Auction-Based Tolling: An Economic Experiment to Address Risk-based Heterogeneous Bidding

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SHORT SUMMARY

Auction-based tolling system is a hypothetical tolling system that could reflect the willingness to pay and value of time of toll users by allowing them to attend an auction and bid their desired toll price when compared to paying a fixed toll price as in traditional tolling system. This study conducted an economic experiment of an auction-based tolling system to observe the bidding behavior and capture heterogeneity in participants' risk profiles. Regression mixture models were estimated, and three risk profiles were classified based on the model results, such as risk-neutral, risk-averse, and risk-seeking. It was found that different risk profiles had a significant impact on the revenue generated in the auction. Additionally, the discriminatory pricing rule as well as the positive incentive payoff structure was found to be more effective in maximizing the revenue compared to the uniform pricing rule and penalization payoff structure.

Keywords: congestion pricing, multiunit auction, managed lanes, bidding behavior, marketplace

1. INTRODUCTION

An auction-based tolling system allows road users to participate in an auction to bid for access to the tolled facility. This mechanism provides an active technique for direct price discovery as opposed to pricing-based on historical data and modeling. Additionally, depending on an auction's design, other policy goals may be obtained such as using discriminatory price rules to vary prices between different road users. Recent studies have proposed various auction-based tolling systems for parallel facilities – where one roadway is non-tolled and a parallel facility is tolled (Collins et al., 2015; Basar & Cetin, 2017; Su & Park, 2015; Liu et al., 2015). However, several behavioral research gaps have been found from these studies.

Firstly, bidding strategies were assumed to follow game-theoretic equilibrium strategies without evidence that this behavior would be expected in practice. (Collins et al., 2015; Liu et al., 2015, Basar & Cetin, 2017; Su & Park, 2015). Secondly, road users in the proposed systems were assumed to be risk-neutral – this clearly follows from the equilibrium strategies chosen (Basar & Cetin, 2017; Su & Park, 2015). Heterogeneous risk profiles exist among road users due to varying socio-demographics, trip purposes, activity schedules, and traffic situations.

To address these gaps, this study conducted an auction experiment and employed a regression mixture model to explore heterogeneity in participants' risk profiles. The auctions are multiunit homogeneous good auctions with single unit demand and consist of two auction rules: uniform price and discriminatory price. The experimentally observed bidding behavior and risk profiles of participants are then compared to the equilibrium bidding strategy. Additionally, the revenue analysis was performed to evaluate the auctions' performance under different pricing rules and risk profiles.

2. METHODOLOGY

Experimental Design

This study conducted single-player auction experiments where participants competed against bots in 10 consecutive auction rounds. The bots were programmed to utilize the equilibrium bidding strategies depending on the experiment's auction pricing rule.

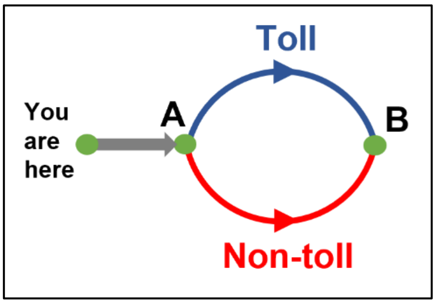

Figure 1: Hypothetical Parallel Facilities Layout

Hypothetical Scenario: The scenario involved a hypothetical network with a single origin-destination pair (AB), connected by both toll road and non-toll road. Road users approaching point A had the choice to take the toll road for not arriving late at B. However, to use the toll road, participants had to participate in an auction and submit a bid indicating their desired toll price.

Payoffs: In each auction round, participants' payoffs were the difference between their valuation of arrival and the Paid Price (if they won). Under Early Arrival Bonus (EAB) scenarios, participants received a bonus if they arrived early (positive arrival valuation). This EAB is a private value known to a participant and randomly selected at the start of each auction round from a uniform distribution on (0,100). Under Late Arrival Penalty (LAP) scenarios, participants were penalized if they arrived late to their destination (negative arrival valuation). This LAP is a private value known to a participant and randomly selected at the start of each auction round from a uniform distribution on (0,100).

Common Information: Common information shared among participants included the group size, available slots, and bid result of the current round. The disclosure of this common information varied between treatments to study its impact on bidding behavior.

Available Slots and Group Size: Participants were randomly placed in groups of 8, 10, or 12 members, with an equal chance of assignment. Excluding the participant, the remaining members in these groups were bots. The number of available toll road slots ranged from 2 to 4, with each option having an equal probability of being selected.

