An incentive strategy for the retention of impatient passengers in ride-sourcing markets

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Short summary

A prominent problem plagued the ride-sourcing service providers is the instantaneous real-time demand-supply imbalance. This entails terrible user experience and causes adverse sustainable results such as customer churn for the platform, which will damage both drivers' income and the platform revenue. To reduce passengers' abandonment due to long waiting time by improving their waiting experience, we propose an incentive-based queue management strategy— a discount method— to retain passengers in a post-peak period. We regard the waiting process for a ride-sourcing service as an M/M/c+M queue system with impatient passengers, and model passengers' reneging behaviors by characterizing the impact of waiting discounts and updated queue information on their travel utilities. Based on such a behavior model, we can analyze the effect of discount strategy on the queuing process which allow us to maximize the platform's profit growth by tailoring the discount strategy. Our result shows that the discount strategy can effectively increase the platform profit. Under the specific condition of the discount strategy, the earlier the strategy is implemented in the post-peak period, the higher platform profit can be achieved. **Keywords**: ride-sourcing market; queueing process; discount strategy; reneging behavior

1 Introduction

Ride-sourcing services provide a suite of strategies for providing travelers effective choices to enhance accessibility and improve travel reliability. However, the enormous growth has exacerbated the imbalance between the vehicle supply and ride-sourcing travel demand, especially during peak commuting hours. Long waiting time further increases order cancellations on the demand side, which not only wastes passengers' time but also reduces the income of drivers and platforms. Moreover, as an important measurement in travelers' mode choices, long waiting time reflects a relatively poor accessibility and quality of ride-sourcing services, leading to a long-term user churn.

In an effort to address this challenge, surge pricing has been widely adopted by most ride-sourcing platforms. This approach increases trip fares to incentivize more drivers to enter the hot areas and meet spiking demand. However, this approach has received criticism from scholars in recent years, who have raised concerns about its potential adverse impact on both riders and drivers.(Castillo, 2023; Dholakia, 2015; Goncharova, 2017). Some scholars have characterized it as a form of price discrimination or price gouging (Dholakia, 2015). This is because surge pricing may lead to higher fares for riders who are willing to pay more, while other riders may be priced out of the market, leading to the passengers abandonment. Others have suggested that surge pricing may damage the interests of drivers, potentially leading to a decrease in their earnings unless they strategically plan their actions in response to surge pricing dynamics (Goncharova, 2017). As a result of these concerns, there has been a growing interest in alternative management approaches that increase the platform's matching rate while considering passenger cost sensitivity. Hence, we propose a novel waiting discount strategy in this paper.

Our strategy aims to reduce passenger's cancellation during peak periods of queuing, thereby increasing the platform's matching rate and revenue. During peak periods, the queue undergoes two stages due to the dynamic arrival rate of users: the queue accumulation stage (pre-peak period) and the queue dissipation stage (post-peak period). To prevent a further exacerbation of supply-demand imbalances, we focus on implementing the discount strategy during the post-peak period, during which the arrival of passengers slows down but there remains a large number of passengers in the queue with the long waiting time. Implementing a discount strategy during this period can increase their willingness to wait, thereby increasing the number of matched orders and fully utilizing supply resources. This not only increases the platform's revenue but also reduces driver idle and cruise time while increasing their earnings. Compared with surge pricing, the discount strategy achieves revenue increment for the platform without leading to riders feeling priced out of the market. This, in turn, promotes long-term customer stickiness to the platform.

We first analyze the discount strategy on queuing process. Discount strategy has dual impact on the passenger behavior. Firstly, the discount directly reduces the economic cost of the trip, thereby decreasing the likelihood of passengers abandoning the queue. This effect is intuitive and aligns with general economic theories of consumer behavior. Besides, the discount indirectly influences the delay announcements. Reduced trip cancellations owing to more attractive fares can lead to increased wait times for subsequent passengers. This, in turn, heightens the probability of these new arrivals abandoning the queue. Therefore, we propose a comprehensive macro-micro modeling framework to evaluate the effect of discounts strategy on queuing process and analyze the steady state of queuing system. Specifically, at a macro level, the queuing process is delineated by an M/M/s+M Markov chain; at a micro level, passenger decision-making is characterized by a logit choice model with random utilities. By integrating the macroscopic model of passenger flow with the microscopic insights into individual abandonment behaviors, we are able to calculate the reneging rate of the queuing process using the fixed-point method across various discount strategies.

