Joint Trajectory Planning for 1-D Snake-like Interruptible Vehicle Platoons in Lane-free Traffic

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SHORT SUMMARY

This paper presents a joint trajectory optimization algorithm for connected and automated vehicles forming 1-D snake-like interruptible platoons in a lane-free traffic environment on highways. A double double-integrator model is employed for the longitudinal and lateral movement, respectively, of each vehicle, considering constant and state-dependent bounds on control inputs, including road boundary constraints. An appropriately defined Euclidian distance to track the front vehicle is utilized to establish and operate a 1-D platoon of vehicles at selectable distance from each other. The distance is used as a soft constraint, which allows for platoons to divide, if required to avoid obstacles, and reunite eventually. A joint multi-objective function is formulated and minimized using an efficient feasible direction algorithm. This leads, beyond the flexible platoon operation, to low fuel consumption, passenger convenience, collision avoidance with external obstacles, tracking of desired speed and prevention of infeasible maneuvers. Challenging scenarios are examined on a lane-free straight motorway stretch, producing promising results for further exploration, like forming 2-D interruptible vehicular flocks with flexible shape or implementation of 1-D platoons and flocks in an MPC framework.

Keywords: Vehicular platooning, lane-free traffic, automated vehicles, joint path planning, snake-like interruptible platoons.

1. INTRODUCTION

Connected and automated vehicles (CAVs) have substantially extended abilities and may potentially improve traffic flow safety and efficiency (Diakaki et al., 2015; Sjoberg et al., 2017). Recently, the TrafficFluid concept was proposed (Papageorgiou et al., 2021), as a novel paradigm for vehicular traffic in the era of CAVs, applicable at high levels of vehicle automation and communication and high penetration rates, which are expected to prevail in the not-too-far future. The TrafficFluid concept is based on two combined principles: (i) Lane-free movement on the 2-D road surface, meaning that vehicles are not bounded to fixed traffic lanes; (ii) Vehicle nudging, i.e. vehicles may be influencing other vehicles in front of them via sensors or communication.

An interesting topic, both for lane-based and lane-free traffic, is vehicular platooning, which has drawn a lot of attention due to its potential advantages in improving safety, elevating roadway capacity and reducing fuel consumption (Axelsson, 2016; Chen et al., 2022; Li, et al., 2022; Shen, et al., 2022; Varaiya, 1993; Wang et al., 2019). Picking few examples from the vast related literature, Chen et al. (2022) employed a cooperative optimal control method for CAV platoons, where a cost function combining the position, speed and acceleration errors of vehicles, is minimized to achieve cooperative performance and reduce energy consumption. A platoon-based cooperative optimal control algorithm for CAVs at highway on-ramps under heavy traffic is discussed by Xue et al. (2023). Another work, addressing lane-free traffic, presents a consensus algorithm for building 2-D CAV flocks (Rostami-Shahrbabaki et al., 2023).

In the lane-free context, Dabestani et al. (2023) proposed a centralized nonlinear constrained Optimal Control Problem (OCP) for joint trajectory optimization of a group of individual CAVs. The OCP is solved numerically via an efficient Feasible Direction Algorithm (FDA) and is applied simultaneously to all involved vehicles. Since multiple vehicles are jointly considered, their goals are accumulated in one common objective function. This leads to optimal decisions that are not "egoistic", as in decentralized approaches, but improve the average performance of all vehicles as one entity. The present work employs the joint optimization approach of (Dabestani et al., 2023), appropriately modified to form and operate a 1-D snake-like interruptible platoon of CAVs that have the same desired speed and move safely at desired distances from each other. The OCP is solved numerically via the efficient FDA and addresses all involved vehicles in a coordinated way. It is demonstrated that the vehicles drive safely at close distances from each other and at the desired longitudinal speed, elegantly avoiding collisions with external obstacles, sometimes at the price of temporarily breaking the platoon formation, if necessary or beneficial. Accelerations are low, yielding passenger convenience and efficient fuel consumption.

