Addressing First-In-First-Out in System Optimal Dynamic Traffic Assignment on Multi Origin-Destination Networks

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SHORT SUMMARY

In this paper, we present a System Optimal Dynamic Traffic Assignment (SO-DTA) framework for multi-destination networks, incorporating hierarchical First-In-First-Out (FIFO) principles at junctions and system-wide levels. Junction level FIFO, also known as local FIFO, ensures orderly traffic flow and proportional distribution at diverging links per destination. System-wide FIFO, or Global FIFO, maintains consistent vehicle-to-destination ratios across all Origin-Destination (OD) pairs. We expand existing mathematical programming frameworks for SO-DTA to include such FIFO constraints. Our model also includes a link-level macroscopic fundamental diagram (MFD) approach for a more accurate representation of link travel times compared to traditional triangular FD and a generic cost function, enhancing the optimization framework's flexibility to accommodate different objectives such as Total System Travel Time (TSTT) and Total System Emissions (TSE). Additionally, we have ensured perfect compatibility between the new FIFO constraints and the usual Non-Vehicle Holding (NVH) ones that are necessary in SO-DTA mathematical programming frameworks.

1. INTRODUCTION:

In the field of transportation network management, optimizing traffic flow and reducing congestion remains a significant challenge, especially within complex, multi-destination networks. Dynamic Traffic Assignment (DTA) is essential in this area, as it aims to dynamically assign routes to vehicles based on time-varying demand and traffic conditions. However, when traffic assignment is extended to dynamic scenarios and formulated as an explicit mathematical program, the solution becomes more complex. This complexity arises because DTA problems aim to reflect real-world traffic behaviors.

Among these realistic traffic behaviors, the FIFO principle is critical. FIFO implies that vehicles should exit links in the same order they enter (Lo and Szeto 2002; Long et al. 2011; Carey et al. 2014). This principle is crucial in managing traffic flow, especially in congested situations, as it prevents the unrealistic scenario of faster vehicles overtaking slower ones in traffic jams. The FIFO concept, while straightforward in theory, presents numerous complexities in practical application, especially in multi destination networks. At the core of this challenge is ensuring FIFO not only at junctions, known as Local FIFO, but also at a more aggregated network-wide level, referred to as Global FIFO. Local FIFO refers to the orderly flow of traffic at individual diverging links in a network, ensuring proportional distribution per destination. Global FIFO, on the other hand, extends this principle to the entire network.

Despite the importance of FIFO in realistic traffic modeling, its explicit integration in mathematical frameworks for System-Optimal DTA (SO-DTA) has been limited. Most existing models focus either on single-destination, single-commodity traffic networks (e.g., Merchant and Nemhauser 1978a, b; Ziliaskopoulos 2000; Zheng and Chiu 2011) or on multi destination networks without explicit FIFO considerations (e.g., Zhu and Ukkusuri 2013; Doan and Ukkusuri 2015). While models such as that by Long and Szeto (2019) contribute significantly to multi-destination network analysis, they specifically focus on minimizing total system travel time (TSTT). This specific focus limits the model's versatility in addressing a variety of cost functions. Additionally, most models often overlook the intricate dynamics of traffic flow, notably the lack of consideration for hierarchical FIFO adherence, marking distinct areas for potential enhancement in these approaches.

In this paper, we introduce an optimization framework to solve SO-DTA problems by integrating the link macroscopic fundamental diagram (MFD) based traffic model, an advancement articulated in our previous work (Shakoori et al. 2022). Contrasting with the Link Transmission Model (LTM) based SO-DTA models developed by Long et al. (2018) and Ngoduy et al. (2016), which focus on networks with a single destination or a single origin-destination (OD) pair, our model addresses networks with multiple origins and destinations. Our SO-DTA model introduces a flexible cost function in its mathematical programming formulation. Unlike previous studies focusing mainly on TSTT, our model accommodates various metrics like TSTT, Total System Emission (TSE), enhancing adaptability and relevance in diverse traffic scenarios. Finally, our approach aims to address FIFO in general networks, enhancing the fidelity of traffic flow modeling under FIFO constraints. This is a key differentiator from models like those of Ziliaskopoulos (2000) and Zheng and Chiu (2011), where FIFO naturally emerges in single-destination contexts, and from Levin’s (2017) multiple-OD SO-DTA model that does not consider FIFO.

