj,pq}, \quad \forall m \in IC, \forall (p, q) \in W, \quad (8d)$$

$$\text{diag}(\boldsymbol{\lambda}) \geq \rho, \quad (8e)$$

$$0 \leq \boldsymbol{\lambda} \leq 1, \quad (8f)$$

$$\mathbf{t}(\mathbf{v}^*(\boldsymbol{\lambda}))^T (\mathbf{v} - \mathbf{v}^*(\boldsymbol{\lambda})) \geq 0, \quad \forall \mathbf{v} \in \Omega'_{\mathbf{v}}. \quad (8g)$$

In Problem (8), Constraint (8b) describes the demand segmentation; Constraints (8c) and (8d) state the flow conservation at the origin nodes and ICs, respectively; Constraint (8e) expresses the non-compliance behaviors by asserting that the diagonal elements in  $\boldsymbol{\lambda}$  (i.e.,  $\lambda_{ww}, \forall w \in W$ ) to be greater or equal than  $\rho$ ; Constraint (8f) gives the feasibility of hopping ratios; and finally, Constraint (8g) states the traffic equilibrium condition.

Since the additional non-compliance constraint is simple and linear, Problem (8) shares the same analytical properties as the original MHCS proposed in Farahani et al. (2021), which is formally stated in the following proposition.

**Proposition 1.** If the link cost function vector  $\mathbf{t}(\cdot)$  is continuously differentiable and increasing with respect to link flows  $\mathbf{v}$ , a solution to the relaxed MHCS problem (8) always exists.

*Proof.* See Farahani et al. (2021).

### Solution algorithms

Problem (8) is solved by a sensitivity-analysis-based (SAB) approach following Farahani et al. (2021). In brief, in each iteration  $k$ , we first solve the equilibrium link flow  $\mathbf{v}^*(\boldsymbol{\lambda}^k)$  at the current solution  $\boldsymbol{\lambda}^k$  and then approximate the derivatives of link flows with respect to the virtual demand, also known as the equilibrium sensitivities. The sensitivity approximates are then used to construct an auxiliary upper-level problem, which is solved by the interior-point algorithm to obtain the hopping ratio for the next iteration. Recognizing that the initial solution could largely affect the final results, Farahani et al. (2021) proposed to first solve the SO state and configure the initial hopping ratios based on the SO path flows. The same algorithm is applied in this study, though a minor adjustment is made to ensure the initial  $\boldsymbol{\lambda}$  meets the non-compliance constraint Eq. (8e).

## 3 RESULTS AND DISCUSSION

The experiments are conducted on the Nguyen-Dupuis network (see Fig. 1), which has been widely used in the literature to produce insights without computational burden Tan et al. (2024); Oszczypała et al. (2023). The network parameters are retrieved from Xu et al. (2011) and reported in Table 1). In total, there are four OD pairs defined in the network:  $d_{12} = 660$ ,  $d_{13} = 495$ ,  $d_{42} = 412.5$ , and  $d_{43} = 495$ .

The performance of MHCS is evaluated using three metrics: (i) the network total travel time ( $TT$ ), (ii) the UE-based unfairness ( $TU_{UE}$ ), and (iii) the perceived unfairness ( $TU_P$ ). Specifically, the UE-based unfairness compares the path travel times under MHCS to those at UE, and the perceived unfairness compares the realized and shortest path travel times under MHCS. Therefore, the former quantifies how much unfairness would be induced by MHCS compared to the status quo, while the latter indicates how likely selfish travelers would deviate from the route recommendation.

The three metrics are mathematically defined as follows:

$$TT = \mathbf{t}(\mathbf{v})^T \mathbf{v}, \quad (9a)$$

$$TU_{UE} = (\mathbf{c} - \boldsymbol{\mu}_{UE})_+^T \mathbf{f}, \quad (9b)$$

$$TU_P = (\mathbf{c} - \boldsymbol{\mu})^T \mathbf{f}, \quad (9c)$$

Table 1: Parameters of Nguyen-Dupuis Network

Link	Free-flow travel time	Capacity	Link	Free-flow travel time	Capacity
1	7	300	11	9	500
2	9	200	12	10	550
3	9	200	13	9	200
4	12	200	14	6	400
5	3	350	15	9	300
6	9	400	16	8	300
7	5	500	17	7	200
8	13	250	18	14	300
9	5	250	19	11	300
10	9	300			

where  $\mu_{UE}$  denotes the vector of shortest path travel times at UE. The operator  $(\mathbf{x})_+ = \max\{0, \mathbf{x}\}$  keeps the positive elements in the vector  $\mathbf{x}$ , thus the UE-based unfairness is always positive.