Pricing Rule: The paid price was the final toll price determined by the auction pricing rule. In the uniform price auction, the paid price was the highest rejected bid, while in the discriminatory price auction, it was winner's exact bid.

Payment to Participants: Payment to participants depended on performance from the total payoff across the ten scenarios. In the EAB scenario, the total payoff served as their performance-based reward bonus. In the LAP scenario, the reward bonus was based on their percentile rank of their payoff among participants. All values in the experiment, including the EAB, LAP, Paid Price, and Payoff are expressed in Experimental Currency Unit Dollars, ECU$, converted to U.S. dollars at a conversion rate of 1 ECU$ = $0.10.

Experimental Treatments:
Table 1 presents the treatment variables in the experiment. The available slots, group size, and bidding results were the information that were disclosed (Known) or not disclosed (Unknown) to the participants depending on the treatment. There was a total of 16 treatment groups corresponding to 16 combinations of treatment variables listed in Table 1. These 16 treatments were identical and employed in both EAB and LAP scenarios.

Table 1: Experimental Treatments

Treatment	Description	Levels
Pricing Rule	How a bid is translated into a price to be paid if a bidder wins an auction	Uniform Price (U) Discriminatory Price (D)
Available Slots	Reveals the total number of toll facility slots available to be won before bidding	Known (1) Unknown (0)
Auction Group Size	Reveals the total number of auctions participants before bidding	Known (1) Unknown (0)
Bidding Results	Reveals the lowest accepted bid/highest rejected bid after bidding	Revealed (1) Not Revealed (0)

Game Theoretic Equilibrium

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Uniform Price Auction: In a multiunit uniform price auction with homogeneous goods and single demand, N participants simultaneously submit a single sealed bid to receive one of K items. These bids are ordered and the winners are the K highest bidders. All winners pay the same price, equal to the highest rejected bid (the $K-1$ highest bid). For the uniform price auction of K identical units and N symmetric bidders (where $K < N$) with single-unit demand, a weakly dominant strategy exists resulting the bidding of one's true valuation x_i (Krishna, 2010):

$$b_i(x_i) = x_i \tag{1}$$

Discriminatory Price Auction: A multiunit discriminatory price auction with homogeneous goods and single demand is conducted similarly to the uniform price auction mentioned above. But the K highest bidders each pay a price that is exactly equal to their bid amount. A symmetric equilibrium bidding strategy for this auction scenario was proven as follows (Krishna, 2010):

$$b_i(x_i) = E[Y_K^{(N-1)} | Y_K^{(N-1)} < x_i] \quad (2)$$

Where $Y_k^{(N-1)}$ denotes the Kth-highest order statistic of (N-1) draws from the distribution of valuation. Assuming that their valuation is in the top K bids, a bidder bids the expected K highest valuation conditional on this assumption.

Modeling Bidding Behavior

As this study seeks to explore heterogeneity in bidding behavior, regression mixture models (Leisch, 2004) were estimated with bidding amount as the dependent variable and individual private value as the independent variable. The primary goal was to explore unobserved risk profiles based on participants' bidding behavior in both uniform and discriminatory price auctions under EAB and LAP settings. In a regression mixture model, M different regression equations are fitted on the data and a probability for an observation belonging to any particular regression is fitted by maximum likelihood estimation. Thus, the expected bid by participant i in class m, where $1 \leq m \leq M$, is:

$$E[b_i | m_i = m] = \beta_0^m + \beta_1^m * Valuation \quad (3)$$

Where:

- β_0^m: constant value for class m
- β_1^m: marginal effect of valuation for class m

Models were fitted using the flexmix package in R (Leisch, 2004). Observations where participants overbid their valuations or underbid by less than 10% of their valuation were excluded from the model estimations, as such behavior was considered irrational for this study. For the uniform price auctions and discriminatory price auctions, two regression mixture models are presented with three latent classes – one for the early arrival bonus and one for the late arrival penalty scenarios. Only observations from the last three to five rounds were included in the model estimations – accounting for learning effects from earlier rounds.

3. RESULTS AND DISCUSSION

Descriptive Statistics

The experiments were conducted online on Prolific. Eligible participants resided in the United States and were aged 18 to 99. Each participant was assigned into an auction treatment group and completed a short pre-questionnaire. The experiment comprised 10 consecutive auction rounds and took about 15 minutes to complete, including the pre-questionnaire. Participants received $3.00 for their participation and earned an additional performance-based reward up to $8.00. Out of the total responses, 401 were valid after excluding dropouts and rejections. Table 2 summarizes the socio-demographics of the participants in the experiment.

