Cost-Efficient Robust Network Design for BEBs: Tackling Energy Uncertainty with Limited Data

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Abstract

This research focuses on electrifying an existing bus network under energy consumption uncertainty between stops, with limited data. The study evaluates the optimal locations and types of charging stations, as well as battery sizes, to minimize electrification costs. Three optimization models are explored: nominal (deterministic model considering expected values for energy consumption), robust optimization with uncertainty budget (BoU), and distributionally robust chance constraint (DRCC), which utilizes observed energy consumption data. Regarding the optimal design, the BoU model opts for more flash-feeding stations to handle greater uncertainties, while DRCC tends to minimize the number of charging stations overall. The performance of the models are compared based on their electrification costs as well as conceived battery longevity in terms of charge-discharge cycle, finding that larger battery capacities in robust models (BoU and DRCC) extend battery life compared to the nominal model. Compared to BoU model, the DRCC achieves comparable improvements in battery life at a lower cost (for similar battery capacity). For the observed energy consumption in this study, nearly 40 data points are found to be sufficient for robust network design which is feasible for 90% of observed energy consumption data. In conclusion, the DRCC model is particularly efficient in designing robust and less conservative network design.
Keywords: Electric bus network, Robust optimization, Distributionally robust chance constraint model, Data-driven models

1 Introduction

The rapid shift towards eco-friendly urban transportation has made utilization of Battery-powered Electric Buses (BEBs) popular (Azadeh et al., 2022). BEBs, however, deal with range limitations due to battery capacity constraints and they are affected by estimated energy consumption. Despite technological improvements, realized energy usage and range often deviate from theoretical models (Zhou et al., 2023), influenced by variables like weather, driving styles, and passenger loads (demand). Accurate estimation of energy consumption for BEBs between stops is vital to design an efficient BEB network. Failure to reach the next stop due to unforeseen energy usage not only interrupts service but also incurs additional costs for relocating the bus to a charging station, adding to the system's costs. As suggested in the literature simulation models, can estimate the energy consumption data (Scarinci et al., 2019; Rios et al., 2014). Their reliance on specific assumptions, can limit their real-world applicability. Hence, collecting real energy consumption data through field experiments can be an alternative which is costly and generates limited data. In this way, utilization of stochastic programming is impractical, since many scenarios are required to be generated to capture the true distribution of energy consumption. Some studies do not explicitly consider energy consumption patterns in the network design e.g., Kunith et al. (2017); Yıldırım & Yıldız (2021). Research on various robust optimization models, to capture energy consumption pattern, such as Bai et al. (2022); Liu et al. (2018), offers insights into robust network design. However, these models only consider the expected and maximum deviations of observations, neglecting that the probability of occurrence of extreme values might be very low. This can lead to overly conservative outcomes, especially when these extreme values are significantly different from other observations (such as skewed distributions).
This study, therefore, poses a critical question: How can we design a robust BEB network with the aim of minimizing the total electrification costs, encompassing charging station locations and types and battery capacities, using limited data that better reflects real-world situations? Furthermore,

what are the differences of such approach with traditional robust optimization methods (in terms of electrification cost and battery longevity)?
This study utilizes data-driven robust methods, specifically Distributionally Robust Chance Constraint (DRCC), as a way to integrate the limited data to the network design. This approach has been applied successfully in diverse fields, including network design optimization for renewable energy generation systems (Alismail et al., 2017), and photovoltaic systems and distribution network management (Zhang et al., 2023). This study applies DRCC to BEB network design, representing one of its first uses in this field.
The paper is structured to discuss three models: a nominal model for baseline comparison, a robust budget of uncertainty model, and the DRCC model. The results section compares models performances and assesses the impact of observational data quantity on the economic feasibility and robustness of BEB network design, concluding with a summary of findings.

2 Methodology

This section outlines the challenge of electrifying an existing bus network and introduces our optimization model. We address the problem of facility location in a network with several bus lines, each with a predefined number of trips and schedules. If the network has shared stop between several lines, we assume their time-table is not conflicting. Our model simultaneously determines three key aspects to minimize total electrification costs: the types and locations of charging stations and the capacity of onboard batteries.
In our approach, BEBs are charged en route at bus stops. There are two types of charging stations: Flash-feeding (FF) and standard. BEBs are fully charged at FF stations, while the charge at standard stations depends on the bus's dwell time and the charging station's power.
We base our model on three assumptions: 1- Each bus line follows a fixed route with a terminal for starting and ending trips. 2- BEBs are fully recharged at the terminal after each service loop. 3- FF and standard charging stations are installed at existing bus stops.
The primary goal of this study is to develop a robust network design for electrifying a bus network, taking into account the uncertainty in energy consumption. We focus on how observations influence the design of such a robust network.