### Full-compliance MHCS with a single IC

We first solve the original MHCS with a single IC, which provides insights into the design of IC sets in the following experiments. Thanks to the small network scale, we are able to enumerate all possible ICs and solve the MHCS with full user compliance for each case. The results are shown in Fig. 2. As seen, Nodes 9 and 10 save the most total travel time while Node 10 contributes to a lower level of unfairness, particularly the perceived unfairness. On the other hand, Nodes 8 and 13 induce the minimum unfairness, though they are not effective in reducing congestion.

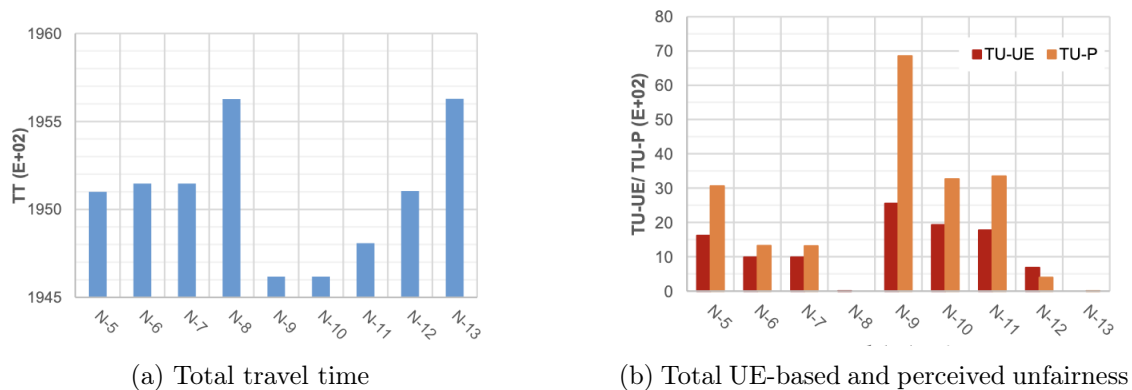


Figure 2: Results of MHCS with a single IC.

### Impact of user compliance on MHCS

Based on the above results, we proceed to design two sets of ICs (see Table 2). Both sets include Node 9, which has been shown to bring the highest system efficiency, Node 8, which leads to the best fairness, and Node 7, which strikes a good balance between efficiency and fairness. The set “R-6” is further augmented with three more nodes following the same rationale. In the experiments, we also vary the non-compliance rate  $\rho$  from zero, where all travelers follow the route guidance, to one, where the system reduces to UE. The results are shown in Fig. 3.

Table 2: Two formations of intermediate checkpoints (ICs) for non-compliance experiments.