Table 1: Notations

Notation	Description
T	Set of discrete time intervals
Δt	Time interval duration
C_R	Set of origin links
C_S	Set of destination links
C	Set of links except for the destination links
Q_i^t	The maximum flow that can get into or out of link i at time interval t
l_i	Length of link i
$d^{O,D,t}$	Demand from origin O to destination D at interval t
$x_i^{O,D,t}$	Number of vehicles in link i during time t oriented from origin O headed to destination D
$y_{i,j}^{O,D,t}$	Number of vehicles moving from link i to link j during time t from origin O to destination D
$k_i^{O,D,t}$	OD Segregated density of link i at time t
k_i^t	Aggregated density of link i at time t
$\Gamma(i)$	Set of successor links of link i
$\Gamma(i)^{-1}$	Set of predecessor links of link i
$\Gamma(i)^{O,D}$	Set of successor links of link i on the paths from origin O to destination D
$\Gamma(i)^{-1,O,D}$	Set of predecessor links of link i on the paths from origin O to destination D
$D(k_i^t)$	Aggregated demand in link i during time t as a function of density
$D(k_i^{O,D,t})$	Segregated demand in link i from origin O to destination D during time t as a function of density
$S(k_i^t)$	Aggregated supply at link i during time t as a function of density

2. METHODOLOGY

SO-DTA without FIFO

In this section, we outline a mathematical programming framework for a link-MFD based SO-DTA model, incorporating a set of essential constraints. These constraints include the network's initial state, mass flow conservation, and flow propagation relations. For integration into the framework, the flow propagation requires transformations, adopting the LP transformation as proposed by Ziliaskopoulos (2000). However, this linearization leads to the well-known vehicle holding (VH) issue in discrete-time SO-DTA models, where vehicles hesitate to move from upstream to downstream links due to exit flows being bounded but not strictly equal to the minimum of demand and supply. To mitigate this issue, we incorporate Non-vehicle Holding (NVH) constraints into our framework. It is important to note that the entire set of constraints are directly adopted from the methodologies presented in the paper by Shakoori et al. (2022). The nomenclature used is summarized in Table 1.

Link MFD-based Dynamic Network Constraints

$$x_i^{O,D,0} = 0,\ y_{i,j}^{O,D,0} = 0 \qquad \forall i \in C, \forall j \in C, \forall O \in C_R, D \in C_S \tag{1}$$

$$x_i^{O,D,t} \geq 0,\ y_{i,j}^{O,D,t} \geq 0 \qquad \forall i \in C \cup C_s, \forall O \in C_R, D \in C_S, \in T \tag{2}$$

$$x_i^{O,D,t} - x_i^{O,D,t-1} + \sum_{j \in \Gamma^{\mathrm{O,D}}(i)} y_{i,j}^{O,D,t-1} = d^{O,D,t-1} \qquad \forall i \in C_R, O \in C_R, D \in C_S, \forall t \in T \tag{3}$$

$$x_i^{O,D,t} - x_i^{O,D,t-1} - \sum_{k \in \Gamma^{-1,\mathrm{O,D}}(i)} y_{k,i}^{O,D,t-1} + \sum_{j \in \Gamma^{\mathrm{O,D}}(i)} y_{i,j}^{O,D,t-1} = 0 \qquad \forall i \in C \backslash C_R, O \in C_R, D \in C_S, t \in T \tag{4}$$

$$\sum_{O,D} \sum_{j \in \Gamma^{O,D}(i)} y_{i,j}^{O,D,t} \leq D(k_i^t) \times \Delta t \qquad O \in C_R, D \in C_S \tag{5}$$

$$\textstyle\sum_{O,D} \sum_{\mathrm{k} \in \Gamma^{-1,O,D}(\mathrm{j})} y_{\mathrm{k},j}^{O,D,t} \leq S(k_j^t) \times \Delta t \qquad O \in C_R, D \in C_S,\ \forall j \in \Gamma(i) \tag{6}$$

$$\sum_{j \in \Gamma^{O,D}(i)} y_{i,j}^{O,D,t} \leq D(k_i^{O,D,t}) \times \Delta t \qquad \forall t \in T, O \in C_R, D \in C_S \tag{7}$$