Nominal modeling formulation

Let K and S be the set of bus lines and bus stops, respectively. The set T denotes all possible charging station types. Here, we present the nominal model, with considering the expected values for BEB energy consumption between stops.

$$\min \sum_{i \in S} \sum_{t \in T} \alpha_{i,t}\ x_{it} + \sum_{k \in K} \beta \gamma_k\ z_k \tag{1}$$

subject to:

$$\begin{aligned}
&\sum_{t \in T} x_{it} \leq 1 && \forall i \in S && (2)\\
&z_k \geq 0 && \forall k \in K && (3)\\
&e_{ki}^{leaving} = b^{upper}\ \ z_k && \forall k \in K, i = o_k && (4)\\
&e_{ki}^{leaving} \leq b^{upper}\ z_k && \forall k \in K, i \in S_k && (5)\\
&e_{ki}^{leaving} \leq e_{ki-1}^{leaving} - \mu_{k,i-1,i} + \sum_{t \in T} P_t\ \Delta_{ki}\ x_{it} && \forall k \in K, i \in S_k && (6)\\
&e_{ki-1}^{leaving} - \mu_{k,i-1,i} \geq b^{lower}\ z_k && \forall k \in K, i \in S_k && (7)\\
&x_{it} \in \{0,1\} && \forall i \in S, t \in T && (8)
\end{aligned}$$

The objective function (1) seeks to minimize total costs, combining the costs of installing charging stations (α_{it}) and batteries for each bus line (β). Parameter γ_k shows the number of buses servicing each line, where x_{it} is a binary variable indicating the installation of a charging station of type t at stop i, and z_k represents the battery capacity for line k. The constraints include:

- Constraint (2) limits each bus stop to one charging station type.
- Constraint (3) ensuring battery capacities are non-negative.

- Battery level constraints (b^{lower}, b^{upper}) to control battery aging, with $e_{k,i}^{leaving}$ showing the remaining power for a BEB leaving stop i. Constraint (4) ensures full charging at the terminal ($i = o_k$), and (5) sets the maximum battery level at non-terminal stops.
- Constraint (6) addresses the battery level changes considering energy consumption and charging at stops. For standard charging stations, power depends on charging station capacity P_t and dwell time Δ_{ki}, while FF stations always fully charge the BEBs (i.e $\Delta_{ki} = 1$).
- Constraint (7) ensures sufficient battery to reach the next stop, and (8) defines the decision variables' domain.

To better align with real-world situation, we will tackle the energy consumption uncertainty in the forthcoming section of our optimization model.

Robust optimization models

To adapt the model for robust optimization, we eliminate the state variables (i.e. $e_{ki}^{leaving}$) since they rely on the realization of uncertainties. Network design decisions must be made prior to actual uncertainty realization, thus requiring "here and now" decisions that are not subject to adjustment. Let $\widetilde{\mu}_{k,i-1,i}$ represent uncertain energy consumption between two successive stops. Since we assume BEB leave terminal fully charged, we can substitute $e_{ki}^{leaving}$ in accumulative manner using constraints (4) and (6):

$$e_{kn}^{leaving} = b^{upper}\ z_k - \sum_{i=m}^{n} \widetilde{\mu}_{k,i-1,i} + \sum_{t\in T}\sum_{i=m}^{n} P_t \Delta_{ki} x_{it} \qquad \forall k \in K, 1 \leq n \leq |S_k|$$

Then, combining them to constraints (5) and (7) yields:

$$\sum_{t\in T}\sum_{i=m}^{n} P_t \Delta_{ki} x_{it} \leq \sum_{i=m}^{n} \widetilde{\mu}_{k,i-1,i}, \qquad \forall k \in K, 1 \leq m \leq n \leq |S_k| \tag{9}$$

$$b^{upper}\ z_k - \sum_{i=m}^{n} \widetilde{\mu}_{k,i-1,i} + \sum_{t\in T}\sum_{i=m}^{n-1} P_t \Delta_{ki} x_{it} \geq b^{lower}\ z_k \qquad \forall k \in K, 1 \leq m \leq n \leq |S_k| \tag{10}$$