Formation	Intermediate Checkpoints (ICs)
R-3	7, 8, 9
R-6	5, 7, 8, 9, 10, 13

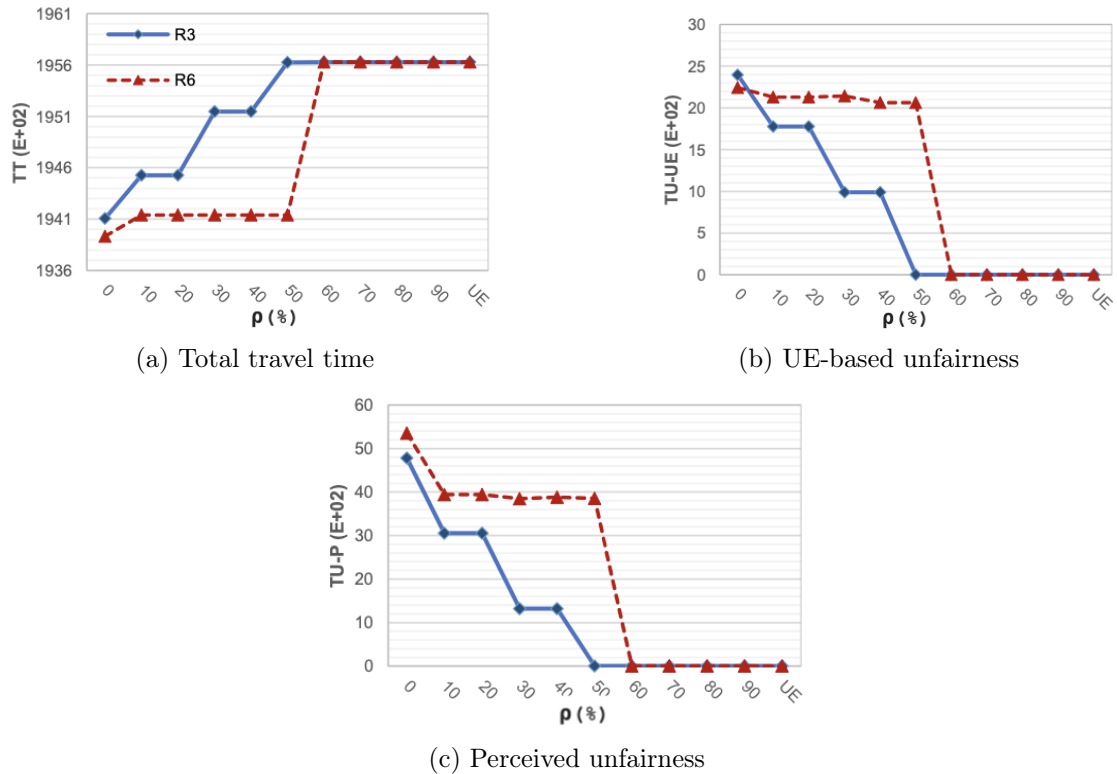


Figure 3: Efficiency and unfairness under MHCS against non-compliance rate.

As expected, the total travel time increases with the non-compliance rate under both IC sets. Besides, with full compliance, R-6 archives more travel time savings compared to R-3 thanks to the extra ICs, though the extra improvement is quite limited. However, the two IC sets behave quite differently in handling the non-compliance. Specifically, R-6 shows a higher capability in managing the selfish behavior of travelers and maintains the network efficiency close to the full-compliance state even though half of travelers disregard the route recommendations. However, it suddenly loses its effectiveness when the non-compliance rate goes beyond a threshold. On the other hand, R-3 is more sensitive to non-compliant behaviors and loses network efficiency in an incremental manner.

A similar trend, though in the opposite direction, is observed in the unfairness measures. As  $\rho$  increases, the unfairness declines gradually in R-3 but suddenly in R-6 as it reaches the threshold. Some other findings are also worth noting. First, a large set of ICs does not necessarily yield higher unfairness under full compliance. As shown in Figs.3 (b) and (c), while R-6 leads to higher perceived unfairness, its UE-based unfairness is less intense than R-3. Nevertheless, R-6 in general incurs more unfairness than R-3 over the various non-compliance rates. Another interesting observation is that perceived unfairness is generally larger than UE-based unfairness. This result implies that MHCS may cause large variations in path travel times, which imposes a risk on compliance in practice.

It is also worthwhile to investigate how the average number of ICs visited by each traveler (AVIC) varies with the non-compliance rate. The results are illustrated in Fig. 4. As expected, AVIC decreases with the non-compliance rate and R-6 tends to pass more travelers through ICs than R-3. Note that, in R-6, as the non-compliance rate increases from 10% to 50%, AVIC declines by about 40% whereas the total travel time remains almost unchanged as shown in Fig. 3(a). This observation implies that the system might be over-controlled within this range. Note that the total perceived unfairness remains almost the same while the non-compliance rate increases from 10% to 50%. This result implies that travelers who remain guided are taking longer detours and paying more to compensate for the efficiency loss due to the non-compliant travelers.

