Modeling Link-level Urban Traffic Dynamics based on Corridor Macroscopic Fundamental Diagram

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SHORT SUMMARY

An efficient and accurate dynamic network loading technique which can reproduce the congestion propagation within an urban road network is essential for the assessment of traffic management strategies. This paper proposes a link-level traffic flow modeling paradigm based on the corridor macroscopic fundamental diagram (CMFD), which describes the relationship between vehicle accumulation and flow production within an urban corridor. With the traffic dynamics depicted by the CMFD, the model formulation is concise. The adopted trip-based modeling approach also ensures first-in-first-out at intersections for multi-commodity flow in congested conditions with spillback. We evaluate the model performance in scenarios with different corridor layouts and origin-destination compositions. To validate the proposed model, the results are compared with microsimulation outcomes. It is anticipated that the model can be conveniently integrated into the optimization of multi-modal urban road space allocation strategy with corridor-level decisions.

Keywords: dynamic network loading; macroscopic fundamental diagram; multi-commodity traffic flow; road space allocation; trip-based modeling; urban network traffic

1 INTRODUCTION

Urban traffic congestion can be interpreted as the reduction of travel speed caused by an excessive amount of vehicles in the network. When a road link reaches the saturated traffic state due to the long vehicle queue, the link travel time increases. Once the link traffic further becomes oversaturated, the spillback queue occurs and hence reduces the flow which can transferred from the upstream intersection. An efficient and accurate macroscopic traffic model or dynamic network loading (DNL) approach which can reproduce the above-mentioned phenomenon and identify the bottleneck locations is crucial for the development of urban traffic management strategies.

Multi-modal road network design is considered a rapid way to facilitate a radical modal shift toward a sustainable urban environment. Reallocating dedicated space to eco-friendly transport modes, such as buses, trams, and bicycles, helps ensure the right-of-way and safety for various road users. However, reducing the available capacity for motorized traffic may lead to more traffic congestion if without careful consideration of network traffic performance. To prevent the severe breakdown of network traffic system, a suitable model which can efficiently simulate the congestion dynamics resulted from the reallocation strategy is required.

However, existing macroscopic urban traffic models were often designed for signal control purposes. Hence, they explicitly include the signal timing plan as parameters or even variables in the model. For instance, there were studies which extended the cell-transmission model (CTM) and link-transmission model (LTM) to consider signal control in the urban context (Adacher & Tiriolo, 2018; van de Weg et al., 2020). These models can be computationally-expensive when applying them to a large-scale network due to the complex model formulation. Moreover, for problems which are unrelated to signal control, these models are considered overly intricate.

For the optimization of dedicated bus lane allocation, Tsitsokas et al. (2021) developed an extended version of the store-and-forward (SaF) model (Aboudolas et al., 2009). Different from CTM and LTM, the model considers horizontal queues and the computation of delay within the link by separating the moving and queuing vehicles, a feature in the S model proposed by Lin et al. (2012).

Therefore, besides the traffic signal timing information, the transfers of flows within a link also needs to be modelled every time step. This increases the number of variables in the model. In addition, the time discretization does not allow the progression of flow when the available space in the downstream link is not enough to accommodate the outflow in one time step, which decreases the modeling accuracy.

On the other hand, the single-commodity version of these models simply simulates turning flow by using the pre-determined split ratios at intersections without considering the detailed routes followed by each origin-destination (OD) pair. Although the multi-commodity versions seek to respond to this limitation by defining the set of routes for each OD instead of specifying turning ratios, they could only attain a certain level of approximation of the first-in-first-out (FIFO) condition (De Souza et al., 2022). The influence of such violation becomes significant when there is congestion within the network (Bar-Gera & Carey, 2022). Moreover, there is still little endeavor which applies such technique to a real urban network with a large number of routes. It is suspected that this complexity may degrade the accuracy of the multi-commodity DNL outcome. A modeling technique which can overcome such an issue is required.