Budget of uncertainty model (BoU)

Our first modeling approach is the robust Budget of Uncertainty optimization model (BoU), which uses expected and maximum deviation data points between stops to find an optimal robust solution. We create a simple uncertainty set for BEB energy consumption, defined by the mean ($\bar{\mu}_{k,i-1,i}$) and maximum deviation ($\hat{\mu}_{k,i-1,i}$) from expected consumption between stops. We assume uncertain energy consumption lies within a symmetric and bounded interval, but only consider positive deviations, ignoring potential negative ones as they don't adversely affect the bus system. This results in an uncertainty box where $\widetilde{\mu}_{k,i-1,i} \in [\bar{\mu}_{k,i-1,i}, \bar{\mu}_{k,i-1,i} + \hat{\mu}_{k,i-1,i}]$.
We standardize the random variable $\widetilde{\mu}_{k,i-1,i}$ with $\varphi_{k,i-1,i}$ for better interpretation. To avoid overly conservative outcomes, we define an uncertainty set $\mathcal{C}(\widetilde{\varphi})$ with a budget constraint Γ_{kmn}, limiting energy consumption deviation along bus line k between stations m and n, which belongs to the interval $[0, n-m+1]$ (Bai et al., 2022). The dual form of our optimization model minimizes $\Gamma_{kmn} u^{kmn} + \sum_{i=m}^{n} v_i^k$ subject to constraints ensuring the robustness against maximum energy consumption deviations. Consequently, Constraints (9) and (10) is reformulated as follows:

$$\sum_{t\in T}\sum_{i=m}^{n} P_t \Delta_{ki} x_{it} \leq \sum_{i=m}^{n} \bar{\mu}_{k,i-1,i} + \Gamma_{kmn} u^{kmn} + \sum_{i=m}^{n} v_i^k, \qquad \forall k \in K, 1 \leq m \leq n \leq |S_k| \tag{11}$$

$$\sum_{b\in B} (b^{upper} - b^{lower})\ z_k + \sum_{t\in T}\sum_{i=m}^{n-1} P_t \Delta_{ki} x_{it} \geq \sum_{i=m}^{n} \bar{\mu}_{k,i-1,i} + \Gamma_{kmn} u^{kmn} + \sum_{i=m}^{n} v_i^k \qquad \forall k \in K, 1 \leq m \leq n \leq |S_k| \tag{12}$$

$$u^{kmn} + v_i^k \geq \hat{\mu}_{k,i-1,i}, \qquad \forall k \in K, m \leq i \leq n \tag{13}$$

$$u^{kmn}, v_i^k \geq 0, \qquad \forall m \leq i \leq n \tag{14}$$

The complete BoU robust optimization counterpart model is (15):

$$\begin{aligned} &\textbf{Objective function}\text{: Equation(1)} \\ &\text{subject to: } (2), (3), (8), (11) - (14) \end{aligned} \tag{15}$$

Although a budget is set to limit the focus on extreme deviations in energy consumption, the BoU model's sensitivity to the estimation of the worst-case scenario is notable. In other words, BoU overlook the probability of occurrence of the worst-case value. A significantly higher worst-case deviation at one stop, with small probability of occurrence, can unduly influence the design of the entire network. Consequently, in the subsequent section, we explore alternative modeling approaches that incorporate a broader range of observed data for system design.

Distributionally robust chance constraint model (DRCC)

To tackle the shortcomings of BoU and reduce the impact of an alienated worst-case scenario on total system's cost, we introduced a tailored DRCC model to develop a robust design for the network. The DRCC approach enables us to modify the optimization model according to the number of data points, achieving a solution that is both robust and less conservative. The logic behind DRCC is shifting from solely focusing on maximum deviation to adjusting observed data points towards the worst possible distribution for the energy consumption based on a distributional distance (i.e. Wasserstein distance) budget. This budget is shown by parameter $\theta \in [0, \infty)$ which is the radius of the Wasserstein ball, representing the confidence level or the degree of risk aversion (Chen et al., 2022; Gao & Kleywegt, 2023).