From the perspective of platform, we further quantify the impact of different discount strategies on platform performances and attempt to design an optimal discount strategy based on the proposed model to improve the performances. By introducing the queue length function, we compare different strategies in terms of the revenue (increasing the passenger's retention) and cost (providing discounts) brought to the platform and calculate the generated profit. The numerical experiment verifies the effectiveness of the proposed discount strategy and reveals some interesting insights for the mechanism design.

2 Methodology

In this section, we introduce a micro-macro modeling framework to understand the interplay between passenger behavior and discount strategies. We also propose a methodology to assess the effectiveness of these strategies, considering both the potential revenue increase through improved passenger retention and the cost implications of implementing discounts.

2.1 Micro-macro framework with discount strategy

In our micro-macro modeling framework, we obtain the impact of discount strategy on the queuing process by integrating insights from individual-level (micro) and queue-level (macro) perspectives by the average reneging rate of passengers λ^R.

2.1.1 Macro queuing process of passengers

We present a queuing model to capture the macro queuing process of passengers. The arrival of passengers and the service of drivers follow different Poisson distributions with the mean of λ and θ respectively. Assuming that the passengers are impatient, we suppose that the patience threshold of each individual is a realization derived from an exponential distribution with rate r, $0 < r < s\theta$, as passengers are more willing to get served. Therefore, in a multi-server queue, the queuing process can be viewed as an $M/M/s+M$ queuing system as shown in Fig 1.
where s represents the number of servers. The death rate of each state depends on the the number of passengers i in the system. When $i \leq s$, the death rate at the ith position is $i\theta$, as passengers exit the queue by being served. When $i > s$, the death rate at the ith position is $s\theta + (i-s)r$, as

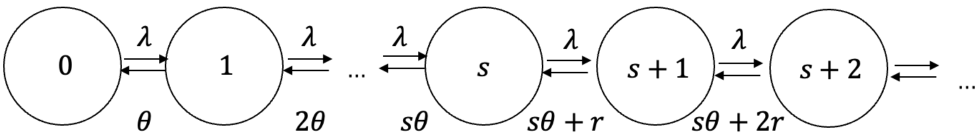

Figure 1: Macro queuing model

passengers leave the queue either by abandoning the line or being served. We focus on the steady-state of the queuing process. When the multi-server queue reaches steady state, the steady-state probability $p(i)$ is formulated as follows,

$$p(i) = \begin{cases} p_0 * \frac{\lambda^i}{i!\theta^i}, & 0 \leq i \leq s. \\ \frac{p_0}{s!\theta^s} * \frac{\lambda^i}{(s\theta+r)(s\theta+2r)...(s\theta+(i-s)r)}, & i > s. \end{cases} \tag{1}$$

where p_0 represents the probability that there is no customer in the system with the formulation, which is given by,

$$p_0 = \frac{1}{[\sum_{i=0}^{s} \frac{\lambda^i}{i!\theta^i} + \frac{\lambda^s}{s!\theta^s}(\frac{\gamma(\frac{s\theta}{r}, \frac{\lambda}{r})*exp(\frac{\lambda}{r})}{(\frac{\lambda}{r})^{\frac{s\theta}{r}}\Gamma(\frac{s\theta}{r})} - \frac{1}{\Gamma(\frac{s\theta}{r}+1)})]} \tag{2}$$

Based on the steady-state probability, the expected number of waiting customers in queue, L_q, is given by the sum of the product of the steady-state probability $p(s+n)$ of there being n customers in the queue, across all possible system states.

$$L_q = \sum_{n=0}^{\infty} p(s+n) * n \tag{3}$$

As the patience time of passengers follows a specific exponential distribution, the average reneging rate of passengers is,

$$\lambda^R = r * L_q \tag{4}$$

2.1.2 Micro queuing behavior of passengers

We characterize a passenger's endogenous decision on abandonment by a dynamic utility function. According to the discount strategy, passengers at different initial positions have varied chances of receiving the discount, hence, dynamic utility of a passenger is not only related to her current position, but also the position she joins the queue. To capture this, we express the dynamic utility function of a passenger as follows