$$-\left[\sum_{a=1}^{m_i} \theta_i^a(t)\right] M \leq \sum_{O,D} \sum_{j \in \Gamma^{O,D}(i)} y_{i,j}^{O,D,t} - D(k_i^t) \times \Delta t \qquad \forall i \in C, \forall t \in T \tag{8}$$

$$-\left[\sum_{a=1}^{m_i} \sigma_a^g - \sum_{a=1}^{m_i} (2\sigma_a^g - 1)\, \theta_i^a(t)\right] M \leq \sum_{O,D} \sum_{\mathrm{k} \in \Gamma^{-1,\mathrm{O,D}}(j_g)} y_{\mathrm{k},j_g}^{O,D,t} - S(k_{j_g}^t) \times \Delta t \qquad \forall i \in C, t \in T, \mathrm{g} \in G_i \tag{9}$$

$$\sum_{a=1}^{m_i} 2^a \theta_i^a(t) \leq 2|\Gamma(i)| \qquad \forall i \in C, \forall t \in T \quad (10)$$

$$\theta_i^a(t) \in \{0,1\}, a = 1, \ldots, m_i \qquad \forall i \in C, \forall t \in T \quad (11)$$

where M is a very large positive value, $G_i = \{1, 2, \ldots, |\Gamma(i)|\}$ is an index set for link i's successor links, j_g is the g-th link in $\Gamma(i)$, $m_i = \text{argmin}_m\{2^{m+1} \geq 2 + 2 \times |\Gamma(i)|\}$, and σ_a^g is 0 or 1, such that $\sum_{a=1}^{m_i} 2^{a-1} \times \sigma_a^g = g$.

Equation (1) captures the network's initial state. The variables' non-negativity is maintained through Equation (2). Link mass conservation is captured by Equations (3)-(4). The constraints for flow propagation are represented in Equations (5)-(7). Lastly, Equations (8)-(11) ensure the NVH conditions.

In the above formulation, the variables $D(k_i^t)$, $S(k_j^t)$, and $D(k_i^{O,D,t})$, denoting aggregated demand, supply, and segregated demand, need precise definitions for linear adaptation. We achieve this through convex combination formulation and the use of Type 2 Special Ordered Sets (SOS2), linearly defining these variables derived from the piece-wise linear link MFDs, as extensively detailed in the paper by Shakoori et al. (2022).

SO-DTA with FIFO:

The FIFO principle suggests that the order in which vehicles enter a road segment or link directly influences their exit order, with earlier entrants leaving before those that arrive later. FIFO operates under the assumption that different vehicle types entering a link around the same time will move at a similar speed. Therefore, vehicles entering a link first are generally expected to exit it first, maintaining an orderly and consistent traffic flow.

In the literature, the FIFO condition for traffic flow on a link is defined with reference to link travel times. As Carey (2004) elucidates, a traffic flow on link i adheres to the FIFO condition if the following holds true for all time instances t_1 and t_2 within the interval [0, T]:

$$t_1 > t_2 \Rightarrow t_1 + \tau_i(t_1) \geq t_2 + \tau_i(t_2) \qquad (12)$$

Here, $\tau_i(t_1)$ *and* $\tau_i(t_2)$ represent the travel times for vehicles entering link i at respective time instants t_1 and t_2.

A direct interpretation of the relation (12) suggests that vehicles entering a link later should exit later than earlier arrivals. However, this also implies simultaneous entrants to link i should experience identical travel times and therefore exit together.

Building on this, consider link i as a diverging junction leading to links j_1 and j_2. As shown in Figure (1.a), total demand D_i at this junction splits into D_{j_1} for link j_1 and D_{j_2} for j_2. If supply is insufficient and not all of D_{j_2} can move forward, a fraction y_{i,j_2} progresses. This leads to a delay δ for vehicles to link j_2 at time t. A similar mechanism applies to link j_1, allowing calculation of y_{i,j_1} (refer to figure 1.b).