Two main definition in DRCC are ambiguity set and safety set. $\mathcal{F}(\theta)$ is a Wasserstein ambiguity set that contains all the plausible probability distributions for the uncertain observation. The worst possible distribution is opted from this set. Safety set $\mathcal{S}(x)$ represents the feasible region based on decision variables. The DRCC model ensures that constraints are met for all probability distributions within the ambiguity set $\mathcal{F}$, with a probability of at least $(1 - \epsilon)$, where ϵ is a predefined risk tolerance level (for chance constraint). Scenario is defined as the energy consumption observation between stops on a bus line, it is indicated by j where N shows the total number of observations. The DRCC model focuses on ensuring that the probability of the uncertain vector for each scenario j (i.e. $\tilde{\mu}^j_{k,i-1,i}$) exceeding the decision-dependent safety set $\mathcal{S}(x)$ is below ϵ for every distribution in the ambiguity set $\mathcal{F}(\theta)$.

In our model, we face the DRCC with joint right-hand side uncertainty. Adapting mixed-integer problem (Chen et al., 2022) to our specific problem, where the energy consumption is calculated in a cumulative way, has led to the development of the following model:

$$\min_{y,r,q,d,x,z} \sum_{k\in K} \beta\ \gamma_k\ z_k + \sum_{i\in S}\sum_{t\in T}\alpha_t\ x_{it} \tag{16}$$

subject to:

$$\sum_{t\in T} x_{it} \leq 1 \qquad \forall i \in S \tag{17}$$

$$\epsilon N q^k - \sum_{i=m}^{n}\sum_{j\in[N]}(r_i^k)^j \geq \theta N \qquad \forall k \in K, m < n \leq |S_k| \tag{18}$$

$$M(1-\sum_{i=m}^{n}(y_i^k)^j) \geq q^k - \sum_{i=m}^{n}(r_i^k)^j \qquad \forall j \in [N], k \in K, m < n \leq |S_k| \tag{19}$$

$$\sum_{i=m}^{n}(d_i^k)^j + M\ \sum_{i=m}^{n}(y_i^k)^j \geq q^k - \sum_{i=m}^{n}(r_i^k)^j \qquad \forall j \in [N], k \in K, m < n \leq |S_k| \tag{20}$$

$$\sum_{t\in T}\sum_{i=m}^{n} P_t \Delta_{ki}\ x_{it} - \sum_{i=m}^{n}(d_i^k)^j \leq \sum_{i=m}^{n}\widetilde{\mu}_{k,i-1,i}^j \qquad \forall j \in [N], k \in K, m < n \leq |S_k| \tag{21}$$

$$(b^{upper} - b^{lower})z_k + \sum_{t\in T}\sum_{i=m}^{n-1} P_t \Delta_{ki}\ x_{it} - \sum_{i=m}^{n}(d_i^k)^j \geq \sum_{i=m}^{n}\widetilde{\mu}_{k,i-1,i}^j \qquad \forall j \in [N], k \in K, m < n \leq |S_k| \tag{22}$$

$$(y_i^k)^j, x_{it} \in \{0,1\};\ (r_i^k)^j,\ q^k, z_k \geq 0 \tag{23}$$

In obtaining mixed-integer programming formulation model, we introduce binary variables to the model. $(y_i^k)^j$ is a binary variable where $(y_i^k)^j = 1$ (respectively $(y_i^k)^j = 0$) corresponds to the situation when sample $\tilde{\mu}_{k,i-1,i}^j$ does not (does) satisfy the chance constraint. Variables r, q and d are the dual variables, and M is a sufficiently large (but finite) number.

3 Results and discussion

In this section, we apply the proposed three models on an existing bus network consists of two bus lines as shown in figure (1).These lines share three stops and feature circular routes, as indicated by the same stops for departures and arrivals. The corresponding expected energy consumption and maximum deviations for bus lines are shown in figure (2). Each line is serviced by 10 buses. The parameters values are shown in table 1. To develop the DRCC model, we generated 100 scenarios using a uniform distribution ranging from $[0.8\bar{\mu}, \hat{\mu}]$ for every stop along each bus line. Gurobi Optimizer version 9.5.2 is used to solve the problem optimally. In the following subsections, we will evaluate the performance of three models, focusing on their immediate costs and projected battery lifespan, along with analyzing the impact of number data points on the DRCC model.