2. JOINT OPTIMAL CONTROL PROBLEM

Problem Variables, State Equations and Constraints

A discrete-time double double-integrator model is considered for each vehicle i, governed by corresponding longitudinal and lateral accelerations as control inputs:

$$x_i(k+1) = x_i(k) + Tv_{x,i}(k) + \tfrac{1}{2}T^2 a_{x,i}(k) \tag{1}$$

$$y_i(k+1) = y_i(k) + Tv_{y,i}(k) + \tfrac{1}{2}T^2 a_{y,i}(k) \tag{2}$$

$$v_{x,i}(k+1) = v_{x,i}(k) + Ta_{x,i}(k) \tag{3}$$

$$v_{y,i}(k+1) = v_{y,i}(k) + Ta_{y,i}(k) \tag{4}$$

where x_i and y_i are longitudinal and lateral positions; $v_{x,i}$ and $v_{y,i}$ are longitudinal and lateral speeds; $a_{x,i}$ and $a_{y,i}$ are control inputs reflecting the longitudinal and lateral accelerations; $k = 0,1,2,...$ is the discrete time index, and T is the step size, related to time t via $t = kT$.

Lower and upper bounds on longitudinal and lateral accelerations are summarized as follows:

$$\begin{aligned} a_{x,i}^{\min}(\mathbf{x}_i(k)) \le a_{x,i}(k) \le a_{x,i}^{\max} \\ a_{y,i}^{\min}(\mathbf{x}_i(k)) \le a_{y,i}(k) \le a_{y,i}^{\max}(\mathbf{x}_i(k)) \end{aligned} \tag{5}$$

where $\mathbf{x}_i = [x_i, y_i, v_{x,i}, v_{y,i}]^T$ is the vector of the state variables mentioned in (1)-(4). The upper bound of the longitudinal acceleration has a vehicle-specific constant value $a_{x,i}^{\max} = A_i^{\max}$ that depends on the vehicle's acceleration characteristics. The lower bound of longitudinal acceleration at time k derives from non-negativity of longitudinal speed at time $k+1$ and a vehicle-specific constant value $A_i^{\min}$, expressing vehicle's decceleration capabilities, as follows (see (Dabestani et al., 2023)):

$$a_{x,i}^{\min}(\mathbf{x}_i(k)) = \max\{-\frac{1}{T} v_{x,i}(k), A_i^{\min}\}. \tag{6}$$

The main requirement regarding the lateral acceleration bounds is that vehicles move within the road boundaries or exactly on a road boundary. In this study, the road boundaries are considered to be straight lines. Every vehicle located within the road boundaries at time steps k and $k+1$

should remain within the road boundaries at time step $k+2$ as well, i.e.:

$$\tilde{y} \le y_i(k+2) \le \hat{y} \tag{7}$$

where $\hat{y}$ and $\tilde{y}$ are the lateral positions of the left and right road boundary, respectively. Whenever the vehicle reaches the left or right road boundary from an admissible lateral position, i.e. the corresponding constraint in (7) is activated, then we must have for the lateral vehicle speed $v_{y,i} = 0$, as otherwise the vehicle would eventually exit the road. Thus, any approach of a road boundary must be asymptotical. Substituting lateral position y_i and lateral speed $v_{y,i}$ from (2) and (4), yields, after some rearrangements, state-dependent bounds on lateral acceleration, which can be interpreted as dead-beat controllers that drive the vehicle towards the corresponding road boundary and its lateral speed to zero in two time-steps. Generalizing those controllers, we get the state-dependent bounds on lateral acceleration (see (Yanumula et al., 2023)):

$$a_{y,i}^{\min}(\mathbf{x}_i(k)) = -K_{lat}\left[y_i(k) - \tilde{y}_i\right] + \left(\frac{T}{2}K_{lat} - 2\sqrt{K_{lat}}\right)v_{y,i}(k) \tag{8}$$

$$a_{y,i}^{\max}(\mathbf{x}_i(k)) = -K_{\text{lat}}\left[y_i(k) - \hat{y}_i\right] + \left(\frac{T}{2}K_{\text{lat}} - 2\sqrt{K_{\text{lat}}}\right)v_{y,i}(k) \tag{9}$$

