Modeling and managing network crowding and congestion in mass transit with a revenue-neutral fare incentive scheme

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SHORT SUMMARY

This study develops a user departure time equilibrium model in a many-to-many transit network and evaluates a revenue-neutral fare incentive scheme where passengers incur a surcharge during the peak hour and obtain a reward during off-peak hours. In the user equilibrium model, passengers' departure time choices are based on the generalized trip costs including crowding costs, queuing costs, schedule delay costs, in-vehicle costs and uniform fare costs. Equilibrium conditions are derived where passengers with the same OD incur identical equilibrium costs. A sequential algorithm is proposed to solve the user equilibrium. A bi-level optimization model is further developed to optimize the proposed incentive strategy based on the network equilibrium model. Case studies are conducted with Copenhagen M2 metro line where results yield the distributions of passengers with different OD in each service run and indicate the reduction of total equilibrium costs with the incentive strategies. We also found that upstream station passengers may incur higher trip costs with the incentive strategies as they have priority to occupy the vehicle space and thus pricing strategies imposes constraints to their choices. The proposed model and incentive schemes can provide tractable references for users and transit authorities in managing demand and relieving commuting congestions.

Keywords: Departure time equilibrium, many-to-many network, commuting, peak-hour congestion, incentives

1. INTRODUCTION

Urban rail transit systems have been developed in most large cities worldwide to generate travel activities and connect communities. Urban rail transit systems encompass a range of mass transit systems including commuter rail, rapid transit, light rail, metro, and streetcar that operate on separate systems. Traveling by rail transit in many metropolitan areas is a primary means of economic travel option for passengers and is an environmentally friendly travel form delivered by the transit agencies and government.

With the vast amounts of passengers and large infrastructure, introducing new expansions, strategies and operations in urban rail transit often involves substantial development costs and time. It is vital to develop decision-support approaches to evaluate tentative planning, operations, and network constructions for optimal decision-making process and resource facilitations. Pioneering studies have been done to develop analytical and mathematical models in general transit network designs comprising trip assignment, routing, and scheduling. Hasselstrom (1982) developed three-stage transit planning involving assignment models, route selections and frequency optimization. A system approach was proposed by Ceder and Wilson (1986) for transit network design and redesign by solving optimization problems. Chua (1984) analyzed quantitative and qualitative approaches to the planning of transit routes and frequencies in bus systems.

A number of studies have also elaborated the interactions between passengers, transit capacity and networks for planning in terms of congestion and crowding costs. Furth and Nielson (1981) considered the travel times and waiting times to optimize the frequencies for a given route structure. Szeto and Jiang (2014) developed a bi-level modeling approach to capture the in-vehicle congestion and passenger transfer behaviors in transit network design and planning. Niu and Zhou (2013) captured the peak hour congestions and optimized the urban rail timetable in oversaturated condition. Högdahl et al. (2019) combined simulation and optimization approaches to minimize railway travel time and delays.

In line with the peak congestion and demand management, transit agencies and governments have a growing need to capture the interim dynamics of the behaviors and operations and develop temporal policy interventions and time-dependent operations such as pricing strategies. Several studies conduct empirical analyses to investigate the effect of time-based differential fare schemes in practice (Yang and Lim, 2018; Zou et al., 2019; Anupriya et al., 2020; Wang et al., 2020a, 2020b). For instance, to smooth public transit demand, a free off-peak trip was provided by the "Travel Early, Travel Free" project in Singapore. Yang and Lim (2018) find this project has effectively encouraged commuters to shift their travel time to off-peak hours, and its effect persisted seven months after the financial incentives' discontinuation. Similarly, a 25 percent fare incentive for pre-peak commuters was offered by "the Early Bird Discount" in Hong Kong to address the overcrowding problems in public transport. Anupriya et al. (2020) evaluate the causal impact of this differential pricing on the trip scheduling of regular commuters based on difference-in-difference methods. The results indicated that the mean arrival time of regular commuters decreased due to the implementation of "the Early Bird Discount" but not at significant level. Hence the results also suggested that a 25 percent fare discount may not be sufficient to incentivize commuters to change their travel scheduling. These results reveal that the effective and efficient design of time-based differential fares is essential to ensure its peak avoidance effects.