Table 1: Parameters of the models

Parameters	Value
T	2 (charging types: S and FF)
α_S	120,000 €
α_{FF}	400,000 €
β	1200 €
b^{lower}; b^{upper}	20%; 80%
Δ	Normal distribution(mean=20 seconds, std. 4 seconds)
M	25 (for DRCC model)

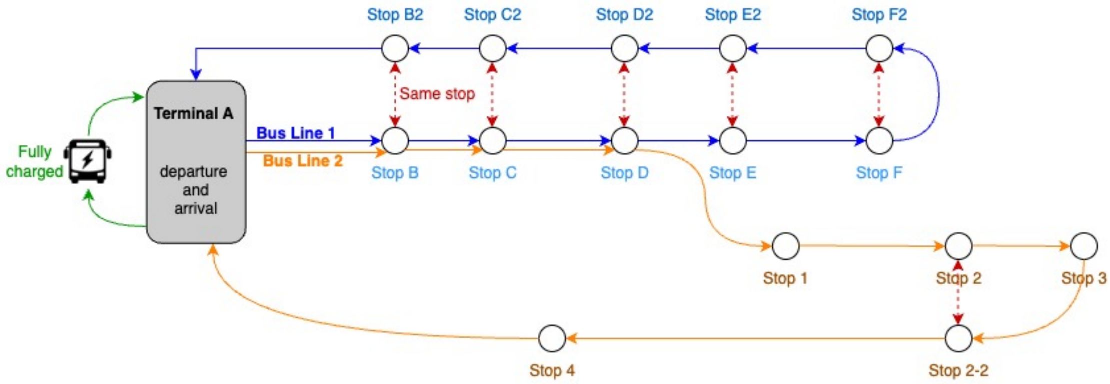
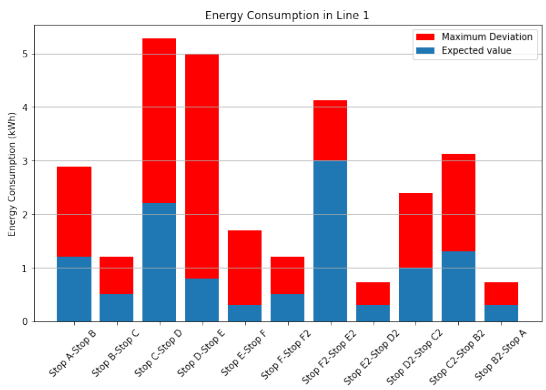
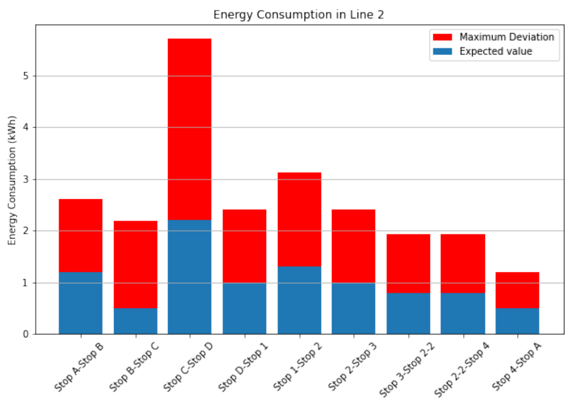


Figure 1: Existing bus network model with terminal charging station



(a) Bus line 1



(b) Bus line 2

Figure 2: Expected and maximum deviation for energy consumption in the studied bus network

Models performance

The outcomes of the three models are presented in table (2). At a zero risk aversion level, the robust models default to the expected energy consumption, mirroring the results of the nominal model. The 'Relative change' columns in (2) highlight the deviations of each model's optimal results from those of the nominal model. An increase in the risk aversion parameter in both robust models correlates with elevated electrification costs and greater battery capacities, as a precaution against extreme energy consumption uncertainties.