Across the 401 experiments ran, the average winning rate by treatment was observed to vary from 0.18 to 0.43 in EAB and 0.25 to 0.47 in LAP for the uniform price auction, while in the discriminatory price auction, it ranged from 0.28 to 0.44 in EAB and 0.28 to 0.45 in LAP.

Table 2: Summary of Socio-Demographics

Variable	Description	EAB	LAP
Age	Mean (Standard deviation)	38.01 (12.47)	39.66 (12.63)
	Median	36	37
Gender	Male	66.67	57.55%
	Female	32.69	39.59%
	Non-binary	0.64	2.45%
	Prefer not to answer	-	0.41%
Work Status	Full-time employment	55.42	61.24%
	Part-time employment	9.64	13.57%
	Currently laid off	6.02	4.26%
	Full-time student	7.23	3.49%
	Part-time student	3.61	3.10%
	Others	16.27	13.18%
	Prefer not to answer	1.81	1.16%
Transportation mode mostly used to travel to work/school	Drive alone	68.26	75.00%
	Carpool with only family/household member(s)	2.38	3.37%
	Carpool with at least one person not in a household	3.17	-
	Bus (public transit)	4.76	4.81%
	Private shuttle bus (e.g., employer)	-	0.48%
	Paratransit	-	0.48%
	Bicycle	2.38	0.96%
	Walk (or jog/wheelchair)	11.11	4.81%
	Uber	-	1.92%
	Other modes	7.94	8.17%
Toll facility usage	Daily or almost every day	2.58	5.31%
	Weekly or at least once a week	5.16	11.02%
	Monthly or at least once a month	9.68	13.06%
	Annually or a few times a year	16.77	21.63%
	Never or rarely	65.81	48.98%
Number of valid participants		156	245

Bidding Behavior: Uniform Price

In the uniform price auction, risk-neutral bidders are expected to bid their valuation (weakly dominant strategy). Bidders have no incentive to overbid their valuation – since they could potentially obtain a negative payoff if the lowest rejected bid is above their valuation. Thus, it was not expected for risk aversion to occur (bidding more to ensure a win) since this would be irrational.

For Early Arrival Bonus scenarios, participants in classes one and two bid nearly their valuations. Since this is the equilibrium strategy, these participants are considered risk-neutral with about 83% of participants exhibiting this behavior. For class three, participants underbid their valuations and bid less than 71% of their valuations. These users exhibited risk-seeking behavior with 17% of participants falling into this class.

For the Late Arrival Penalty scenarios, participants in class one (42%) bid approximately their valuations, thus exhibiting risk neutral behavior. For class two in the LAP, bidders bid less than 90% of their valuations, and thus these participants exhibited slightly risk-seeking behavior. This was observed among 47% of participants. About 11% of participants exhibited highly risk-seeking behavior with class three participants only bidding 30% of their valuations.

Table 3: Risk Profiles in Uniform Price Auction

Uniform Price Auction	Early Arrival Bonus	Late Arrival Penalty
Class 1	$-0.26 + 1.00x$ (45%)	$-0.23 + 1.00x$ (42%)
Class 2	$-4.12 + 0.99x$ (38%)	$-3.20 + 0.90x$ (47%)
Class 3	$-6.16 + 0.71x$ (17%)	$5.52 + 0.30x$ (11%)
Number of Observations	167	203
Log-likelihood	-459.44	-586.02
Note: The models were estimated with only the observations in rounds 8, 9, and 10 for both scenarios. The prior probability for membership in a class is given in parentheses.		

Bidding Behavior: Discriminatory Price

Note that risk-neutral participants under symmetric equilibrium conditions are expected to bid K highest expected bid conditional on their bid being in the top K. Participants were not observed to exhibit this equilibrium behavior. Regression models were run using this calculated conditional value, but model results were counterintuitive and did not fit the data better than using a simpler formulation based on linearly with valuation. Model results for the discriminatory price auction are included in Table 4.

For the early arrival bonus auctions, class one participants (48%) exhibited risk averse behavior and bid close to their true valuation, less than 96% of their valuations. This results in a smaller loss when the participant wins than statistically expected. Class two participants bid about 83% of their valuation and were considered risk neutral. Although not exhibiting the exact equilibrium behavior, their regression line was closest to the expected curve. Lastly, 23% of respondents were risk seeking. These class three participants bid about 64% of their valuations.