Existing studies mainly focused on qualitative analyses and data-oriented to evaluate the post-impacts of pricing strategies. Transit agencies and governments are facing challenges to assess the real-time passenger flows in vehicles, platforms, and stations under various pricing and operations to ensure an informed decision-making process. Moreover, different from other systems such as road traffic, urban rail transit systems with human interactions exhibit crowding effects where queuing and congestion increase nonlinearly with the size of the agents and the scale of the interactions.

To this end, this study developed a novel user departure time choice model considering a transit network with many-to-many origin-destination pairs, and proposed a surcharge-reward scheme to tackle the peak hour congestion and reduce transit crowding. The rest of the paper is organized as follows. Section 2 describes the problem settings of a transit network and passengers' generalized trip costs. Section 3 devises the user equilibrium conditions and solving algorithms. Section 4 introduces the concepts and designs of the surcharge-reward scheme. Section 5 conducts case studies with Copenhagen metro system to validate the proposed model and pricing schemes.

2. PROBLEM SETTINGS

Consider a mass transit network (many-to-many network) with multiple lines and stations, as shown in Figure 1. A service run departs from the furthest station ST_1 and stops by $ST_2 \dots ST_k$ stations to the last station D. Each passenger has a pair of origin station k and destination station s. The demand for each origin-destination (OD) pair is denoted by N_{kr}, $k = 1,2, \dots, K, r > k$. The in-vehicle time between station k and station $k + 1$ is denoted by d_k.

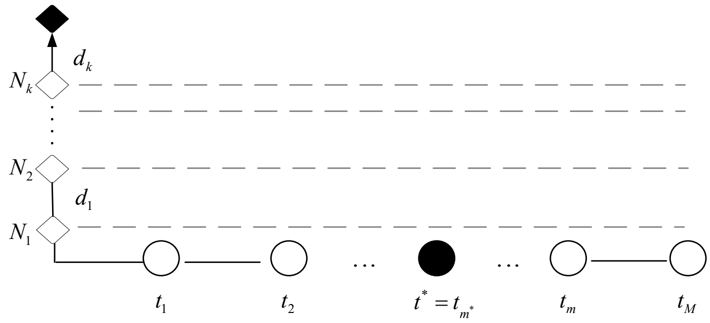

Figure 1. The many-to-many transit system

In this case, commuters with the same OD pair $k - r$ who takes a service run labeled as m at station ST_k encounter the generalized costs as below:

$$TC_{kr}\left(t_m^k\right) = \delta_{kr}\left(t_m^k\right) + q_k\left(t_m^k\right) + c_{kr}\left(t_m^k\right) + p + \varphi_{kr},$$
$$k = 1,2,\dots,K-1, s > k, m = 1,2,\dots,M \quad \textbf{(1)}$$

where t_m^k denotes the departure time of service run m at station k, p^k is the fare cost at station k, $q\left(t_m^k\right)$ is the queuing cost at platform if the service run is full, $c\left(t_m^k\right)$ is a crowding cost, φ^k is the in-vehicle time cost, and $\delta\left(t_m^k\right)$ is a schedule delay cost if they arrive at work earlier or late.

The crowding costs is expressed as

$$c_{kr}\left(t_m^k\right) = \sum_{w=k}^{r-1} g\left(\sum_{z=1}^{w}\sum_{l=w+1}^{K} n_m^{zl}\right) d_w \text{ }, k = 1,2, \dots K-1, r > k, m = 1,2,\dots,M \quad \textbf{(2)}$$

where Δn_m^z is the number of additional in-vehicle commuters from downwards stations starting from station k taking service run m.

The in-vehicle time cost is expressed as

$$\varphi_{kr} = \alpha \cdot (d_k + d_{k+1} + \cdots + d_{r-1}), k = 1,2, \dots K-1, r > k \quad \textbf{(3)}$$

3. DEPARTURE TIME EQUILIBRIUM IN MANY-TO-MANY NETWORK

3.1. Network Equilibrium Conditions

At equilibrium, commuters with the same origin-destination supposed to have identical equilibrium cost. Meanwhile, commuters with the same origin-destination incur constant fare costs and in-vehicle time costs which have no effect on the equilibrium departure time distribution and are thus ignored.