To mathematically represent this concept, we introduce two equations detailing the relationship between total traffic demand on each successor link and the fraction of traffic advancing during congestion:

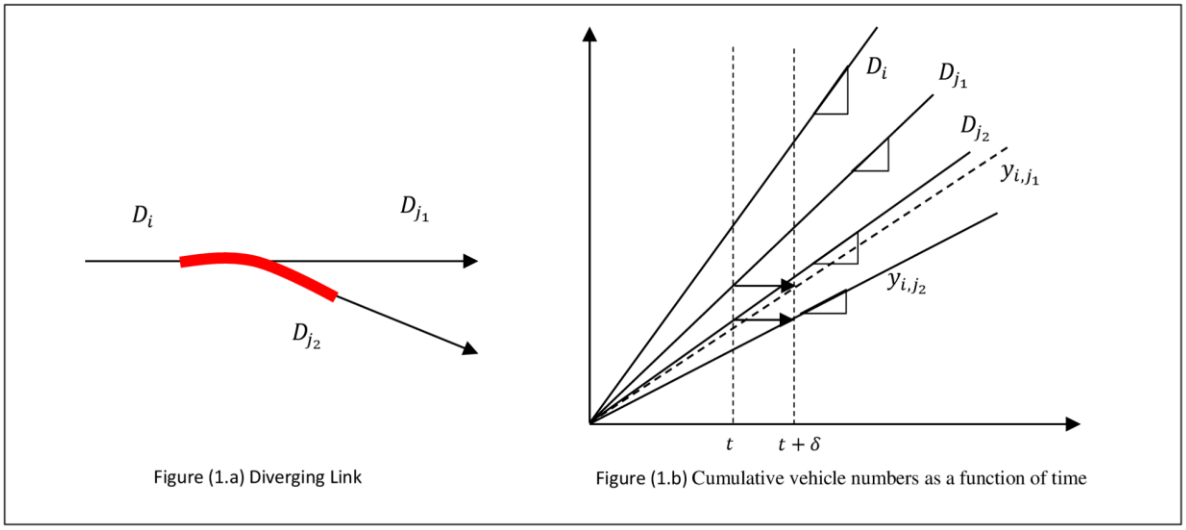

Figure 1. Traffic Distribution and Delay Dynamics at a Diverging link

For successor link j_2: $D_{j_2} \times t = \ y_{i,j_2} \times (t + \delta)$ (13.a)
For successor link j_1: $D_{j_1} \times t = \ y_{i,j_1} \times (t + \delta)$ (13.b)

These equations capture the traffic flow dynamics at the diverging junction, demonstrating how total demand at each link adapts to delays by adjusting the fraction of traffic that progresses through congestion.

These formulations lead to a key insight about flow proportions on the links. The ratio of traffic that can proceed on each link relative to its total demand remains constant, as indicated by the following relationship:

$$\frac{y_{i,j_2}}{D_{j_2}} = \frac{y_{i,j_1}}{D_{j_1}} \quad (14)$$

This equation indicates that the reduction in traffic flow due to congestion is proportionally equal for both links. This adheres to the FIFO principle, promoting equal travel times during congestion. In equation (14), FIFO compliance in congestion means equalizing travel time across all diverging junction's successor links, which translates to keeping a steady flow-to-demand ratio for each successor link.

In a multi-destination network, this method introduces a hierarchical FIFO Principle, involving local and global FIFO compliance. Local FIFO focuses on preserving flow-to-demand ratios near each diverging link, ensuring orderly traffic for each destination. Conversely, global FIFO expands this concept across the network, managing traffic ratios and sequences across multiple destinations to promote efficient, organized movement throughout the entire network.

Local FIFO: Local FIFO is applied at each diverging link for individual OD pairs. A diverging link i splits into multiple successor links, denoted as $\{j_1, j_2, \ldots, j_n\} = \Gamma^{O,D}(i)$, forming paths for a specific OD pair. We define partial OD segregated demand as the OD pair's demand distributed among a diverging link's successors. The total demand at link i for each OD pair is split across various paths via successor links j, termed $D_{i,j}^{OD}$, representing the allocated portion of total OD demand for each successor at the diverging link.