Similar behavior was observed among the late arrival penalty participants. Risk averse class one participants (43%) bid less than 96% of their valuations. Risk neutral class two participants bid about 78% of their valuation, while risk seeking class three participants bid more than 49% of their valuations. Class two and three included 31% and 26% of respondents respectively.

Table 4: Risk Profiles in Discriminatory Price Auction

Discriminatory Price Auction	Early Arrival Bonus	Late Arrival Penalty
Class 1	$-1.98 + 0.96x$ (48%)	$-1.68 + 0.96x$ (43%)
Class 2	$-0.91 + 0.83x$ (29%)	$0.00006 + 0.78x$ (31%)
Class 3	$-0.04 + 0.64x$ (23%)	$0.73 + 0.49x$ (26%)
Number of Observations	426	574
Log-likelihood	-1364.22	-2064.57
Note: The models were estimated with only the observations in rounds 5 to 10 for both scenarios. The prior probability for membership in a class is given in parentheses.		

Revenue Maximization

This revenue analysis assumes a central toll operator who controls access to a capacity constrained toll facility and thus creates a difference in travel time between the parallel tolled and non-tolled routes to encourage toll road access.

The objective function aims to maximize the generated revenue by adjusting the percentage of vehicles accepted to the toll road:

$$\max_{\lambda} \Delta T(\lambda) \cdot \int_0^1 R_b(x, \lambda, n) f(x) dx \tag{4}$$

Where,

- $\Delta T(\lambda)$: travel time saved by using toll-road with toll road volume proportion λ
- $R_b(x, \lambda, n)$: expected individual-level revenue (rate)
- $f(x)$: valuation distribution's pdf
- λ: proportion of accepted vehicles to the toll road
- x: individual valuation

The integral in the objective function calculates the total expected toll revenue from all road users with accepted bids, where the bids are in proportion to the users' value of time. Note that essentially the distribution of bids is used, and its cumulative distribution function can be used to obtain expected prices. Because for any one valuation, three different bids are possible, the bid distribution function takes a stepwise uniform distribution rather than the uniform distribution of the valuation distribution. By multiplying this rate of expected toll revenue with the time savings achieved using the toll road, the optimization problem yields the maximum expected revenue per 1-hour of travel time saved.

The travel time on the tolled and non-tolled routes follow the BRP link performance function (Bureau of Public Roads, 1964):

$$t(q) = t_{ff} \left[1 + 0.15 \left(\frac{q}{c}\right)^4\right] \tag{5}$$

The following assumptions were made:

- Both roads have the same capacity c and free flow travel time t_{ff}
- c and t_{ff} normalized to 1
- Total traffic flow between both roads normalized to 1, $Q = q_{toll} + q_{nontoll} = 1$

Then, the travel time saved $\Delta T(\lambda)$ has a closed form as follows:

$$\Delta T(\lambda) = t_{toll} - t_{non-toll} = 0.15[(1-\lambda)^4 - (\lambda)^4] \tag{6}$$

The derivation used here follows similarly to Collins et al. (2015). Thus, the revenue is presented as a percentage of the maximum value of time times the number of road users.

Due to the revenue equivalence theorem and the construction of the equilibrium bidding strategy for multiunit auctions with single unit demand, the uniform and discriminatory price auctions generate the exact same revenue (Krishna, 2010). But the experimentally derived bidding strategies above do not exhibit this property. Also note, a risk neutral, rational actor would bid similarly whether payoffs involve a bonus or penalty.

Table 5 presents the results of the optimization problem under three auction settings: Equilibrium, Experimental Uniform Price, and Experimental Discriminatory Price. This is presented separately for EAB and LAP. The Equilibrium was considered as a benchmark for evaluating revenue generation of the experimental bidding results saw in the EAB and LAB. Under the equilibrium bidding behavior, 16.563% (optimal λ) of participants bids were accepted. The optimal λ in Equilibrium case maximized the revenue generated regardless of employed pricing rules or incentive structures (EAB or LAP).

Table 5: Revenue Generated in Each Auction

Auction	Objective	EAB	LAP
Equilibrium (Uniform and Discriminatory Price)	λ	0.16563	
	Revenue	0.01000 (100%)	
Experimental Uniform Price	λ	0.15812	0.15896
	Revenue	0.00934 (93.40%)	0.00894 (89.40%)
Experimental Discriminatory Price	λ	0.17334	0.17080
	Revenue	0.00958 (95.80%)	0.00935 (93.50%)
Note: Values in parentheses are in reference to the revenue under equilibrium. Revenue is expressed as a percentage of the maximum value of time times the total number of road users.			