The equilibrium departure time model in transit network can be mathematically expressed as follows:

$$\sum_{t=k}^{r-1} g\left(\sum_{z=1}^{t}\sum_{l=t+1}^{r} n_m^{zl}\right) d_t$$

$$\min_{n} L(\boldsymbol{n}) = \sum_{w=k}^{r-1}\left(\sum_{m=1}^{M} G\left(\sum_{z=1}^{w}\sum_{l=w+1}^{r} n_m^{zl}\right)\right) d_w + \sum_{m=1}^{M}\left(\sum_{z=1}^{w}\sum_{l=w+1}^{r} n_m^{zl}\right)\delta_{kr}(t_m^k) \quad \textbf{(4)}$$

subject to

$$\sum_{m=1}^{M} n_m^{kr} = N^{kr}, k = 1,2,\dots,K \quad \textbf{(5)}$$

$$\sum_{z=1}^{k}\sum_{l=k+1}^{r} n_m^{kr} \leq \mathrm{S}, m = 1,2,\dots,M, k = 1,2,\dots,K = 1, r > k \quad \textbf{(6)}$$

$$n_m^{kr} \geq 0, m = 1,2,\dots,M, k = 1,2,\dots,K \quad \textbf{(7)}$$

The first term of the objective function (4) is the integral of crowding cost function which has no economic interpretation. The second term $\delta(m)$ is the aggregate schedule delay cost and T^k is the in-vehicle time of commuters in the transit system respectively. The third term is the transit revenue. Constraint (5) is the demand constraint for each station and constraint (6) represents the rigid capacity. The commuter departure time equilibrium is given by the first order condition of the problem (4):

$$n_m^{kr}\left(\sum_{t=k}^{r-1} g\left(\sum_{z=1}^{t}\sum_{l=t+1}^{r} n_m^{zl}\right) d_t + \delta_{kr}(t_m^k) + q_m^{kr} - v^{kr}\right) = 0 \quad \textbf{(8)}$$

$$\sum_{t=k}^{r-1} g\left(\sum_{z=1}^{t}\sum_{l=t+1}^{r} n_m^{zl}\right) d_t + \delta_{kr}(t_m^k) + q_m^{kr} - v^{kr} \geq 0 \quad \textbf{(9)}$$

$$q_m^{kr}\left(N^{kr} - \sum_{m=1}^{M} n_m^{kr}\right) = 0 \quad \textbf{(10)}$$

$$q_m^{kr} \geq 0 \quad \textbf{(11)}$$

$$\sum_{z=1}^{k}\sum_{l=k+1}^{r} n_m^{kr} \leq \mathrm{S} \quad \textbf{(12)}$$

$$\sum_{m=1}^{M} n_m^{kr} = N^{kr} \quad \textbf{(13)}$$

$$n_m^{kr} \geq 0 \quad \textbf{(14)}$$

where q_m^{kr} and v^{kr} are the Lagrange multipliers. q_m^{kr} represents the equilibrium queuing time costs with origin station ST_k and destination station ST_r in the service run departing at t_m^k. v^{kr} represents the equilibrium trip costs of commuters with the origin-destination pair ST_k and ST_r.

Eqs. (8) and (9) state the equilibrium condition that at station ST_k, if the service run is non-empty, passengers' generalized trip costs are the same as the equilibrium trip costs and if the service run is empty, then the generalized trip costs are no less than the equilibrium trip costs. The equilibrium condition can be mathematically expressed as:

$$\begin{cases} v_m^{kr} = v^{kr}, if\ n_m^{kr} > 0 \\ v_m^{kr} \geq v^{kr}, if\ n_m^{kr} = 0' \end{cases} = 1,2,\dots,M, k = 1,2,\dots,K = 1, r > k \quad \textbf{(15)}$$

Eqs. (10) - (11)indicate the equilibrium queuing condition by the rigid capacity constraint. Passengers incur the queuing time costs if the service run is saturated. The queuing condition can be mathematically expressed as:

$$\begin{cases} q_m^{kr} = 0, if \sum_{z=1}^{k} \sum_{l=k+1}^{r} n_m^{kr} < S \\ q_m^{kr} \geq 0, if \sum_{z=1}^{k} \sum_{l=k+1}^{r} n_m^{kr} = S \end{cases}, k = 1,2, \dots, M, k = 1,2, \dots, K = 1, r > k \quad \textbf{(16)}$$

Therefore, passengers can board the service run immediately if the arriving service run is unsaturated or have to wait for the next available service runs at the station if the arriving service run is saturated. Eqs. (12) and (13) are demand and supply constraints.