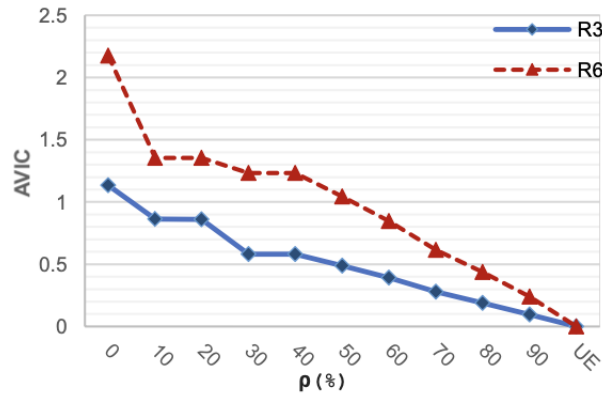


Figure 4: Average number of visited ICs (AVIC) against non-compliance rate.

## 4 CONCLUSIONS

This paper investigates the impact of non-compliance behavior on the effectiveness of the Multi-Hop Control Scheme (MHCS) recently proposed for traffic management. Our numerical findings suggest that as travelers become less compliant, system efficiency inevitably drops, though the decreasing speed and magnitude highly depend on the set of intermediate checkpoints (ICs). In general, having more ICs in the network brings MHCS a larger capacity to absorb non-compliant behaviors without compromising system efficiency. Yet, as the non-compliance rate increases beyond a certain threshold, the effectiveness of MHCS cascades rapidly.

Regarding the trade-off between efficiency and fairness, it is also revealed that the gain in network efficiency may not justify the cost of enforcing full compliance of travelers. Instead, it is possible to achieve a nearly optimal state while assuming few travelers comply with the route recommendation, or equivalently, few travelers are controlled. Nevertheless, the compromise made by these guided travelers should not be overlooked. The result that the system efficiency remains the same while fewer users are guided implies these users may suffer from much longer detours. In this regard, the central planner needs to make a trade-off between implementing comprehensive control over all travelers and targeting a certain group of travelers, who should be compensated more for their contribution to the social good. A plausible strategy is to alternate the role of travelers between guided and unguided on a daily basis (e.g., daily commuting). In this way, every user can benefit from the routing scheme instead of sacrificing for society all the time. Another idea is to assign route guidance to AVs in mixed traffic (Zhang & Nie, 2018), anticipating the wide adaptation of AVs and the less sensitivity of AV travelers towards detours.

## REFERENCES

- Afrin, T., & Yodo, N. (2020). A survey of road traffic congestion measures towards a sustainable and resilient transportation system. *Sustainability*, 12(11), 4660.
- Bonsall, P., & Parry, T. (1991). *Using an interactive route-choice simulator to investigate drivers' compliance with route guidance advice* (No. 1306).
- Chellapilla, H., Sivanandan, R., Chilukuri, B. R., & Rajendran, C. (2023). Bi-objective optimization models for mitigating traffic congestion in urban road networks. *Journal of Traffic and Transportation Engineering (English Edition)*, 10(1), 86-103. Retrieved from <https://www.sciencedirect.com/science/article/pii/S2095756423000041> doi: <https://doi.org/10.1016/j.jtte.2021.09.006>
- Farahani, H. R., Rassafi, A. A., Zhang, K., & Nie, Y. M. (2021). A multi-hop control scheme for traffic management. *Transportation Research Part C: Emerging Technologies*, 130, 103278. Retrieved from <https://www.sciencedirect.com/science/article/pii/S0968090X21002904> doi: <https://doi.org/10.1016/j.trc.2021.103278>