$$U_n^k(a_n^k) = \varphi_n^k(a_n^k) + \epsilon_n^k(a_n^k) \tag{5}$$

where k represents the the position she join in the queue and n represents her current position in the queue, $n \leq k$. U_n^k depends on her decision, a_n^k, where $a_n^k = 0$ for abandonment and $a_n^k = 1$ for staying in line. Eq. 5 is composed of the deterministic utility φ_n^k and a random idiosyncratic term ϵ_n^k including external factors that influence the passenger's preference for either choice. We assume that the random utility term follows a Gumbel distribution. Consequently, the reneging probability q_n^k of the passenger at position n given she join in the kth position can be derived by a logit model,

$$q_n^k = \frac{e^{\varphi_n^k(a_n^k=0)}}{e^{\varphi_n^k(a_n^k=1)} + e^{\varphi_n^k(a_n^k=0)}} \tag{6}$$

And the probability that a passenger will abandon given she joins at the k^{th} position Q_k can be formulated as,

$$Q_k = 1 - S_k = 1 - \prod_{i=0}^{k}(1 - q_i^k) \tag{7}$$

where S_k represents the probability that a passenger, who initially joins the queue at position k, will remain in the queue until being served.
The deterministic utility function can be formulated as,

$$\varphi_n^k = \begin{cases} V - \beta * E[D_n] - [(1-\phi^k)R + \phi^k * R * (1-d)], & if\ a_n^k = 1 \\ 0, & if\ a_n^k = 0 \end{cases} \tag{8}$$

where V denotes a representative passenger's valuation of accomplishing the trip, $\beta * E[D_n]$ denotes the cost of waiting at position n, in which β is the value of time and $E[D_n]$ is the expected delay announcement. The expression $(1-\phi^k)R + \phi^k * R * (1-d)$ denotes the expected discounted trip fare, accounting for possible discounts. Within this, ϕ^k indicates the probability that a passenger can receive the discount when she joins the queue in the kth position, d is the discount size, and R is the original trip fare.

The platform estimates the distribution of the delay announcement D_n based on the passenger's position and the service condition $s\theta$. The time until a passenger gets served in the nth position is the time it takes for all of the passengers waiting ahead enter service plus the time required for a service completion (when all servers are busy). Therefore, the delay announcement can be captured by a pure-death process and D_n represents the downcrossing time from state $s+n$ to absorption state s. Given a constant death rate, D_n is distributed according to an Erlang distribution. This distribution results from the convolution of n identical and independent exponential distributions, each with a rate of $s\theta$. The density distribution and cumulative distribution are given by,

$$g_n(t) = \frac{1}{(n-1)!}(s\theta)^n t^{n-1} e^{-s\theta t} \tag{9}$$

$$G_n(t) = 1 - \sum_{k=0}^{n-1} \frac{1}{k!} e^{-s\theta t}(s\theta t)^n \tag{10}$$

The expectation of the delay announcement D_n is $E[D_n] = \sum_{k=0}^{n} \frac{1}{s\theta} = \frac{n}{s\theta}$. Under our discount strategy, passengers facing an initial delay announcement exceeding a predefined waiting time threshold T are eligible for a discount. Thus, $\phi^k = P(D_k > T)$.

The average reneging rate is calculated by adding up the probabilities of new arrivals abandoning the service at each state.

$$\lambda^R = \sum_{k=1}^{\infty} \lambda p(s+k) Q_k \tag{11}$$

2.1.3 Solution algorithm

Until now, with the average rate of abandoning passengers connecting the queue process at the macro level and individual behavior at the micro level, we complete the framework of the delay-reaction system where the patience threshold parameter r is an endogenous variable, and the other parameters are exogenous. Under this kind of circumstance, the exact value of the patience time distribution parameter can be derived through the fixed-point method which can be formulated as $f(r) = r$.

2.2 The performance of the discount strategy

In this section, we compare the profit of the platform during the queuing process under different discount strategies. For the platform, offering discounts diminishes the revenue of each order, yet it also increases the number of completed orders. Consequently, the impact of providing discounts on the platform's profit is uncertain. To quantify the revenues and costs brought by a discount strategy, we introduce an expected queue length function with time in the queuing process. In an M/M/s+M queuing model the arrival rate, service rate, and reneging rate are λ, θ, and r respectively. The expected queue length function of time can be expressed as follows:

$$X(t+h) = \begin{cases} X(t)+1, & \text{with probability } \lambda h + o(h). \\ X(t)-1, & \text{with probability } [s\theta + rX(t)]h + o(h). \\ X(t), & \text{with probability } 1 - [\lambda + s\theta + rX(t)]h + o(h). \end{cases} \tag{12}$$