The optimization model (4) - (7) is to be solved for departure time distribution equilibrium in a many-to-many network. It should be noted that the simultaneous solution of equilibrium for all stations may not be unique. However, for each station ST_k, the commuter departure time equilibrium model has a convex function and a convex set of constraints given fixed variables of other stations, hence a unique equilibrium solution can be obtained for each station.

An algorithm is proposed to solve the departure time equilibrium model sequentially for each station with unique solutions.

Algorithm: Compute the departure time equilibrium in a many-to-one network

Step 1: Initialization. Set $l = 1$ as iteration index. For each station and service run, set an initial boarding flow $\mathbf{n}_m^{00(l)} = \{1 \leq m \leq M, 1 \leq k \leq K\}$

Step 2: Update of boarding flow in stations

 2.1 Initialization. Set $k = 1$

 2.2 Set boarding flow in station ST_k as variable and fixed the boarding flow for all the other stations by $\mathbf{n}^{kr(l)} = \left\{\mathbf{n}_m^{1r,(l+1)}, \dots, \mathbf{n}_m^{(k-1)r,(l+1)}, \mathbf{n}_m^{(k+1)r,(l+1)}, \dots, \mathbf{n}_m^{KR,(l)} | 1 \leq m \leq M, 1 \leq k \leq K, r > k\right\}$, Solve the minimization problem (4) - (7) and obtain the equilibrium boarding flow of each OD pair $k - r$ at station ST_k, denoted by $\mathbf{n}_m^{kr}$

 2.3 If $k = K$, then stop and go to step 3. Otherwise, set $k = k + 1$ and return to step 2.2.

Step 3: Update boarding flow $\mathbf{n}^{kr,(l+1)} = \left\{\mathbf{n}_m^{kr,(l+1)} | 1 \leq m \leq M, 1 \leq k \leq K, r > k\right\}$ and go to step 4.

Step 4: Iteration stops if $\sum_{k=1}^{K-1} \sum_{r=k}^{K} \sum_{m=1}^{M} \left| n_m^{kr,(l+1)} - n_m^{kr,(l)} \right| < \varepsilon$ ($\varepsilon = 0.1$here), otherwise, set $l = l + 1$ and $\mathbf{n}^{(l+1)} = \left\{(\mathbf{n}^{(l+1)} + \mathbf{n}^{(l)})/2\right\}$, and return to step 2

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4. OPTIMIZATION OF INCENTIVES TO MANAGE COMMUTING

4.1. The surcharge-reward scheme

The surcharge-reward scheme is an individual-based incentive strategy to smooth out the peak hour flow in the transport systems. The concept of the surcharge-reward scheme is to redesign a fare strategy which split the peak period into one central period and two shoulder periods (Tang et al., 2020; Yang & Tang, 2018). Passengers incur a surcharge Δs in the central period which will be all refunded to their personal account. And they can use a reward Δr from the account balance if they travel in the two shoulder periods. Passengers are not allowed to use the reward if they do not have sufficient account balance. The control ratio is defined as $\lambda = (\Delta s/\Delta r)/(1 + \Delta s/\Delta r) = \Delta s/(\Delta s + \Delta r)$, which can be regarded as the ratio of number of trips with rewards to the total number of trips made by one passenger.

A design criteria is introduced for the incentive spirit of the surcharge-reward scheme such that for each station, the equilibrium trip costs with a reward are no greater than those with a surcharge. Therefore, the surcharge-reward scheme is able to encourage passengers to shift their departure times to shoulder periods if they have the rewards. The design criteria is expressed mathematically as:

$$v_{RFI}^{kr} \leq v_{RFI}^{kr} + \Delta s/\lambda\, , k = 1,2, \dots, K, r > k \quad \textbf{(17)}$$

where v_{RFI}^{kr} represents the equilibrium trip costs in reward fare intervals (namely, the shoulder periods) for each OD pair $k - r$ at station ST_k and v_{RFI}^{kr} represents the equilibrium trip costs in the surcharge fare intervals (namely, the central period) at station ST_k.
For practicality and simplicity, an identical surcharge-reward scheme design is considered in all stations. Therefore, the above design criteria can be transferred to

$$\Delta s/\lambda \geq \max\left(v_{RFI}^{kr} - v_{RFI}^{kr}\right), k = 1,2, \dots, K, r > k \quad \textbf{(18)}$$

The design mechanism of the surcharge-reward scheme indicates that passengers with reward will travel in shoulder periods while passengers without reward will travel in the central periods. Hence the SRS will introduce segregated travel patterns during the peak period as shown in Figure 2.