In this paper, we propose a link-level traffic modeling paradigm by exploiting the merit of the corridor macroscopic fundamental diagram (CMFD) to describe the traffic dynamics on road links along an urban corridor. The use of CMFD in the traffic model also guarantees easy integration with the optimization algorithm for urban road space allocation problems with corridor-level decisions, while the trip-based formulation is adopted to enable continuous-time modeling and ensure exact FIFO. This also allows the incorporation of route choice behavior at the network-scale. The remainder of the paper is structured as follows. Section 2 introduces the proposed model. Section 3 describes the experimental design. The results are then presented in section 4, while section 5 again summarizes the contribution and discusses the future research direction.

2 Model Description

MFD has been a convenient tool for urban traffic modeling at the aggregated-level. For instance, the traffic dynamics on a corridor can be captured by its CMFD. This means that the effect of signal control, including the green-to-cycle ratio and the offset, is already accounted for. Hence, when modeling the corridor traffic dynamics, the signal timing information would not need to be considered anymore, which greatly reduces the number of variables. In the proposed model, we assume that the traffic dynamics on every link along the corridor is unified and can all be described by the CMFD. The outcome of such an approximation will be discussed in section 4.

The shape of the CMFD used by the model is estimated with the variational theory (VT) method proposed in Leclercq and Geroliminis (2013). In the VT method, an analytical CMFD is composed of a set of practical cuts in the density-flow (k, q) plane. The cuts are derived from the mean speeds and mean costs of several shortest path moving observers in the VT graph, which is a time-space (x, t) plane containing the signal timing plans of intersections along the corridor. For detailed information regarding the VT method, the reader is referred to the original paper.

This paper adopts the event-based resolution method of the trip-based MFD modeling approach originally proposed in Mariotte et al. (2017) so that the movement of each individual trip is tracked. Different from the network-level modeling, the formulation is applied to model the transmission of vehicles between links in this paper. Figure 1 explains the modeling principle for a single link. The movement of vehicles within the link is governed by the remaining travel distance and the mean speed of the link v which can be derived from the CMFD according to the flow q and the density k of the link, as given by Equation 1.

$$v(k) = q(k)/k \tag{1}$$

As shown in the illustrated CMFD in Figure 1, let Q^{cap} denotes the flow capacity of the link. The traffic state is considered saturated between the critical density K^{cr} and the oversaturated density K^{os}. N_i^{in} and N_i^{out} are the order of the vehicles which are about to enter and exit the link, respectively. The entry flow into the link is limited by an entry supply function I_i, which will be elaborated below, while the exit flow is constrained by both the outflow capacity Qcap and the

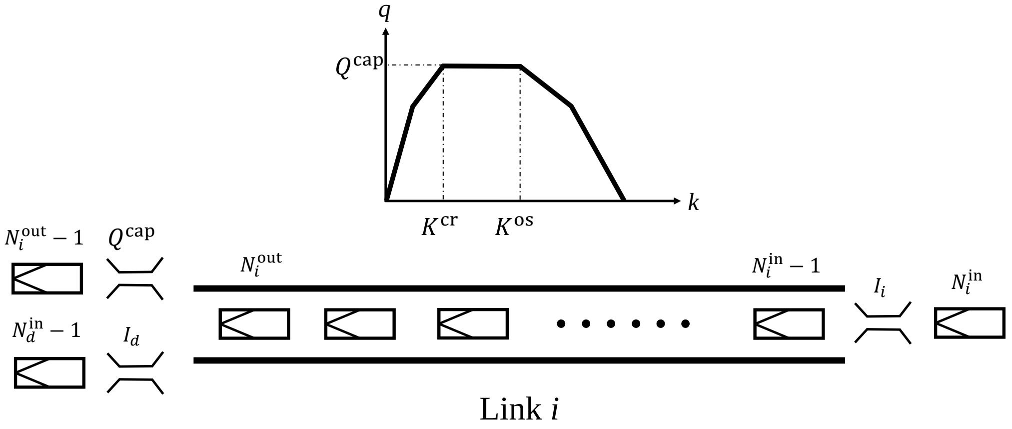

Figure 1: Representation of the link flow modeling based on the CMFD

entry supply of the downstream link d of the next exiting vehicle N_i^{out}.