In the BoU model with $\Gamma = 0.1$, the network design remains unchanged, suggesting that the nominal design can accommodate some uncertainty by simply augmenting battery capacities. For higher Γ values, a more cautious network design is adopted, characterized by a greater number of FF charging stations and fewer standard ones. Conversely, the DRCC model tends to result in fewer charging stations of both types. The DRCC model's results at $\theta = 0.6$ and $\theta = 1$ imply that the distribution identified at $\theta = 0.6$ is sufficiently robust to a broad spectrum of energy consumption data, including those markedly different from the nominal. Extending the ambiguity set does not reveal new distributions with significant deviations. This indicates that the solution obtained for $\theta = 0.6$ is already 'safe' across a wide range of scenarios. Despite the same optimal network design for both $\theta = 0.6$ and $\theta = 1$, the estimated charge-discharge cycles, as shown in figure (3c), differ, influencing the perceived battery lifespan.

Comparing the most conservative results of the DRCC and BoU models, the DRCC model achieves higher battery capacity with notably fewer charging installations. The analysis shows that the DRCC model, by considering the energy consumption data, remains robust in extreme scenarios with a significantly reduced number of charging installations and related costs.

Table 2: Models performance comparison

Model		Electrification Cost		Battery Capacity		Network design
		OFV(Euro)	Relative change	Capacity (kWh)	Relative change	
Nominal		$2.72 * 10^6$	-	z_{line1}=6.5; z_{line2}=6.5	-	**S:** {F, 2, 3}; **FF:** {D, E2}
BoU	$\Gamma = 0$	$2.72 * 10^6$	-	z_{line1}=6.5; z_{line2}=6.5	-	**S:** {F, 2, 3}; **FF:** {D, E2}
	$\Gamma = 0.1$	$3.12 * 10^6$	14%	z_{line1}=8.53; z_{line2}=7.85	z_{line1}=31%; z_{line2}=21%	**S:** {F, 2, 3}; **FF:** {D, E2}
	$\Gamma = 0.6$	$3.82 * 10^6$	40%	z_{line1}=8.5; z_{line2}=8.99	z_{line1}=31%; z_{line2}=38%	**S:** {B}; **FF:** {D, F, D2, 2}
	$\Gamma = 1$	$4.19 * 10^6$	54%	z_{line1}=9.7; z_{line2}=10.86	z_{line1}=49%; z_{line2}=67%	**S:** {C}; **FF:** {D, F, D2, 3}
DRCC	$\epsilon = 0.1,\ \theta = 0$	$2.72 * 10^6$	-	z_{line1}=6.5; z_{line2}=6.5	-	**S:** {F, 2, 3}; **FF:** {D, E2}
	$\epsilon = 0.1,\ \theta = 0.1$	$3.19 * 10^6$	17%	z_{line1}=8.67; z_{line2}=9.29	z_{line1}=33%; z_{line2}=43%	**S:** {2, D2}; **FF:** {D, F2}
	$\epsilon = 0.1,\ \theta = 0.6$	$3.38 * 10^6$	24%	z_{line1}=10.02; z_{line2}=11.54	z_{line1}=54%; z_{line2}=77%	**S:** {-}; **FF:** {D, F2}
	$\epsilon = 0.1,\ \theta = 1$	$3.38 * 10^6$	24%	z_{line1}=10.02; z_{line2}=11.54	z_{line1}=54%; z_{line2}=77%	**S:** {-}; **FF:** {D, F2}

S: standard charging station; **FF**: flash-feeding charging station; **Network design column contains stop names (shown in figure 1) that specific charging station types is installed**

Conceived bus battery life

After determining the optimal design for each model, we explored the longevity of the bus batteries in these models by examining the number of cycles until failure (in terms of battery aging). Battery depreciation occurs in each charge-discharge cycle, and depth of discharge (DOD) in a cycle has a strong effect on it. In this research each of three models results in different charge-discharge cycles (shown in 3) that affects the battery life time. The number of charge-discharge cycles, denoted by N_{cycle}, during the battery life decreases with changes in charge-discharge cycle. The DOD_i^k of BEB for stop i in line k is expressed as Zang et al. (2022):

$$\text{DOD}_i^k = \frac{(z_k - y_i'^k)}{z_k},\ y_i'^k \in [0, b^{lower} z_k] \tag{24}$$