In a transport system, it is therefore important to optimize the surcharge-reward scheme to reduce the congestion including the queuing and crowding costs. The implementation of the surcharge-reward scheme focuses on the amount of the surcharges, the control ratio and how to split the peak period to central period and shoulder periods. Based on this, we introduce four decision variables which are

- The surcharge, Δs
- The control ratio, λ
- The index of the starting service run in central period, $e, 1 \leq e \leq M$
- The number of service runs in central period, $l, 1 \leq l \leq M - e$

While the surcharge and the control ratio are continuous variables, the index of starting service run and the number of service runs are integer variables. We further established a bi-level optimization model to optimize the performance of the surcharge-reward scheme in a many-to-many network. The optimization is formulated as follows:

Upper level:

$$\min_{\lambda,e,l,n^{RFI},n^{SFI}} TEC = \sum_{k=1}^{K-1} \sum_{r=k+1}^{K} \left[v_{SFI}^{kr} \cdot (1 - \lambda) \cdot N^{kr} + v_{RFI}^{kr} \cdot \lambda \cdot N^{kr}\right] \quad \textbf{(19)}$$

subject to

$$\frac{\Delta s}{\lambda} \geq \max\left\{v_{RFI}^{kr} - v_{SFI}^{kr}\right\}, k = 1,2, \dots, K, r > k \quad \textbf{(20)}$$

$$1 \leq e \leq M, e \in \mathbb{Z}^{+} \quad \textbf{(21)}$$

$$1 \leq l \leq M - e, l \in \mathbb{Z}^{+} \quad \textbf{(22)}$$

$$\Delta s \geq 0 \quad \textbf{(23)}$$

$$\lambda \geq 0 \quad \textbf{(24)}$$

Lower level:

$$\min_{\mathbf{n^{kr}_{RFI}},\mathbf{n^{kr}_{SFI}}} TEC(\mathbf{n}) = \sum_{w=k}^{r-1}\left(\sum_{m=1}^{M} G\left(\sum_{z=1}^{w}\sum_{l=w+1}^{r} n^{zl}_{m,RFI}\right)\right)d_w + + \sum_{m=1}^{M}\left(\sum_{z=1}^{w}\sum_{l=w+1}^{r} n^{zl}_{m,RFI}\right)\delta_{kr,RFI}(t^k_m) + \sum_{w=k}^{r-1}\left(\sum_{m=1}^{M} G\left(\sum_{z=1}^{w}\sum_{l=w+1}^{r} n^{zl}_{m,SFI}\right)\right)d_w + + \sum_{m=1}^{M}\left(\sum_{z=1}^{w}\sum_{l=w+1}^{r} n^{zl}_{m,SFI}\right)\delta_{kr,SFI}(t^k_m)$$

subject to

$$\sum_{m=1}^{M} n^{kr}_{m,SFI} = (1-\lambda)N^{kr}, k = 1,2,\dots,K-1, r > k \quad \textbf{(25)}$$

$$\sum_{z=1}^{k}\sum_{l=k+1}^{r} n^{kr}_{m,SFI} \leq \mathrm{S}, m = 1,2,\dots,M, k = 1,2,\dots,K-1, r > k \quad \textbf{(26)}$$

$$n^{kr}_{m,SFI} \geq 0, m = 1,2,\dots,M, k = 1,2,\dots,K-1, r > k \quad \textbf{(27)}$$

$$\sum_{m=1}^{M} n^{kr}_{m,RFI} = \lambda N^{kr}, k = 1,2,\dots,K-1, r > k \quad \textbf{(28)}$$

$$\sum_{z=1}^{k}\sum_{l=k+1}^{r} n^{kr}_{m,RFI} \leq \mathrm{S}, m = 1,2,\dots,M, k = 1,2,\dots,K-1, r > k \quad \textbf{(29)}$$

$$n^{kr}_{m,RFI} \geq 0, m = 1,2,\dots,M, k = 1,2,\dots,K-1, r > k \quad \textbf{(30)}$$

The objective of the upper level model is to minimize the total equilibrium cost for all commuters in the system. Since the design of the surcharge-reward scheme is identical among all the stations, constraint (20) indicates that the difference of fare cost with SRS is set to be no less than the maximum difference of equilibrium cost so to incentivize commuters in all stations. The lower level model is to solve for departure time distribution equilibrium in a many-to-one network. The first and second terms in the objective function along with constraints (25) - (27) solve for the equilibrium in central period. Similarly, the third and the fourth terms in the objective function along with constraints (28) - (30) solve for the equilibrium in shoulder periods.