To consider the spillback effect at the entry of the link, the concept of entry supply limitation described in Mariotte and Leclercq (2019) is included. Equation 2 formulates the entry supply function I_i which is a function of the link density k_i. When the link is oversaturated, i.e., the link density exceeds K^{os}, only the remaining production can be utilized by the inflow demand. The earliest possible (supplied) entry time of vehicle N_i^{in} is then calculated by Equation 3.

$$I_i(k_i) = \begin{cases} Q^{\text{cap}}, & \text{if } k_i \leq K^{\text{os}} \\ Q(k_i), & \text{otherwise} \end{cases} \tag{2}$$

$$t_{\text{entry supply}}^{N_i^{\text{in}}} = t_{\text{entry}}^{N_i^{\text{in}}-1} + \frac{1}{I_i(k_i)} \tag{3}$$

In undersaturated conditions, the time of the next exit demand is determined by the remaining distance of vehicle N_i^{out} on the link and the link speed computed from the CMFD. In Equation 4, L_i denotes the total length of link i, while $L^{N_i^{\text{out}}}$ is the remaining distance for vehicle N_i^{out} to travel on link i. For saturated or oversaturated conditions, the extension to account for the maximum exit demand during saturation is also applied. When the link density exceeds K^{cr}, the last vehicle on the link N_i^{out} would be pushed to exit the link so that the exit flow can fulfill the maximum outflow Q^{cap}. On the other hand, besides the capacity constrain, the exit time is also limited by the entry supply of the downstream link of vehicle N_i^{out}, which is denoted by d here. The supplied exit time of vehicle N_i^{out} can hence be expressed as Equation 5.

$$t_{\text{exit demand}}^{N_i^{\text{out}}} = \begin{cases} t + (L_i - L^{N_i^{\text{out}}})/v(k_i), & \text{if } k_i \leq K^{\text{cr}} \\ \max(t, t_{\text{exit}}^{N_i^{\text{out}}-1} + \frac{1}{Q^{\text{cap}}}), & \text{otherwise} \end{cases} \tag{4}$$

$$t_{\text{exit supply}}^{N_i^{\text{out}}} = \max(t_{\text{exit}}^{N_i^{\text{out}}-1} + \frac{1}{Q^{\text{cap}}}, t_{\text{entry supply}}^{N_d^{\text{in}}}) \tag{5}$$

The procedure of the event-based trip-based modeling scheme for the proposed link-level traffic flow model is summarized below:

1. Initialize all the vehicle entry requests at every inflow point by assigning the desired entry time, selected route, and length of the first link of the route (remaining link travel distance).
2. Determine the next time step and time step size by finding the event with the minimum entry or exit time on all links following the demand and the supply limitations described in Equation 3, Equation 4, and Equation 5. Note that it is possible to have multiple events taking place at the same time step.
3. Update the remaining link travel distance for all existing vehicles based on the link speed derived from the CMFD and the time step size determined in step 2.
4. Insert and/or remove vehicle for each link according to the events identified in step 2. Afterward, update the accumulation on each link.

5. Compute the new link speed for each link in the next time step using the updated density.
6. Return to step 2.

3 EXPERIMENT

The model performance is tested in two scenarios. A homogeneous corridor with a uniform block length of 250 m is implemented in the first scenario, as shown in Figure 2. The second scenario simulates the traffic dynamics of a real urban corridor with heterogeneous layout so that the validity of the model can be further demonstrated. For testing purposes, a corridor in the city center of Zurich consisting of Mühlegasse, Rudolf-Brun-Brücke, Uraniastrasse, and part of the Sihlstrasse is selected without considering whether the lane configuration in reality is consistent across the corridor or not, as shown in Figure 3.