Using the experimental uniform price bid functions, fewer road users were accepted into the toll facility and less revenue was generated. Revenue was 93% less under EAB and 89% less under LAP. Under EAB, 15.8% of road users were accepted, while 15.9% were accepted under LAP. Using the experimental discriminatory price bid functions, more road users were accepted into the toll facility and less revenue was generated. Revenue was 96% less under EAB and 94% less under LAP. Under EAB, 17.3% of road users were accepted, while 17.1% were accepted under LAP. More revenue was observed under the discriminatory price auction when using experimentally derived bid functions. This is to be expected due to risk aversion observed in the discriminatory price auctions and since road users pay their own bid.

Sensitivity Analysis

A sensitivity analysis was performed to evaluate how varying the proportion of EAB and LAP participants within an auction affected the generated revenue and its corresponding optimal λ. This analysis was to give an insight into the situation with the presence of mixed risk profiles in the auction.

Figure 2 visualized the results from the uniform price auction, showing a consistent trend: as the proportion of EAB participants increased (and conversely, LAP decreased), the generated revenue also increased. Interestingly, the optimal λ remained relatively consistent across different compositions of EAB and LAP, with a very small reduction from 0.15896 to 0.15812. This suggested that while the revenue was sensitive to the proportion of EAB participants, the acceptance rate remained relatively stable, unaffected by the mix of EAB and LAP bidders.

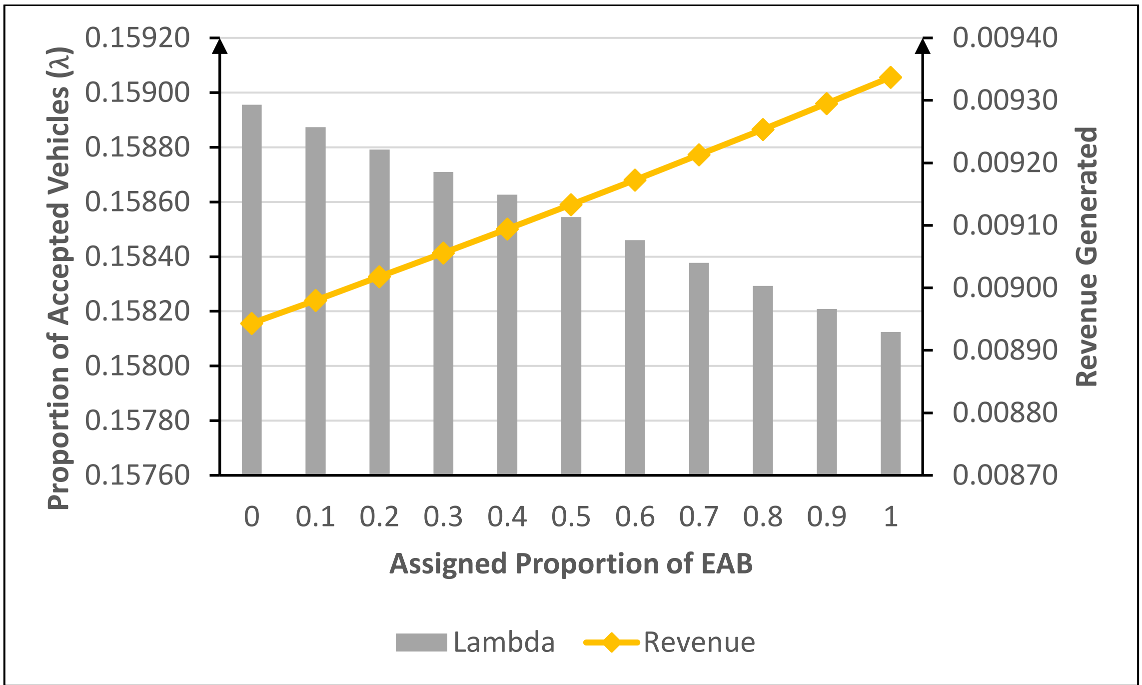

Figure 2: Revenue Generated in Uniform Price Auction by Varying the Proportion of EAB and LAP

For the discriminatory price auction, Figure 3 displayed a similar pattern in revenue, which increased proportionately to the EAB proportion. However, unlike the uniform price auction, the optimal λ in the discriminatory price auction exhibited a slightly upward trend as the EAB proportion increased. This trend could be due to the bidding strategy of submitting lower bids to take advantage of the EAB, which increased their overall payoff if winning. This strategy was captured in the second class in the discriminatory price auction. On the other hand, the auctioneer would possibly accept more bids. Thus, both the optimal acceptance rate and the generated revenue increased proportionately to the proportion of EAB.