5. CASE STUDY IN COPENHAGEN COMMUTER LINE

The many-to-many case considers five stations from Vanlose station to Christianshavn station in the rail line M2 in the Copenhagen metro system, supposing in the morning peak period that travelers having the origin and destination pairs between the five stations. The OD demand matrix is presented in Table 5.1. The system characteristics are summarized in Table 5.2.

Table 5.1. OD demand matrix

	Vanlose	Fasanvej	Norreport	Kongens Nytorv	Christianshavn
Vanlose	/	150	700	950	250
Fasanvej	/	/	700	500	400
Norreport	/	/	/	1900	900
Kongens Nytorv	/	/	/	/	1000
Christianshavn	/	/	/	/	/

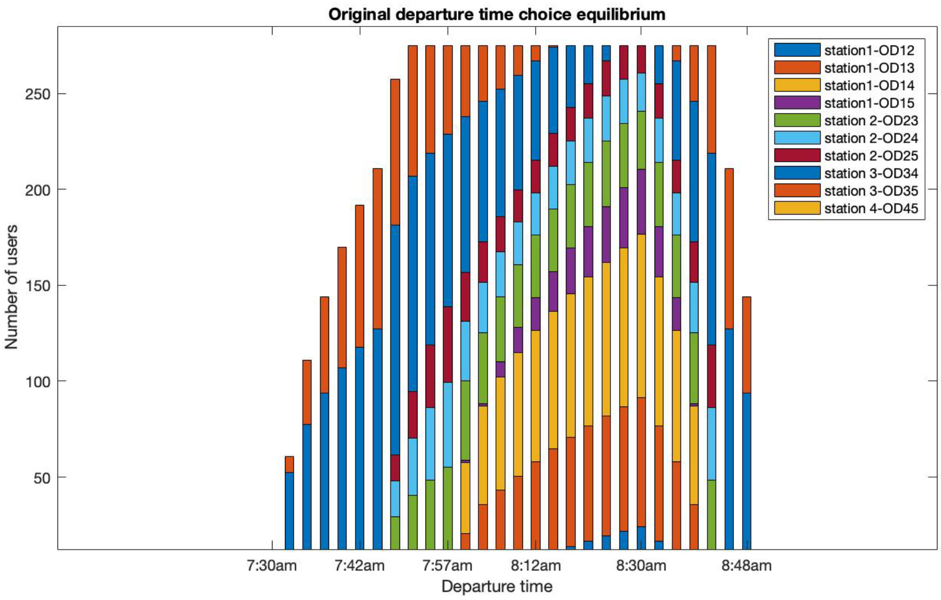
Table 5.2. System characteristics

Station	In-vehicle time between two stations	Off peak price (DKK)	Peak Price (DKK)
(1) Vanlose	6 mins	29.6	37
(2) Fasanvej	5 mins	29.6	37
(3) Norreport	3 mins	18.8	23.5
(4) Kongens Nytorv	2 mins	14.4	18
(5) Christianshavn	/	14.4	18

The transit service and social factors are listed as follows:

$$h = 3mins, S = 275, seat = 120, standing = 155, \beta = 60\frac{DKK}{h}, r = 60\frac{DKK}{h}, t^* = 8:30am$$

As shown in Figure 5.1 and Table 5.3, the original departure time duration is from 7:33 am to 8:48 am, with surcharge reward scheme, the optimal surcharge interval is set from 7:00 am to 7:39 am, the reward ratio is 0.4579, and the lower bound of the optimal surcharge varies with the station ranging from 11.206 DKK to 13.536 DKK. Since the equilibrium cost difference of SFI and RFI is different in each station, to assure that the surcharge-reward scheme is effective for all travelers, the lower bound of the optimal surcharge depends on the maximum difference value of SFI and RFI equilibrium cost among all the stations, as indicated in constraint (20).