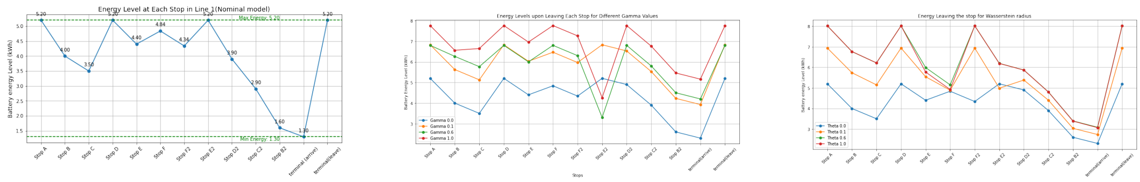
where $y_i'^k$ $(i \in S)$ is the battery remaining when arriving to stop i in bus line k. Similarly, the effect of charging process at charging stations$i$ in bus line k is captured by:

$$\text{charge}_i^k = \frac{z_k - Y_i^k}{z_k},\ Y_i^k \in [0, b^{upper} z_k] \tag{25}$$

where Y_i^k $(i \in S)$ is the battery level when leaving stop i in bus line k (with a charging station installed). It's worth noting that $(Y_i - y_i')$ is the charged quantity in stop i. Therefore, N_{cycle} considering the charge-discharge cycle for each bus line k can be deduced by Wang & Hong (2015):

$$N_{cycle}^k = g_1 \cdot \sum_i ((\text{DOD}_i^k)^{-g_2} + (\text{charge}_i^k)^{-g_2}) \tag{26}$$

where g_1 and g_2 are fixed constants related to the type of battery. Based on Dallinger (2013), for a typical Li-ion battery, we set $g_1 = 1331$, $g_2 = 1.825$. Our analysis is then extended to focus on



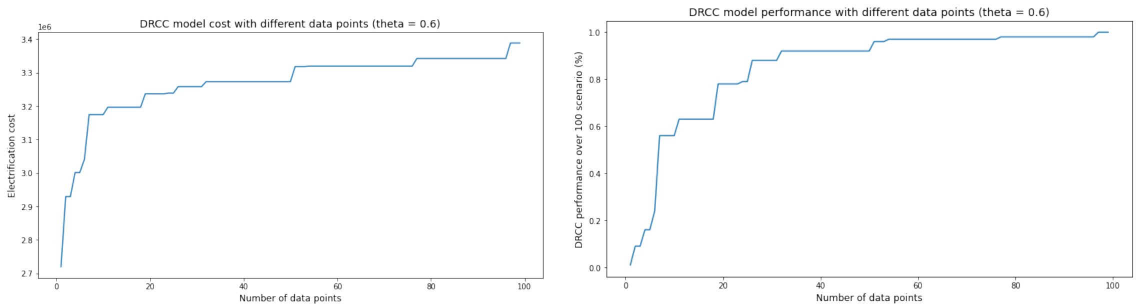
(a) Nominal model (b) BoU model (c) DRCC model

Figure 3: Energy level in BEB upon leaving each stop, comparing three different models

bus line 1, with the understanding that the same methodology could be applied to bus line 2. We selected the BoU model with $\Gamma = 0.6$ and DRCC model with $\epsilon = 0.1$ and $\theta = 0.1$ as our primary scenarios for evaluating N_{cycle}^{line1}, due to their comparable installed battery capacities (see in table 2). The results revealed that the battery longevity in terms of cycles was 513, 650, and 618 for the nominal, BoU, and DRCC models, respectively. The robust models, both BoU and DRCC, demonstrated an enhanced cycle count compared to the nominal model, attributed to their larger battery capacity and a network design that mitigates severe charge depletion. In the BoU model, the placement of chargers regulates the DOD, yet this leads to a 40% cost increase compared to the nominal model. Conversely, the DRCC model leverages observational data to achieve a similar cycle lifespan with a substantially smaller cost increment.

Impact of observations

One of the primary goals of this research is to investigate the effects of energy consumption data and the frequency of data collection on the optimal network design (shown in figure 4). We analyzed how the number of data points influences both the costs of network electrification (4a) and the feasibility of solutions across 100 scenarios (4b). Our findings in (4a) reveal a correlation where an increase in data points correlates with higher electrification costs. This is due to the model accounting for more severe scenarios and increasing battery capacity, leading to elevated costs. However, we observed steady cost lines in certain intervals, indicating that additional data points in these ranges do not add worse energy consumption observations, suggesting a robust design for these observation periods.
Furthermore, the study examines if the decisions derived from a specific set of data points remain viable for daily operations across all observations. This aspect is evaluated by the number of feasible cases out of 100 scenarios (4b), where a higher count indicates more effective robust solutions. A significant insight from our research is that nearly 40 data points collected are sufficient to derive an optimal network design that is robust in approximately 90% of cases. Consequently, the DRCC model has been shown to be capable of finding the robust distribution for energy consumption with fewer data points, contingent on the scenarios characteristics.