By solving this equation, we obtain the function of time-dependent queue length as follows,

$$X(t) = C_1 e^{-rt} + \frac{\lambda - s\theta}{r} \tag{13}$$

where C_1 is determined by the specific queue length at the initial time of our interest.
In the post-period, given a discount strategy is triggered when the queue length reaches I, the queuing process then is divided into two distinct phases. The pre-discount phase covers the duration from $0 \le t \le t_I$, and the post-discount phase spans $t_I < t \le t_e^I$, where t_e^I denotes the time when the queue dissipates or the time that queue length reaches to 0. The moment t_I is defined as the point in time when the queue length reaches I. Throughout both the pre-discount and post-discount queuing process, the rates of passenger arrival λ and driver service θ remain constant. However, the introduction of the discount incentive leads to a change in the passenger reneging rate from r to a new value r'. Both r and r' can be determined from the model described in Section 4.1. With these considerations in mind, the queue length $X(t)$ at any given time t can be represented as a two-part piecewise function as follows. The initial queue length for the pre-discount phase is assumed to be I_0 at $t = 0$ while for the post-discount phase, it is I at $t = t^I$, $t_I = X^{-1}(I)$.

$$X(t) = \begin{cases} (I_0 - \frac{\lambda - s\theta}{r})e^{-rt} + \frac{\lambda - s\theta}{r}, & 0 \le t \le t_I. \\ (I - \frac{\lambda - s\theta}{r'})e^{r'(t_I - t)} + \frac{\lambda - s\theta}{r'}, & t_I < t \le t_e^I. \end{cases} \tag{14}$$

We further model the queue length as a discrete variable. Therefore, the queue dissipation time can be expressed as the sum of the time durations the queue spends at each length, i.e., $t_e^I = T_{I_0} + T_{I_0-1} + ... + T_{I+1} + T'_I + T'_{I-1} + ... + T'_1$. T'_n and T_n represents the time duration that the queue spends at length n with and without the discount strategy respectively.

$$T_n = \frac{1}{r} ln(1 + \frac{r}{r(n-1) + s\theta - \lambda}) \tag{15}$$

We then calculate the platform's revenue and associated discount costs based on the duration for which the queue remains at each specific length. Specifically, let RV^I denote the revenue generated by the platform when a discount is offered at a queue length of I, and TD^I represents the revenue loss incurred by the platform for the same discount offered length. Thus, the profit derived from offering this discount, prior to the dispersal of the queue, is quantified as $RV^I - TD^I$. The expression of RV^I and TD^I are shown as follows,

$$RV^I = t_e^I * s\theta R * c = (T_{I_0} + T_{I_0-1} + ... + T_{I+1} + T'_I + T'_{I-1} + T'_1) * s\theta R * c \tag{16}$$

$$TD^I = \sum_{i=1}^{I} T'_i \lambda(\phi^i dR) S_i \tag{17}$$

For Eq.(16), the platform revenue is the product of the number of completed orders and the commission fee per order. Here, the former is represented as the total service rate multiplied by the queue dissipation time. The latter is determined by the order price R and the commission rate c. For Eq.(17), the revenue loss (discount cost) is calculated by summing the product of the number of passengers arriving at different queue lengths and the corresponding revenue loss of the platform at those queue lengths. This is based on the understanding that only those passengers who are ultimately served will actually receive the discount.

Once we have established a method to quantify the effectiveness of a discount strategy, our focus is to identify the optimal discount strategy with an objective function of platform profit growth represented by the metric of percentage increase in revenue. The specific objective function is formulated as follows,

$$\max_{(d,T,I)\ge 0} \frac{(RV^I - TD^I) - (RV^0 + (t_e^I - t_e)\lambda Rc)}{RV^0} \tag{18}$$

where $(RV^I - TD^I) - (RV^0 + (t_e^I - t_e^0)\lambda Rc)$ is the profit increment of the platform between the scenarios with and without the discount. $(RV^I - TD^I)$ is the platform's profit before the dispersion of the queue (i.e., for $t < t_e^I$) given the discount, RV^0 is the platform's profit before the dispersion of the queue (i.e., for $t < t_e^0$) without the discount strategy. After the queue dissipates, the platform's profit depend on the passenger arrival rate. Therefore, platform's profit without the discount from $t = 0$ to $t = t_e^I$ is $RV^0 + (t_e^I - t_e^0)\lambda Rc$, where $(t_e^I - t_e^0)\lambda Rc$ is the profit of the platform from t_e^0 to t_e^I.