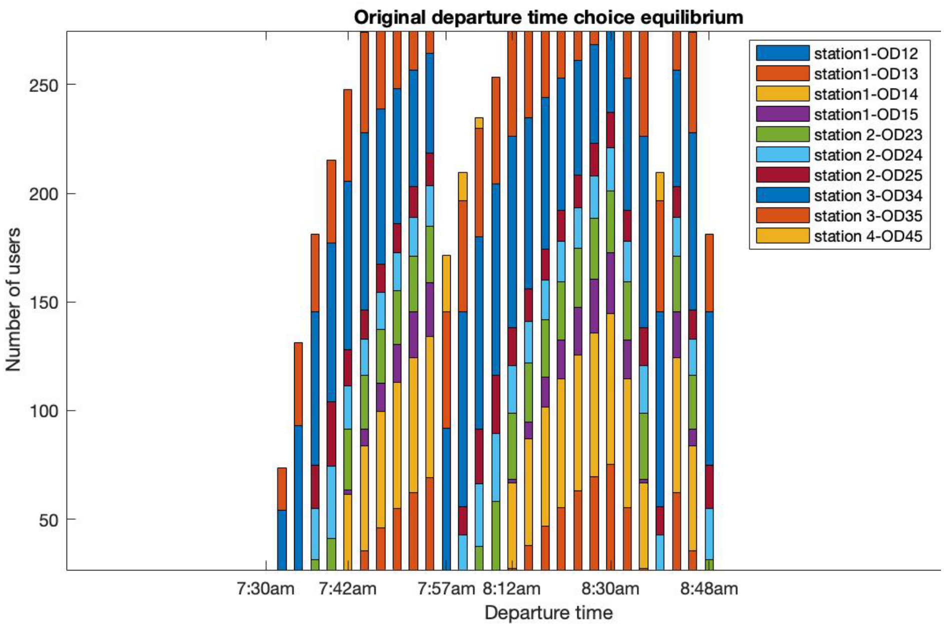


Figure 5.1. Travelers departure time distribution before and after the surcharge fare scheme in a many-to-many network

Table 5.3. The optimal solution

Boarding Stations	Optimal reward ratio	Optimal surcharge interval	Lower bound of the optimal surcharge	Rewards
Vanlose	0.4579	8:00 -8:39 am	13.536	31.240
Fasanvej			13.015	30.627
Norreport			11.206	29.815
Kongens Nytorv			13.188	27.594

Results indicate that the optimal surcharge-reward scheme overall reduces the system equilibrium costs by 11.58%. The performance also varies with the stations as shown in Table 4. For the first upstream station Vanlose, the optimal design results in higher costs. This is because commuters in upstream stations have priority to occupy the vehicle space. Implementing the surcharge-reward scheme constrained their free choices of service runs. Hence in the many-to-many network case, a cost reduction with the surcharge-reward scheme in one station cannot guarantee the same property in the other stations, although the overall total equilibrium costs are minimized. The results also imply the implementation of reward scheme that considers reduction of equilibrium costs for each station.

Table 5.4. Performance of surcharge-reward scheme

Stations	Total equilibrium costs reduction
Vanlose	7.33%
Fasanvej	-5.33%
Norreport	-20.67%
Kongens Nytorv	-27.63%
Total	-11.58%

6. CONCLUSIONS

This study develops a network-wide user departure time choice model and incentive strategy optimization to relieve commuting congestion in a transit network considering many-to-many origin-destination patterns. The case study is conducted considering a subset of the Copenhagen M2 metro line which is the most congested commuting metro line in Copenhagen metropolitan area. The user equilibrium results indicate that downstream passengers are mostly boarding the service runs at the shoulder of the peak period depending on the availability of vehicle capacity. The incentive strategy model also yields the reduction of the total equilibrium costs in the system yet reveals the increase of the trip costs in the upstream stations.

The present study can provide tractable information and guidance for transit authorities and users to quantify the time-varying crowding, queuing and trip costs in the transit system. The proposed solution algorithms also provide practical applications for the implementation of large-scale networks. There exist limitations of the study where we consider the homogeneity of passengers. Future research will explore the impacts of heterogeneity on passengers' departure time choices and incentive strategies performances as well as include the full M2 line.

Our study has also opened potential avenues for further research such as the integration of large-scale networks, the differentiated incentives for each station and the optimizations of the transit operations. One may also take into account commuter heterogeneity in terms of preferred arrival time in each station (commute and early/late arrival).

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