where $0 < K_{\text{lat}} \le 1/T^2$ is a feedback controller gain that may be tuned appropriately for smooth boundary approach, while asymptotic behavior is guaranteed by construction.
The current problem with state-dependent control bounds is transformed to a problem with constant control bounds by replacing the original control variables $a_{x,i}$ and $a_{y,i}$ with the following equations:

$$\begin{aligned} a_{x,i}(k) &= (1-u_{x,i}(k))a_{x,i}^{\min}(\mathbf{x}_i(k)) + u_{x,i}(k)a_{x,i}^{\max} \\ a_{y,i}(k) &= (1-u_{y,i}(k))a_{y,i}^{\min}(\mathbf{x}_i(k)) + u_{y,i}(k)a_{y,i}^{\max}(\mathbf{x}_i(k)). \end{aligned} \tag{10}$$

Then, the new control variables $u_{x,i}$ and $u_{y,i}$ have constant bounds as follows:

$$0 \le u_{x,i}, u_{y,i} \le 1. \tag{11}$$

Objective Function

In this study, a group of n vehicles are subjected to joint trajectory planning under the requirement of forming a 1-D platoon, which moves, whenever possible, longitudinally aligned with a pre-specified inter-vehicle distance and with a given desired speed. The platoon is flexible in various respects to cope with current circumstances:

(i) Individual vehicle speeds may deviate from the desired speed.
(ii) Lateral snake-like movements may emerge, e.g. to avoid obstacles.
(iii) Longitudinal inter-vehicle distances may vary.
(iv) The platoon is interruptible, i.e. it may divide temporarily into two or more parts, if necessary or beneficial, before re-uniting again.

Without loss of generality, the longitudinal ordering of the platoon vehicles is (from upstream to downstream) $1, 2, ..., i, i+1, ..., n$. Though the OCP controls only the n platoon vehicles, other vehicles (obstacles), with known (predicted) trajectories, may be encountered, and collision with them must be avoided. We denote obstacles with sub-index $o \in O$.
To achieve these and other regular requirements, an appropriate joint objective function for all platoon vehicles, to be minimized, is defined, which is composed by several sub-objectives that are designed to: keep each platoon vehicle at a safe and flexible gap from the vehicle in front (thus creating and maintaining the 1-D platoon), avoid collisions with other circulating vehicles, mitigate fuel consumption and passenger discomfort, track the desired speed and suppress infeasible maneuvers. Each sub-objective must be continuous and differentiable.

The sub-criteria answering to fuel consumption and passenger comfort are given by the squares of the longitudinal and lateral accelerations $(a_{x,i}(k))^2$ and $(a_{y,i}(k))^2$, respectively.

Two quadratic cost terms are considered for tracking the common desired speeds, as $(v_{x,i}(k)-v^*_x)^2$ and $(v_{y,i}(k)-v^*_y)^2$, with v^*_x and v^*_y being longitudinal and lateral desired speeds, respectively.

In extreme cases, particularly at very low longitudinal speeds, the vehicle dynamics described in (1)-(4) might lead to infeasible maneuvers for the real vehicle, e.g. moving only laterally. To suppress such infeasible maneuvers, the following sub-objective is considered, which limits the ratio of lateral over longitudinal speeds ($\beta > 0$):

$$f_i^c(\mathbf{x}_i(k)) = \begin{cases} (\beta v_{x,i}(k) - |v_{y,i}(k)|)^2 & \text{if } |v_{y,i}(k)| > \beta v_{x,i}(k) \\ 0 & \text{otherwise} \end{cases}. \tag{12}$$