Local FIFO ensures a balanced and proportional vehicle distribution for each OD pair at every diverging link. It maintains a uniform flow-to-demand ratio across all successors originating from the same predecessor, guaranteeing consistent link travel times for each specific destination, regardless of the successors used:

$$\frac{y_{i,j_1}^{OD}}{D_{i,j_1}^{OD}} = \frac{y_{i,j_2}^{OD}}{D_{i,j_2}^{OD}} = \cdots = \frac{y_{i,j_n}^{OD}}{D_{i,j_n}^{OD}} \qquad \forall i \in Diverging\ links, \forall OD \qquad (15)$$

Global FIFO: Across the entire network, the ratio of the number of vehicles heading to a specific destination to the demand for that destination must be identical for all destinations. Unlike local FIFO, which applies only to diverging links, global FIFO applies to all network links, regardless of successor count. Global FIFO in a network with m OD pairs is mathematically formulated as:

$$\frac{\sum_{j\in\Gamma^{(OD)_1}(i)} y_{i,j}^{(OD)_1}}{D_i^{(OD)_1}} = \frac{\sum_{j\in\Gamma^{(OD)_2}(i)} y_{i,j}^{(OD)_2}}{D_i^{(OD)_2}} = \cdots = \frac{\sum_{j\in\Gamma^{(OD)_m}(i)} y_{i,j}^{(OD)_m}}{D_i^{(OD)_m}} \qquad \forall i \qquad (16)$$

In equation (16), the numerators denote the total flow for a specific OD pair exiting link i through all its successors $j \in \Gamma^{OD}(i)$, representing the collective flow from link i to various paths to the designated destination. The denominator indicates the OD segregated demand at each link i.

This comprehensive FIFO strategy ensures a balanced demand distribution network-wide for each destination, aiming for equitable traffic flow relative to each destination's demand across all links. This results in equal travel times for vehicles to different destinations, maintaining the FIFO principle throughout the network.

In both local and global FIFO considerations within a network, it's crucial to observe a conditional constraint. Let us consider a diverging link with two successors as an example to illustrate the conditional local FIFO, and a network with two OD pairs to demonstrate the conditional global FIFO:

Conditional Local FIFO: At a diverging link, if the partial OD segregated demand for any exit is zero, indicating no designated vehicles for that exit for a specific OD pair, local FIFO needs careful consideration and becomes conditional. Local FIFO applies only if both exits have nonzero demand. If either successor link at the divergence has zero demand, which happens when the optimal solution for SO-DTA steers the flow to a restrictive set of destinations, local FIFO is disregarded. In such scenarios, where a successor is unused for an OD pair, the diverging junction essentially acts as an ordinary link, making global FIFO the main principle, as the divergence becomes irrelevant for that OD pair.

To clearly define this approach, consider a diverging link, i, with two successor links $j_1$and j_2. Let D_{i,j_1}^{OD} and D_{i,j_2}^{OD} represent the demand for exits $j_1$and j_2 for a specific OD pair. The condition for applying the local FIFO can be formulated as:

Local FIFO Application Condition:

$$\text{If } D_{i,j_1}^{OD} \times D_{i,j_2}^{OD} > 0, \text{ then enforce local FIFO: } \frac{y_{i,j_1}^{OD}}{D_{i,j_1}^{OD}} = \frac{y_{i,j_2}^{OD}}{D_{i,j_2}^{OD}} \quad (17)$$

Local FIFO Relaxation Condition:

$$\text{Otherwise, if } D_{i,j_1}^{OD} \times D_{i,j_2}^{OD} = 0, \text{ then local FIFO considerations are ignored.} \quad (18)$$

This formalizes the conditions under which the local FIFO rule is either applied or ignored, based on the demand at each exit.

Conditional Global FIFO: Similarly, if the OD segregated demand at link i for a particular OD pair is zero, indicating all demand targets a single destination, the global FIFO condition needs careful consideration. Its enforcement is conditional, relevant only when all destinations have nonzero demand. If any OD pair has zero demand, global FIFO can be set aside. In such cases, focus shifts to local FIFO, as traffic unifies towards one destination, rendering global FIFO redundant and making local FIFO the governing traffic management principle.