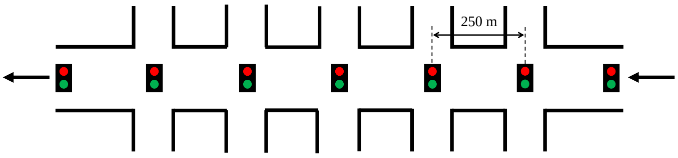

Figure 2: Layout of the homogeneous corridor

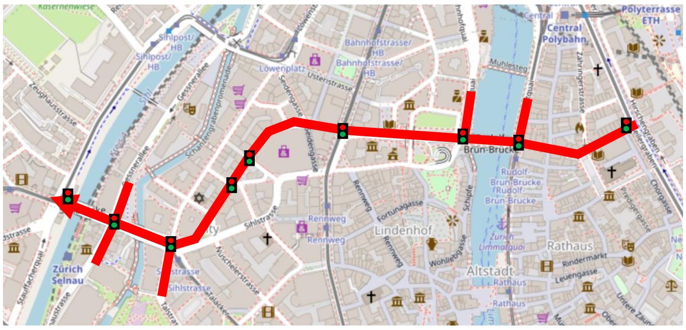

Figure 3: Layout of the selected heterogeneous corridor in the city center of Zurich (source: OpenStreetMap)

It is assumed that the signals along the corridor have a uniform green length of 25 s and cycle length of 60 s. There is no endogenous bottleneck since the lane configuration and signal timing plan are both uniform across the corridor. Considering the free-flow condition, the offsets are implemented in a way that approximately half of the vehicle platoon which starts from the upstream intersection can pass through the downstream intersection if there is no queuing vehicle, i.e., block length / free flow speed / 2.

A 3-hour scenario with varying inflow demand is designed to mimic a typical peak period. Figure 4 plots the demand profile of the OD which enters from the first intersection and exits at the last intersection of the corridor (hereinafter referred to as the main OD). In the homogeneous corridor case, one turning OD is simulated. Turning vehicles enter the corridor from the fourth intersection and exit via the seventh intersection. The demand profile of every turning OD is set to one-fifth of the main OD. For the heterogeneous corridor, we implement more turning ODs to test the ability of the model to simulate a complicated flow composition. At each intersection with cross-roads, there is a turning OD exiting the corridor, while there is also an OD entering from there. In this scenario, the demand profile of every turning OD is set to one-tenth of the main OD.

The open-source microscopic traffic simulation tool, SUMO (Lopez et al., 2018), is used to compute the groundtruth simulation outcome. The desired maximum speed parameter in the car-following model is also determined based on the setup in Table 1. No stochasticity and heterogeneity in the driving behavior is included.

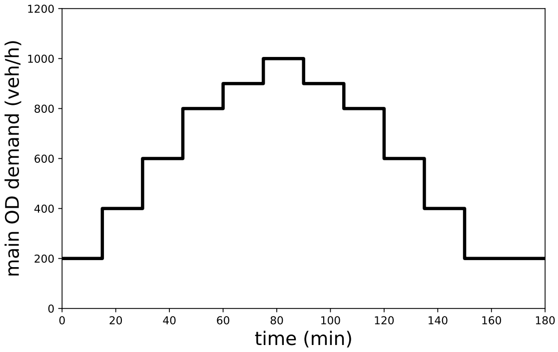

Figure 4: Demand profile of the OD which passes through the entire corridor

Parameters used in the VT method are summarized in Table 1. It is worth noting that the intersection capacity (saturation flow rate) and jam wave speed are larger than the values typically used in practice since a homogeneous driving behavior is considered to align with the setting in the microsimulation tool for validation purposes.

Table 1: Parameters used in the VT method

Parameter	**Value**
free flow speed v_f	12.5 m/s
jam wave speed w	7.5 m/s
intersection capacity q_m	0.62 veh/s

4 RESULTS AND DISCUSSION

The modeling and simulation outcomes are discussed by showing the evolution of the main OD inflow, corridor accumulation, and mean speed of the corridor over time. The corridor MFDs are also presented to demonstrate that whether the internal congestion caused by spillback queues can be reproduced.