An ellipsoid function is created to penalize vehicles approaching too close to obstacles:

$$c_{io}(\mathbf{x}_i,\mathbf{x}_o) = \{1-\tanh[(\frac{x_i-\delta_{x_o}}{0.5d_1})^{p_1} + (\frac{y_i-y_o}{0.5d_2})^{p_2}] + \frac{1}{[(\frac{x_i-\delta_{x_o}}{0.25d_1})^{p_3} + (\frac{y_i-y_o}{0.25d_2})^{p_4}+1]^{p_5}}\} \tag{13}$$

where

$$\delta_{x_o} = x_o - (\omega_x(v_{x,i}-v_{x,o})/2) \tag{14}$$

and d_1 and d_2 are given by

$$\begin{aligned} d_1 &= L_i + \omega_x v_{x,i} + \omega_x v_{x,o} \\ d_2 &= W_i + \omega_y[(\tanh(y_o-y_i))(v_{y,i}-v_{y,o}) + \sqrt{(\tanh(y_o-y_i))^2(v_{y,i}-v_{y,o})^2+\varepsilon_\omega}] \end{aligned} \tag{15}$$

where ε_ω is a small positive value, L_i is the average length of vehicle i and the obstacle o and W_i is the average width of vehicle i and the obstacle o. Thus, the complete collision avoidance sub-objective is $\sum_{o\in O} c_{io}(\mathbf{x}_i,\mathbf{x}_o)$. Further details regarding the sub-objectives can be found in (Dabestani et al., 2023).

To form and operate a 1-D platoon, we softly impose a safe longitudinal distance D_i and zero lateral distance between each vehicle i and the vehicle $i+1$ located in front of it. Specifically, each vehicle located at (x_i, y_i) targets a location $(x_{i+1}-D_i, y_{i+1})$, where $D_i = D_0 + 0.5\omega_{\text{at}}(v_{x,i}+v_{x,i+1})$ depends on both vehicles' speeds, a time gap and a constant D_0, and the square of the Euclidian distance from this target location is considered for the cost term. However, as safety is of high importance, it is crucial for the vehicles in the platoon not only to be attracted to each other but also to keep the safe longitudinal distance of D_i from one another. If two vehicles get closer to each other than half of D_i, the above Euclidian distance is more strongly penalized to prevent any collisions. To this end a nonlinear attraction function based on the Euclidean distance is formulated as follows:

$$E(\mathbf{x}_i,\mathbf{x}_{i+1}) = (\frac{c_1-c_2}{1+e^{-(x_i-(x_{i+1}-\frac{D_i}{2}))}} + c_2)((x_i-(x_{i+1}-D_i))^2 + (y_i-y_{i+1})^2) \tag{16}$$

where c_1 and c_2 are positive constants.

Gathering all sub-objectives, the OCP is defined as minimization of the following collective objective criterion subject to vehicle dynamics (1)-(4) and constraints (10):

$$J = \sum_{k=0}^{K-1} \{ \sum_{i=1}^{n} \left(\frac{1}{2} b_1 (a_{x,i}(k))^2 + \frac{1}{2} b_2 (a_{y,i}(k))^2 + \frac{1}{2} b_3 (v_{x,i}(k) - v^*{}_x)^2 + \frac{1}{2} b_4 (v_{y,i}(k) - v^*{}_y)^2 \right.$$
$$\left. + \frac{1}{2} b_5 f_i^c(\mathbf{x}_i(k)) + b_6 \sum_{o \in O} c_{io}(\mathbf{x}_i, \mathbf{x}_o) \right) + \frac{1}{2} b_7 \sum_{i=1}^{n-1} E(\mathbf{x}_i, \mathbf{x}_{i+1}) \} \tag{17}$$

where b_1 to b_7 are weighting factors for the sub-objectives to be chosen appropriately, and K is the time horizon.

Numerical solution

The general form of the considered OCP is:

$$J = \sum_{k=0}^{K-1} \Phi[\mathbf{x}(k), \mathbf{u}(k)] \tag{18}$$

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k)] \tag{19}$$

$$\mathbf{u}^{\min} \le \mathbf{u}(k) \le \mathbf{u}^{\max} \tag{20}$$

where $\mathbf{u}^{\min}$ and $\mathbf{u}^{\max}$ are constant lower and upper bounds on controls, respectively. The Hamiltonian function of this OCP reads (Papageorgiou et al., 2015):

$$H[\mathbf{x}(k), \mathbf{u}(k), \boldsymbol{\lambda}(k+1)] = \boldsymbol{\lambda}(k+1)^T \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k)] + \Phi[\mathbf{x}(k), \mathbf{u}(k)] \tag{21}$$

where $\lambda(k)$ are the co-states, associated with the state equations. The necessary conditions for a local minimum are given below. The state equation is:

$$\mathbf{x}(k+1) = \frac{\partial H}{\partial \boldsymbol{\lambda}(k+1)} = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k)] \tag{22}$$