To provide a clear definition of this approach, consider a network with only two OD pairs, $(OD)_1$and $(OD)_2$. Let the $D_i^{(OD)_1}$ and $D_i^{(OD)_2}$ denote the OD segregated demand at each link i. The condition for applying the global FIFO can be formulated as:

Global FIFO Application Condition:

$$\text{If } D_i^{(OD)_1} \times D_i^{(OD)_2} > 0, \text{ then enforce global FIFO: } \frac{\sum_{j \in \Gamma^{(OD)_1}(i)} y_{i,j}^{(OD)_1}}{D_i^{(OD)_1}} = \frac{\sum_{j \in \Gamma^{(OD)_2}(i)} y_{i,j}^{(OD)_2}}{D_i^{(OD)_2}} \quad (19)$$

Global FIFO Relaxation Condition:

$$\text{Otherwise, if } D_i^{(OD)_1} \times D_i^{(OD)_2} = 0, \text{ then global FIFO considerations are ignored.} \quad (20)$$

It's important to note that the concept of conditional local and global FIFO, while illustrated using a diverging link with two successors and a network with two OD pairs, is applicable to any diverging link regardless of the number of successors and any transportation networks regardless of number OF OD pairs. The general principle is as follows:

Local FIFO: If a specific successor's demand at a diverging link is zero, that successor should be omitted from local FIFO considerations. This applies to any diverging link with multiple successors, permitting local FIFO constraint relaxation for paths lacking demand.

Global FIFO: If OD segregated demand at a link for a particular OD pair is zero, meaning no traffic flow for that pair at the link, that OD pair should be excluded from global FIFO considerations. This holds regardless of the network's OD pair count.

Finally, by adopting linear (e.g., TSTT) or linearized non-linear (e.g., TSE through PWL approximations) cost functions, we will be able to establish a robust optimization framework. The inclusion of specific traffic features and constraints can shift the problem from a Mixed Integer Linear format (excluding FIFO) to a non-convex format (including FIFO), posing a challenge for further enhancement within this framework.

3. RESULTS AND DISCUSSION

The above mathematical model is applied to a small network, as shown in Figure 2, under a time-dependent demand. The study spans 9 steps, with each step lasting 20 seconds. The network contains two OD pairs. Vehicle entry rate is 0.4 vehicles/s for each OD pair.

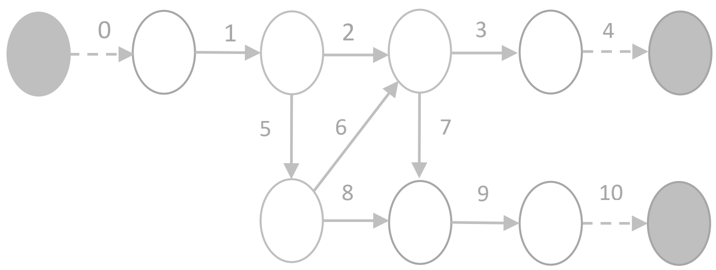

Figure 2. Simulated network

The PWL MFD for each link consists of 5 breakpoints. The demand function for link i begins at $(0,0)$ and ends at (k_{jam}^{i}, Q_i), while the supply function starts at $(0, Q_i)$ and concludes at $(k_{jam}^{i}, 0)$. Table 2 summarizes the link characteristics and other breakpoints for the demand and supply functions of each link. All links have a jam density assumed to be 0.17 vehicles/m.

Table 2: Characteristics of the links in the simulated network

Link	Length (m)	Max flow (veh/s)	Speed limit (m/s)	Breakpoints on the MFD: Demand function			Supply function		
1	250	0.5	12.5	(0.02,0.25)	(0.04,0.40)	(0.06,0.5)	(0.08,0.5)	(0.12,0.35)	(0.14,0.24)
2	250	0.3	12.5	(0.01,0.13)	(0.02,0.20)	(0.04,0.3)	(0.06,0.3)	(0.10,0.23)	(0.14,0.12)
3	250	0.5	12.5	(0.02,0.25)	(0.04,0.40)	(0.06,0.5)	(0.08,0.5)	(0.12,0.35)	(0.14,0.24)
5	250	0.5	12.5	(0.02,0.25)	(0.04,0.40)	(0.06,0.5)	(0.08,0.5)	(0.12,0.35)	(0.14,0.24)
6	250	0.5	12.5	(0.02,0.25)	(0.04,0.40)	(0.06,0.5)	(0.08,0.5)	(0.12,0.35)	(0.14,0.24)
7	250	0.5	12.5	(0.02,0.25)	(0.04,0.40)	(0.06,0.5)	(0.08,0.5)	(0.12,0.35)	(0.14,0.24)
8	300	0.5	15	(0.01,0.15)	(0.03,0.38)	(0.05,0.5)	(0.07,0.5)	(0.10,0.40)	(0.14,0.20)
9	250	0.5	12.5	(0.02,0.25)	(0.04,0.40)	(0.06,0.5)	(0.08,0.5)	(0.12,0.35)	(0.14,0.24)