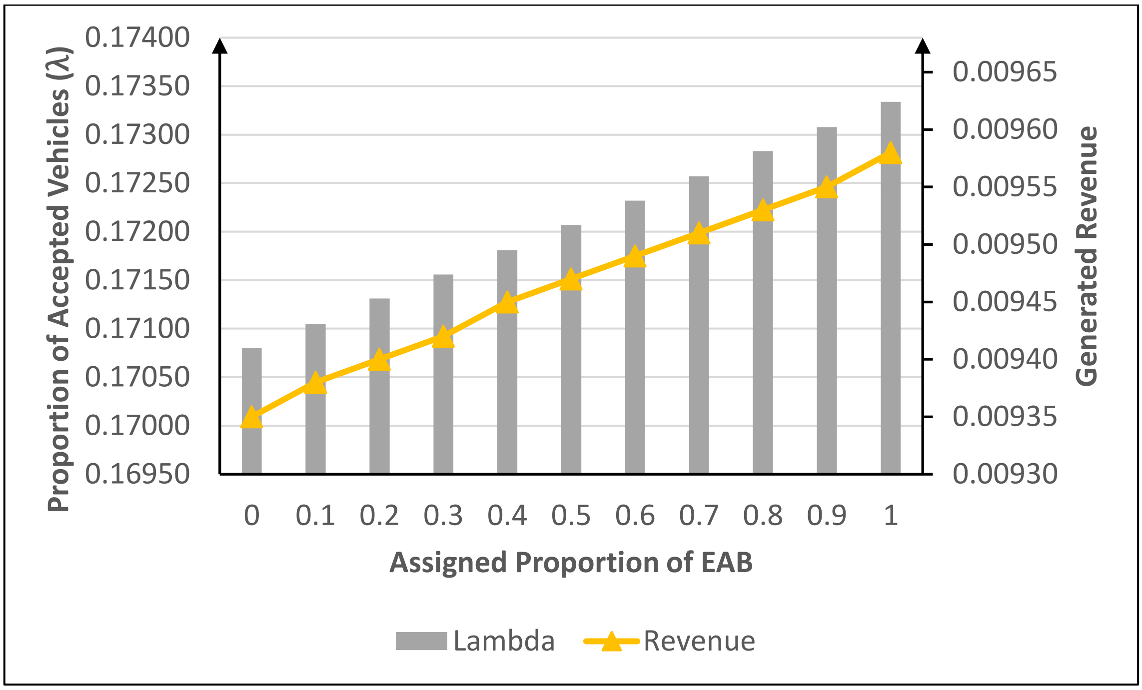

Figure 3: Revenue Generated in Discriminatory Price Auction by Varying the Proportion of EAB and LAP

4. CONCLUSIONS

The estimated regression mixture models in this study captured the heterogeneity in bidding behavior, categorizing bidding behavior into three primary risk profiles: risk-neutral, risk-averse, and risk-seeking. Risk-averse participants were observed to bid very close to their true valuation in discriminatory price auctions, exhibiting an overly cautious bidding strategy to avoid losses – bidding so close to their valuation that they on average only received a payoff of about 4% of their valuation. It was observed that the different risk profiles considerably impacted the generated revenue compared to equilibrium revenue. These results suggest that policy makers may consider discriminatory price auctions to achieve higher revenues, but that this result exploits risk averse road users which may impact lower income and more time-sensitive users disproportionately.

Additionally, risker bidding (smaller bids) is observed in the LAP scenarios as compared to the EAB scenarios. These road users were unlikely to win auctions due to this behavior but understanding the profiles of these users may be important when considering equity and fairness aspects of auctions.

In conclusion, the exploration of risk profiles through regression mixture models has explained the strategic bidding behaviors of auction participants under different pricing rules and positive (EAB) or negative (LAP) incentive structure, as well as the impacts of different risk profiles on the revenue generation. The advantage of discriminatory pricing rule in revenue maximization, combined with the effectiveness of EAB incentives, could offer a valuable insight for auction designers and toll operators. Future work could focus more on the factors affecting individual risk profile as well as analyzing how the disclosure of information impacts the individual risk profile.

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