The control condition is as follows:

$$\frac{\partial H}{\partial u_i(k)} = \begin{cases} < 0 & \text{if} \quad u_i(k) = u_i^{\max} \\ = 0 & \text{if} \quad u_i^{\min} \le u_i(k) \le u_i^{\max} \\ > 0 & \text{else} \quad u_i(k) = u_i^{\min} \end{cases} . \tag{23}$$

Taken for all control variables $u_i(k)$. The co-state equation is:

$$\boldsymbol{\lambda}(k) = \frac{\partial H}{\partial \mathbf{x}(k)} \tag{24}$$

and the boundary conditions are given by:

$$\begin{aligned} \mathbf{x}(0) &= \mathbf{x}_0 \\ \boldsymbol{\lambda}(K) &= 0 \end{aligned} \tag{25}$$

A very efficient feasible direction algorithm (FDA) (Papageorgiou et al., 2016) is employed to solve this OCP numerically. FDA is an iterative algorithm that starts with a feasible initial guess of the control trajectories. At each iteration, using reduced gradient information, an appropriate step in the *mK*-dimensional control space is taken to improve control trajectories. In this work, the potentially improved control trajectory is derived by using Resilient backpropagation (RPROP) (Dabestani et al., 2023; Kotsialos, 2014; Papageorgiou et al., 2016).

Figure 1. Scenario 1, initial vehicle arrangement

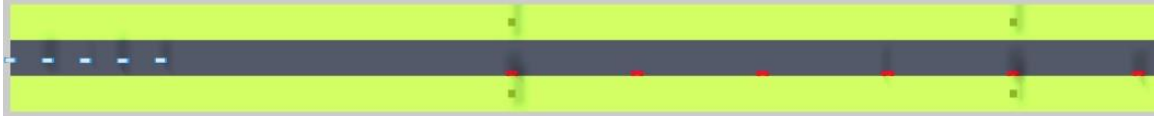

Figure 2. Scenario 2, initial vehicle arrangement

3. PRELIMINARY RESULTS

Scenario Setup

Two challenging scenarios, each one addressing a platoon of 5 vehicles (see Table 1 for their characteristics) driving on a straight lane-free motorway with 14.2 m width, are tested to demonstrate the efficiency of the proposed approach. In Scenario 1, the platoon faces 6 obstacle vehicles in front, that are moving with constant longitudinal speeds of 20 m/s and zero lateral speeds (see Fig. 1). These 6 vehicles, accounted for in the collision avoidance term, are not optimized, but drive independently. At the initial state, the obstacles are positioned at $(x_1(0), y_1(0)) = (160,5)$ m , $(x_2(0), y_2(0)) = (210,9)$ m , $(x_3(0), y_3(0)) = (260,2)$ m , $(x_4(0), y_4(0)) = (310,5)$ m , $(x_5(0), y_5(0)) = (360,9)$ m and $(x_6(0), y_6(0)) = (410,2)$ m . In Scenario 2, the same platoon faces 6 obstacles in front, which are moving with sinusoidal lateral trajectories and constant longitudinal speeds of 20 m/s (see Fig. 2). At the initial state, the obstacles are positioned at $(x_1(0), y_1(0)) =$ $(200,1)$ m , $(x_2(0), y_2(0)) = (250,1)$ m , $(x_3(0), y_3(0)) = (300,1)$ m , $(x_4(0), y_4(0)) = (350,1)$ m , $(x_5(0), y_5(0)) = (400,1)$ m and $(x_6(0), y_6(0)) = (450,1)$ m .
All vehicles have same dimensions of 5 m length and 2 m width. In (17), the penalty weights are $\{b_1, b_2, b_3, b_4, b_5, b_6, b_7\} = \{1, 0.1, 0.01, 0.01, 0.1, 5, 1\}$ and the time step is $T = 0.25$ s. The constant longitudinal acceleration bounds are $A^{\max} = 1.5$ m/s^2 , $A^{\min} = -2$ m/s^2 and the time-gap is $\omega_{at} = 0.6$ s while $D_0 = 0$. The parameters used for the collision-avoidance ellipsoid are $p_1 = 6$, $p_2 = p_3 = p_4 = p_5 = 2$ in (13) and $\omega_x = 0.5$ s, $\omega_y = 0.5$ s, $\varepsilon_\omega = 0.1$ in (15). In (16) we have $c_1 = 10^{-3}$ and $c_2 = 10^{-4}$; while in (12), β is set to 0.03 (see (Yanumula et al., 2023)).