Numerical Example and Results

Optimal solution analysis with and without FIFO: In this analysis, we scrutinize the implementation of the FIFO principle in optimizing routing decisions. For simplicity, we limit our discussion to TSTT, though it's noteworthy that the same analysis is applicable to the TSE cost function. Table 3 and Figure 3 serve as references for understanding the implications of these principles. Initially, in the absence of FIFO, there's a noticeable inconsistency in travel times across various destinations, indicative of a non-standardized traffic flow. However, the incorporation of FIFO principles modifies this pattern, ensuring that demand-to-flow ratios for successor links align, as detailed in Table 3 and travel times for different destinations become consistent, as depicted in Figure 3.

Table 3: Comparative Analysis of Traffic Flow: Local FIFO Validation

Principle	t	D_1^t	$D_1^{0,4,t}$	$D_1^{0,10,t}$	$D_{1,2}^{0,4,t}$	$D_{1,5}^{0,4,t}$	$D_{1,2}^{0,10,t}$	$D_{1,5}^{0,10,t}$	$y_{1,2}^{0,4,t}$	$y_{1,5}^{0,4,t}$	$y_{1,2}^{0,10,t}$	$y_{1,5}^{0,10,t}$	Local FIFO OD: 0-4 $\frac{y_{1,2}^{0,4,t}}{D_{1,2}^{0,4,t}}$	Local FIFO OD: 0-4 $\frac{y_{1,5}^{0,4,t}}{D_{1,5}^{0,4,t}}$	Local FIFO OD: 0-10 $\frac{y_{1,2}^{0,10,t}}{D_{1,2}^{0,10,t}}$	Local FIFO OD: 0-10 $\frac{y_{1,5}^{0,10,t}}{D_{1,5}^{0,10,t}}$
Non-FIFO	1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-	-	-	-
	2	8,00	2,80	5,20	2,52	0,28	1,04	4,16	2,52	0,28	1,04	4,16	1	1	1	1
	3	8,80	7,83	0,97	5,91	1,92	0,09	0,88	5,91	0,66	0,09	0,34	1	0,34	1	0,39
	4	10,00	3,64	6,36	3,28	0,36	1,27	5,08	3,28	0,36	1,27	5,08	1	1	1	1
	5	10,00	1,22	8,78	0,98	0,24	1,76	7,02	0,98	0,24	1,76	7,02	1	1	1	1
	6	10,00	0,46	9,54	0,37	0,09	1,91	7,63	0,37	0,09	1,91	7,63	1	1	1	1
	7	10,00	3,84	6,16	3,45	0,38	0,62	5,55	3,45	0,38	0,62	5,55	1	1	1	1
	8	10,00	6,62	3,38	5,95	0,67	0,05	3,33	5,95	0,66	0,05	0,46	1	1	1	0,14
	9	10,00	1,85	8,15	1,67	0,19	0,81	7,33	1,67	0,19	0,81	7,33	1	1	1	1
FIFO	1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-	-	-	-
	2	8,00	4,00	4,00	4,00	0,00	0,00	4,00	4,00	0,00	0,00	4,00	1	-	-	1
	3	8,80	4,40	4,40	4,40	0,00	0,00	4,40	4,40	0,00	0,00	4,40	1	-	-	1
	4	9,28	4,64	4,64	4,64	0,00	0,00	4,64	4,64	0,00	0,00	4,64	1	-	-	1
	5	9,57	4,78	4,78	4,78	0,00	0,00	4,78	4,78	0,00	0,00	4,78	1	-	-	1
	6	9,74	4,87	4,87	4,87	0,00	0,00	4,87	4,87	0,00	0,00	4,87	1	-	-	1
	7	9,84	4,92	4,92	4,92	0,00	0,00	4,92	4,92	0,00	0,00	4,92	1	-	-	1
	8	9,91	4,95	4,95	1,13	3,82	0,00	4,95	1,13	3,82	0,00	4,95	1	1	-	1
	9	9,94	4,97	4,97	1,32	3,65	4,68	0,29	1,32	3,65	4,68	0,29	1	1	1	1

Table 3 specifically affirms the adherence to Local FIFO for Link 1, a representative diverging link, by revealing aligned demand-to-flow ratios at this specific link for each destination. Figure 3 further illustrates the impact of FIFO at the network level (Global FIFO). Figure 3.a displays the cumulative inflow and outflow on Link 1 when FIFO is not applied, resulting in diverse travel times for different destinations. This contrasts with the scenario in Figure 3.b, where FIFO guarantees consistent travel times for Link 1 across all destinations.