(a) Observation quantity and system cost (b) Observation quantity and scenario feasibility

Figure 4: Effect of number of observations in the DRCC modeling on system's costs and performance

4 Conclusions

In summary, this study addresses the challenge of designing a BEB network with limited data, focusing on the placement and type of charging stations and battery capacity decisions. We compare the effectiveness of different models, including nominal and robust optimization models, under real-world energy consumption uncertainties. The findings demonstrate that the DRCC model offers a more cost-effective and robust approach for BEB network design under energy consumption uncertainties, outperforming nominal models and budget of uncertainty robust model, utilizing limited data. Future analysis can include exploring the effects of different energy consumption distribution scenarios on the optimization of network design and to reassess the results accordingly.

References

Alismail, F., Xiong, P., & Singh, C. (2017). Optimal wind farm allocation in multi-area power systems using distributionally robust optimization approach. *IEEE Transactions on Power Systems*, *33*(1), 536–544.

Azadeh, S. S., Vester, J., & Maknoon, M. (2022). Electrification of a bus system with fast charging stations: Impact of battery degradation on design decisions. *Transportation Research Part C: Emerging Technologies*, *142*, 103807.

Bai, Z., Yang, L., Fu, C., Liu, Z., He, Z., & Zhu, N. (2022). A robust approach to integrated wireless charging infrastructure design and bus fleet size optimization. *Computers & Industrial Engineering*, *168*, 108046.

Chen, Z., Kuhn, D., & Wiesemann, W. (2022). Data-driven chance constrained programs over wasserstein balls. *Operations Research*.

Dallinger, D. (2013). *Plug-in electric vehicles integrating fluctuating renewable electricity* (Vol. 20). kassel university press GmbH.

Gao, R., & Kleywegt, A. (2023). Distributionally robust stochastic optimization with wasserstein distance. *Mathematics of Operations Research*, *48*(2), 603–655.

Kunith, A., Mendelevitch, R., & Goehlich, D. (2017). Electrification of a city bus network—an optimization model for cost-effective placing of charging infrastructure and battery sizing of fast-charging electric bus systems. *International Journal of Sustainable Transportation*, *11*(10), 707–720.

Liu, Z., Song, Z., & He, Y. (2018). Planning of fast-charging stations for a battery electric bus system under energy consumption uncertainty. *Transportation Research Record*, *2672*(8), 96–107.

Rios, M., Munoz, L., Zambrano, S., & Albarracin, A. (2014). Load profile for a bus rapid transit flash station of full-electric buses. In *Ieee pes innovative smart grid technologies, europe* (pp. 1–6).

Scarinci, R., Zanarini, A., & Bierlaire, M. (2019). Electrification of urban mobility: The case of catenary-free buses. *Transport Policy*, *80*, 39–48.

Wang, T.-H., & Hong, Y.-W. P. (2015). Learning-based energy management policy with battery depth-of-discharge considerations. In *2015 ieee global conference on signal and information processing (globalsip)* (pp. 992–996).

Yıldırım, Ş., & Yıldız, B. (2021). Electric bus fleet composition and scheduling. *Transportation Research Part C: Emerging Technologies*, *129*, 103197.

Zang, Y., Wang, M., & Qi, M. (2022). A column generation tailored to electric vehicle routing problem with nonlinear battery depreciation. *Computers & Operations Research*, *137*, 105527.

Zhang, S., Ge, S., Liu, H., Li, J., & Wang, C. (2023). Model and observation of the feasible region for pv integration capacity considering wasserstein-distance-based distributionally robust chance constraints. *Applied Energy*, *347*, 121312.

Zhou, Y., Ong, G. P., Meng, Q., & Cui, H. (2023). Electric bus charging facility planning with uncertainties: Model formulation and algorithm design. *Transportation Research Part C: Emerging Technologies*, *150*, 104108.