Figure 5 presents the simulation outcome of the homogeneous corridor. It can be seen from the comparison between the two approaches that the results of the proposed model shows good compliance with the SUMO microsimulation output. In Figure 5a, the limited main OD inflow from approximately 4000 s caused by the spillback blockage can be successfully captured. This can also be explained by the plateau of accumulation shown in Figure 5b. In addition, the reduction and recovery of the corridor mean speed are well-reproduced, as plotted in Figure 5c. The simulated CMFDs are plotted in Figure 5d. The shape of the two figure-8 hysteresis loops caused by the inhomogeneous density distribution within the corridor closely match each other, as can be seen from the maximum corridor flow, maximum corridor density, and the speed of the congestion dissipation (slope of the recovery branch of the MFD).

The outcome of the homogeneous corridor demonstrates the ability of the proposed method to model traffic dynamics in a network with a uniform layout (block length) and simple OD composition. To test the robustness of the method, a more complex flow composition is simulated in the heterogeneous corridor case. Here, the threshold density values K^{cr} and K^{os} controlling the entry supply and exit demand on each link are fine-tuned by running several trials. The results are presented in Figure 6.

The results of the main OD inflow and accumulation on the heterogeneity corridor presented in Figures 6a and b indicate that the overall trend of the traffic performance evolution can again be captured at the aggregated level. However, Figure 6c shows that there is a smaller mean speed drop amplitude between 2000 s and 4000 s simulated by the proposed model. This can also be observed from the simulated MFD shown in Figure 6d. The proposed method fails to model the corridor traffic state deviation from the steady state theoretical MFD envelope during the

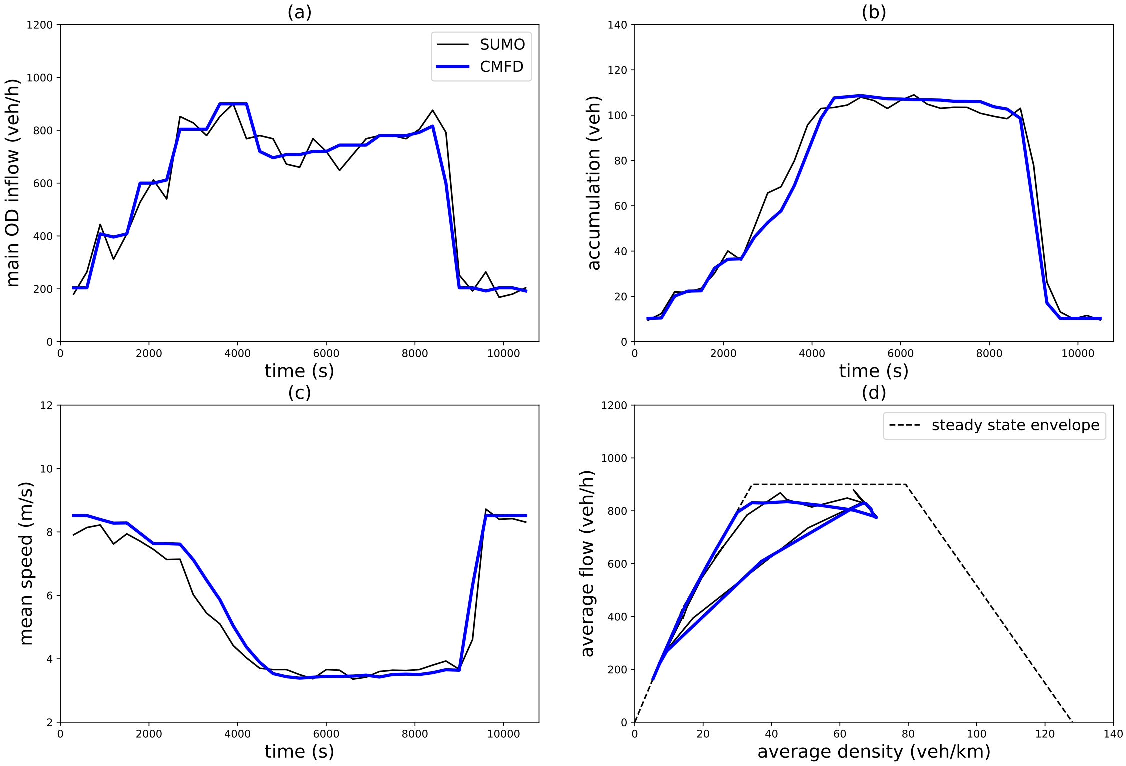

Figure 5: Simulated outcomes of the (a) main OD inflow (b) accumulation (c) main OD mean speed (d) MFD of the homogeneous corridor using SUMO and the proposed model