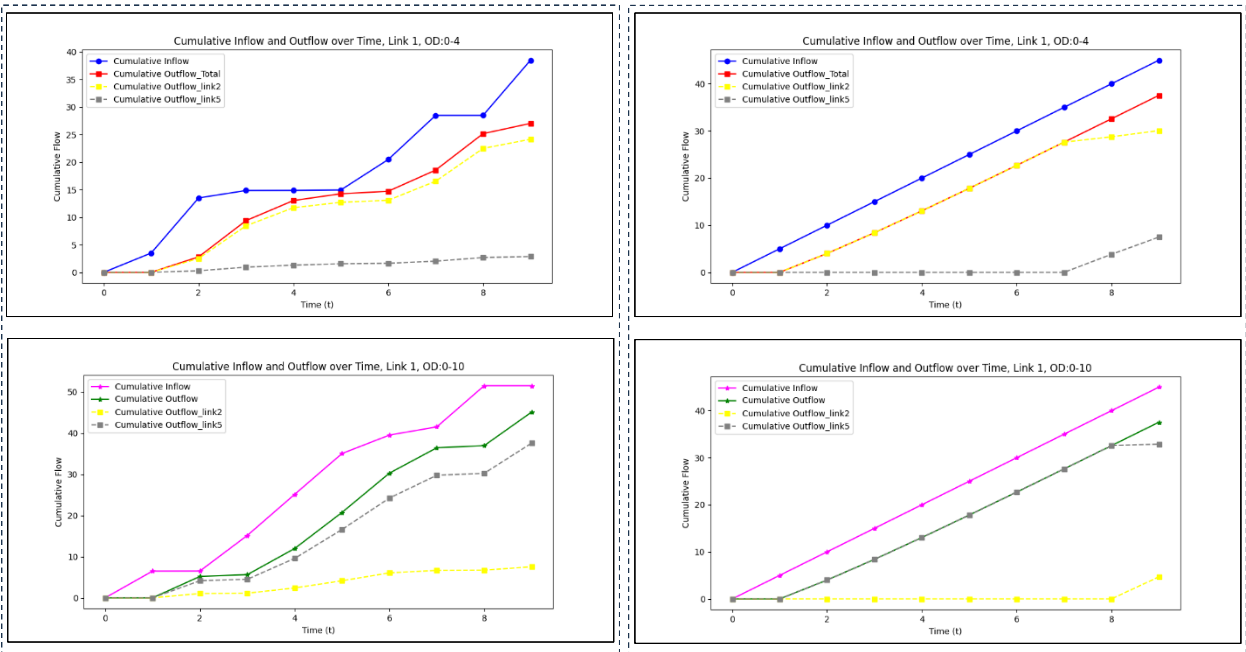

Figure (3.a) Non-FIFO

Figure (3.b) FIFO

Figure 3. Comparative Analysis of Traffic Flow: Global FIFO Validation

Comparison of Cost Functions in Different Routing Scenarios: This section compares cost function outcomes in two scenarios: the baseline, reflecting typical user choices without traffic information, and the SO-MFD-DTA based on our optimization approach. In the baseline scenario, vehicles follow the shortest path in distance for each OD pair, leading to increased cost function and reduced system efficiency due to individualistic routing. Conversely, the SO-MFD-DTA scenario optimizes routes considering network-wide equilibrium, resulting in a notable decrease in cost function.

Table 4: Cost Function Comparison

	Cost Function	
Scenario	TSTT(s)	TSE(g)
FIFO-Baseline	13306	12224
FIFO-TSTT	12117	13010
FIFO-TSE	13247	12022
NON-FIFO-TSTT	11750	12740
NON-FIFO-TSE	13058	11965

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