Table 1: Scenarios 1/2 Characteristics

Vehicle No.	**1**	**2**	**3**	**4**	**5**
$\mathbf{x}(0)$ in (m)	0	15	30	45	60
$\mathbf{y}(0)$ in (m)	6.2	6.1	6.05	6	6.15
$\mathbf{v}_x^*$ in (m/s)	25	25	25	25	25
$\mathbf{v}_x(0)$ in (m/s)	25	25	25	25	25

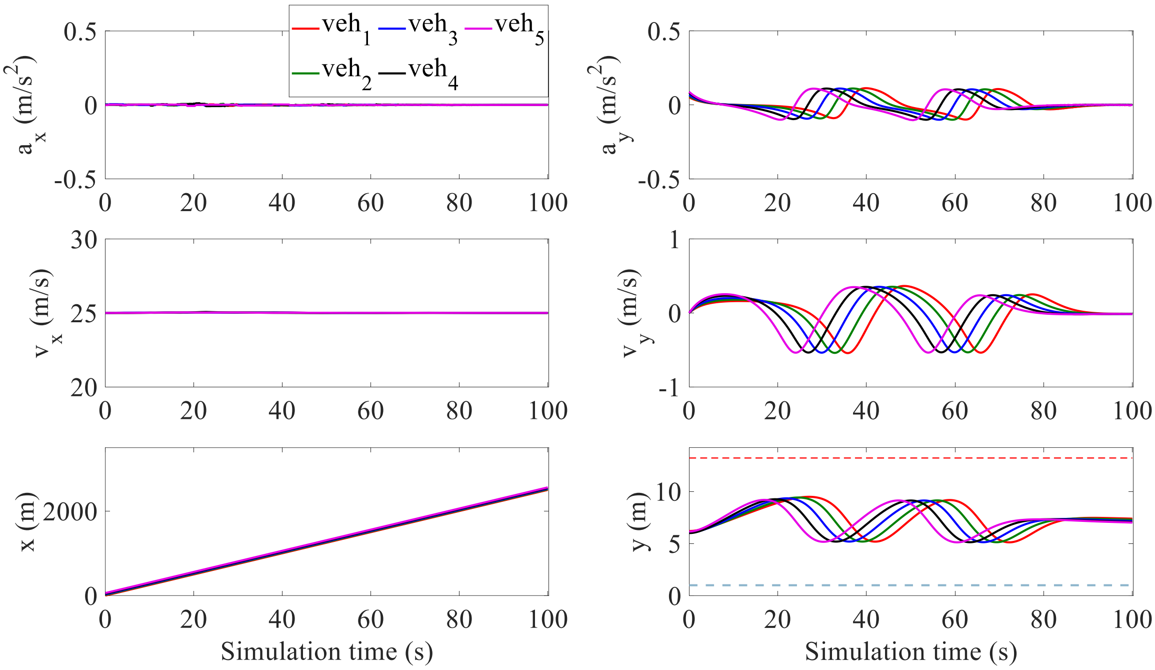

Figure 3. Optimal longitudinal and lateral trajectories for all 5 vehicles in Scenario1