- Huo, Y., Delahaye, D., & Sbihi, M. (2023). A dynamic control method for extended arrival management using enroute speed adjustment and route change strategy. *Transportation research part C: emerging technologies*, 149, 104064.
- INRIX. (2023). *INRIX Global Traffic Scorecard*. <https://inrix.com/scorecard/>. (Accessed: [December 2023])
- Jahn, O., Möhring, R. H., Schulz, A. S., & Stier-Moses, N. E. (2005). System-optimal routing of traffic flows with user constraints in networks with congestion. *Operations research*, 53(4), 600–616.
- Jiang, S., Tran, C. Q., & Keyvan-Ekbatani, M. (2024). Regional route guidance with realistic compliance patterns: Application of deep reinforcement learning and mpc. *Transportation Research Part C: Emerging Technologies*, 158, 104440. Retrieved from <https://www.sciencedirect.com/science/article/pii/S0968090X23004308> doi: <https://doi.org/10.1016/j.trc.2023.104440>
- Krölller, A., Hüffner, F., Kosma, Ł., Krölller, K., & Zeni, M. (2021). Driver expectations toward strategic routing. *Transportation research record*, 2675(11), 44–53.
- Nguyen, S., & Dupuis, C. (1984). An efficient method for computing traffic equilibria in networks with asymmetric transportation costs. *Transportation Science*, 18(2), 185-202. Retrieved from <https://doi.org/10.1287/trsc.18.2.185> doi: 10.1287/trsc.18.2.185
- Oszczypała, M., Ziółkowski, J., Małachowski, J., & Łegas, A. (2023). Nash equilibrium and stackelberg approach for traffic flow optimization in road transportation networks: A case study of warsaw. *Applied Sciences*, 13(5). Retrieved from <https://www.mdpi.com/2076-3417/13/5/3085> doi: 10.3390/app13053085
- Ringhand, M., & Vollrath, M. (2018). Make this detour and be unselfish! influencing urban route choice by explaining traffic management. *Transportation research part F: traffic psychology and behaviour*, 53, 99–116.
- Rocha Filho, G. P., Meneguette, R. I., Neto, J. R. T., Valejo, A., Weigang, L., Ueyama, J., ... Villas, L. A. (2020). Enhancing intelligence in traffic management systems to aid in vehicle traffic congestion problems in smart cities. *Ad Hoc Networks*, 107, 102265.
- Sheffi, Y. (1984). *Urban transportation networks: Equilibrium analysis with mathematical programming methods*. Prentice-Hall. Retrieved from <https://books.google.ch/books?id=zx1PAAAAMAAJ>
- Tan, Y., Sun, Z., Zhu, B., Qin, Z., Zhao, Y., & Wang, X. (2024). Minimize population exposure to vehicle-generated emissions by road pricing. *Transport Policy*, 148, 15-30. Retrieved from <https://www.sciencedirect.com/science/article/pii/S0967070X23003591> doi: <https://doi.org/10.1016/j.tranpol.2023.12.025>
- van Essen, M., Thomas, T., van Berkum, E., & Chorus, C. (2020). Travelers' compliance with social routing advice: evidence from sp and rp experiments. *Transportation*, 47, 1047–1070.
- Vreeswijk, J. D., Landman, R. L., van Berkum, E. C., Hegyi, A., Hoogendoorn, S. P., & van Arem, B. (2015). Improving the road network performance with dynamic route guidance by considering the indifference band of road users. *IET intelligent transport systems*, 9(10), 897–906.
- Wang, J., Zheng, Y., Xu, Q., Wang, J., & Li, K. (2020). Controllability analysis and optimal control of mixed traffic flow with human-driven and autonomous vehicles. *IEEE Transactions on Intelligent Transportation Systems*, 22(12), 7445–7459.
- Wardrop, J. G. (1952). Road paper. some theoretical aspects of road traffic research. *Proceedings of the institution of civil engineers*, 1(3), 325–362.
- Xu, H., Lou, Y., Yin, Y., & Zhou, J. (2011). A prospect-based user equilibrium model with endogenous reference points and its application in congestion pricing. *Transportation Research Part B: Methodological*, 45(2), 311-328. Retrieved from <https://www.sciencedirect.com/science/article/pii/S0191261510001062> doi: <https://doi.org/10.1016/j.trb.2010.09.003>



Zhang, K., & Nie, Y. M. (2018). Mitigating the impact of selfish routing: An optimal-ratio control scheme (orcs) inspired by autonomous driving. *Transportation Research Part C: Emerging Technologies*, 87, 75–90.

Zhou, Z., Yang, M., Sun, F., Wang, Z., & Wang, B. (2021). A continuous transportation network design problem with the consideration of road congestion charging. *Sustainability*, 13(13), 7008.