3 Results and discussion

We first identify the ideal discount strategy with the proposed platform profit growth maximization model for a ride-sourcing platform. We plot the profit growth under the strategy to explore the implications of the three discount-related variables. We assume that the common exogenous parameters are defined as follows[1]: $\lambda = 1\, pax/min$, $s\theta = 2\, pax/min$, $R = 3{+}0.7{*}\, trip\, time(USD)$, $trip\ time = 13min$, $\beta = 3\ usd/min$, $V = R * v(USD)$, $v = 1.5$, $c = 0.3$.

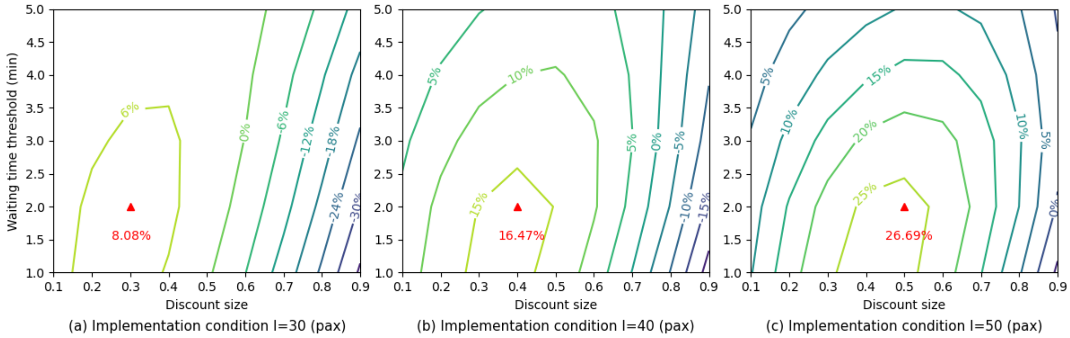

Figure 2: Platform profit growth under the discount strategy

From the contour plots, we can observe that the profit growth under the discount strategy can be positive when the discount strategy is implemented. According to common sense, providing incentives to passengers is meaningless when the demand exceeds the supply because the number of answered orders is limited by the supply resources. However, the results of this study show that a well-designed discount strategy can lead to an increase in the platform's profit by maximizing the utilization of the supply.

By comparing the profit growth under different scenarios shown in Fig 2 (a), (b) and (c), we can observe when the discount strategy is implemented earlier (i.e., when the initial queue length is longer), the platform obtains more profit. The result indicates the timing of implementing the discount strategy is crucial to the success of the strategy, if the discount strategy is implemented too late, the platform may miss out on potential revenue opportunities. Besides, the earlier the discount strategy is used, the greater the discount intensity that the platform needs to provide. This is because longer waiting times require larger discounts to reduce cancellations. However, the optimal waiting time threshold has hardly changed, which may be because the waiting time threshold reflects a reasonable waiting time that is unrelated to the number of people queuing and is instead related to supply and demand.

Additionally, we can observe from the contour plot that both too small (decreasing d or increasing T) and too large (increasing d or decreasing T) discount sizes lead to a decrease in platform profit given the implementation condition. This is because excessively small discounts cannot effectively retain passengers, while excessively large discounts result in discount costs that exceed the increased revenue brought by the strategy. The contour lines corresponding to smaller discount sizes are sparser, while those corresponding to larger discount sizes are denser, indicating that platform profit changes more rapidly as discount size increases. In other words, as discount size increases, its impact on platform profit becomes more significant.

4 Conclusions

In this study, we intend to characterize such interactions and determine whether an incentive strategy for passenger retention should be implemented, as well as how to design appropriate incentives to maximize the platform's revenue under various conditions. Our study fills the research gap of characterizing the interactions between passengers' queuing behaviors, incentives and updating queue information in on-demand mobility systems. Besides, it also offers a reference to the ride-sourcing platform in how to design an incentive strategy according to the actual supply and

[1]The data is from New York City Taxi Limousine Commission https://www.nyc.gov/site/tlc/index.page.

demand conditions and allocate resources reasonably, so as to improve its revenue without affecting the passengers' utility.

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