Results

Fig. 3 displays the longitudinal and lateral movements of all 5 platoon vehicles in Scenario 1. The optimization time horizon for this scenario is 100 s ($K = 400$), and FDA needed 8.24 s of CPU time to converge, coded in C and run on an Intel(R) Core TM i5-10500 CPU @ 3.10GHz with 8.0 GB installed RAM. It should be noted that the employed convergence test to reach the reported results was quite strict. Dabestani et al. (2023) suggest that if the FDA iterations are stopped earlier, the obtained results are similarly good from an application point of view, while the computation time is sensibly reduced. The simulation time horizon was taken sufficiently long to ensure that the platoon has overtaken all the obstacles. In this scenario, the platoon needs to move in a snake-wise manner to overtake all the obstacles safely and efficiently (see Fig. 3).
For Scenario 2, with 112.5 s of time horizon ($K = 450$), FDA needed 6.96 s computational time to converge to a solution, and Fig. 4 presents the optimal vehicle trajectories. At times, the platoon had to split to overtake an obstacle before reuniting again. An illustration of such behavior is shown by vehicle #3 for the period [20, 80] s, when it is constantly interrupted by obstacles, overtakes them and then re-joins the platoon. Such safe and intelligent behavior emerges for each vehicle and the platoon as a whole from the OCP formulation.
The accelerations in both scenarios are smooth and low in magnitude, yielding passenger convenience. In particular, the longitudinal accelerations are close to zero and vehicles are moving at their longitudinal desired speeds which implies low and efficient fuel consumption. Furthermore, as depicted in Fig. 5, the platoon vehicles finally converge to the desired longitudinal distance of $\Delta x = 15$ m (the desired distance is based on vehicle's speeds and time-gap) and 0 m lateral deviations, despite many interruptions by obstacles. It is worth noting that no collisions occur.

Corresponding videos, demonstrating the performance of the proposed approach for both scenarios, are available in (https://bit.ly/3zjWcnN)

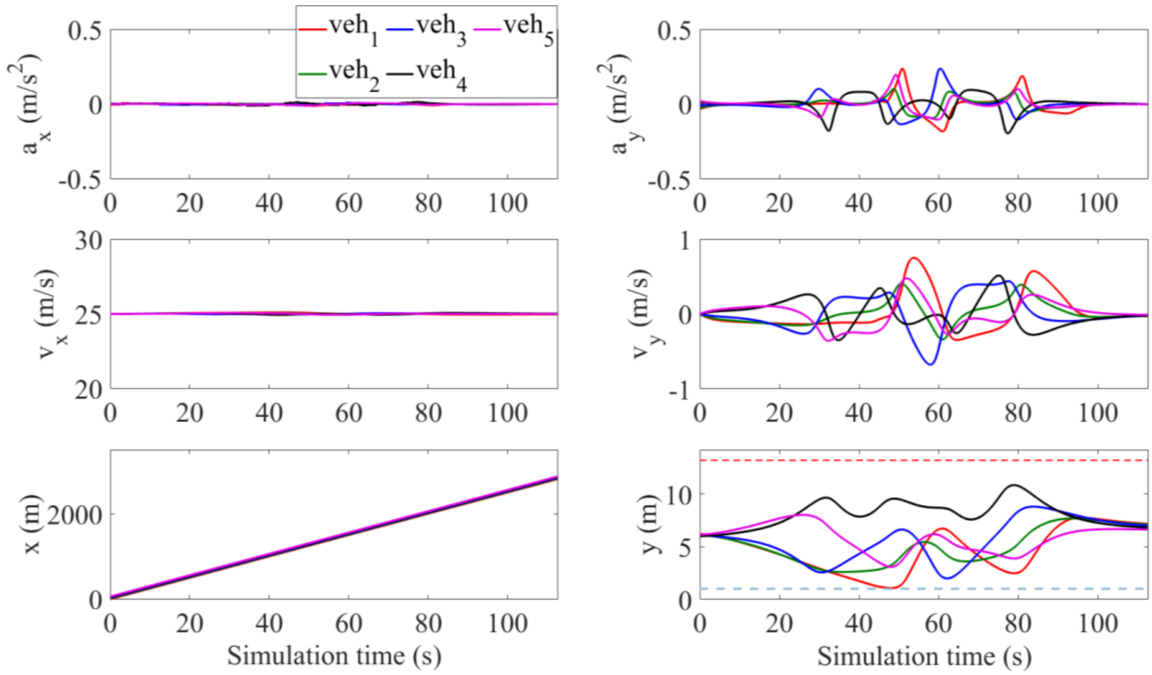

Figure 4. Optimal longitudinal and lateral trajectories for all 5 vehicles in Scenario2

4. CONCLUSIONS

An optimal control-based joint path-planning approach is developed to create platoons of vehicles and have the vehicles drive at close distances in lane-free traffic. In the proposed approach, an objective function is formulated for the platoon, considering driving efficiently, safely and comfortably for all platoon vehicles. The problem is solved numerically with a feasible direction algorithm, using the resilient backpropagation (RPROP) method. Two challenging scenarios demonstrate that vehicles create 1-D platoons and move efficiently, flexibly and safely, tracking the desired speed, respecting the road boundaries and avoiding external obstacles. Current and future work is focused on:

- Joint optimization of deformable 2-D vehicle flocks in lane-free traffic.
- Implementation of the proposed approach in an MPC framework.

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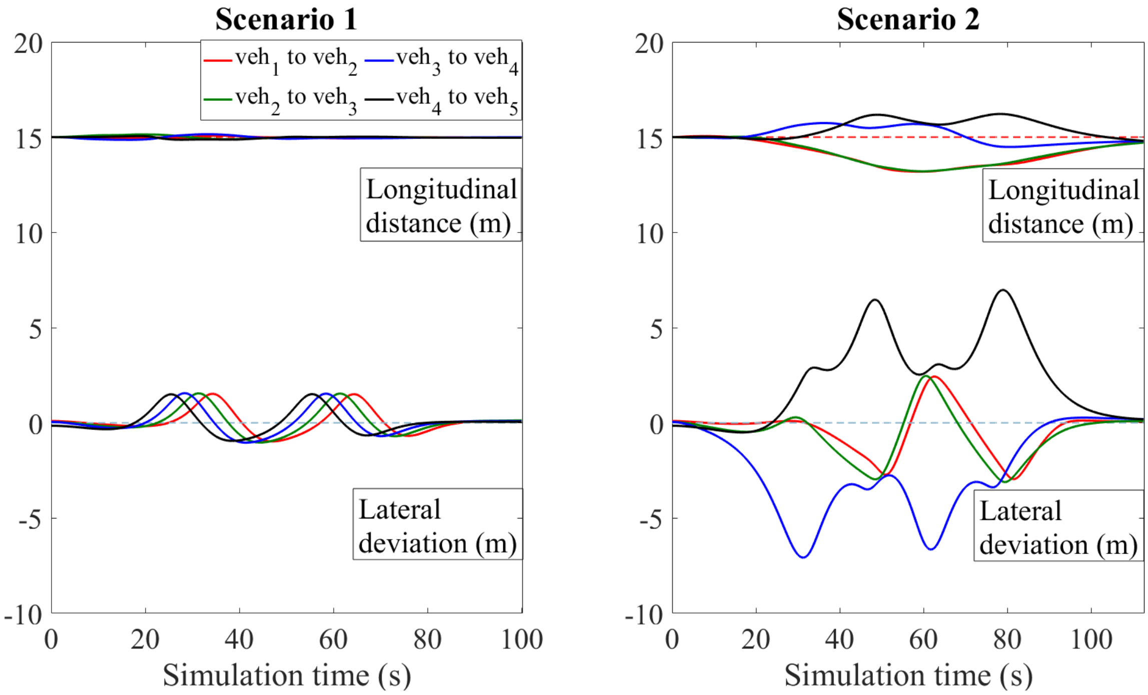

Figure 5. Inter- vehicle longitudinal distances and lateral deviations for both Scenarios.

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