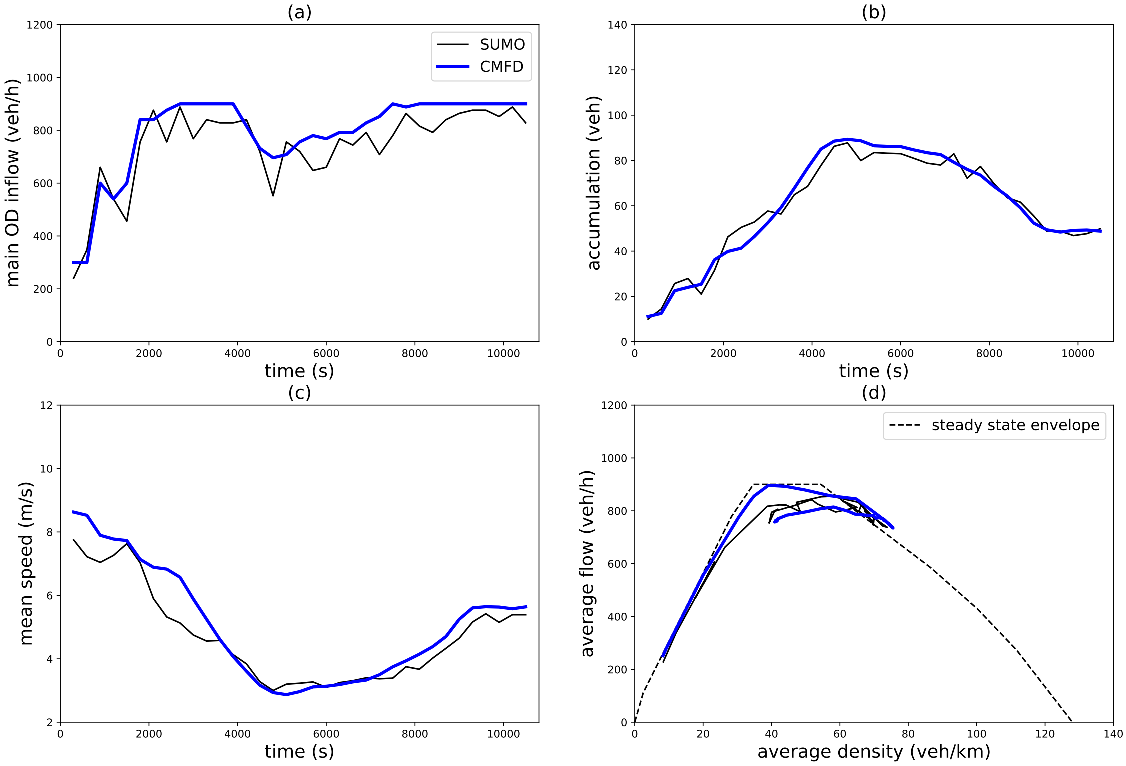

Figure 6: Simulated outcomes of the (a) main OD inflow (b) accumulation (c) main OD mean speed (d) MFD of the heterogeneous corridor using SUMO and the proposed model

onset of congestion. This is caused by the fact that CMFD does not precisely describe the traffic dynamics on each link. It is, instead, an approximation from the corridor-level. Figure 7 provides a detailed examination of model performance by showing the accumulation evolution on each link.

Discrepancies can be observed from some of the links as the progression dynamics is heterogeneous within the corridor due to the different block lengths and signal offsets. Further fine-tuning of the parameters K^{cr} and K^{os} for each link can be considered.

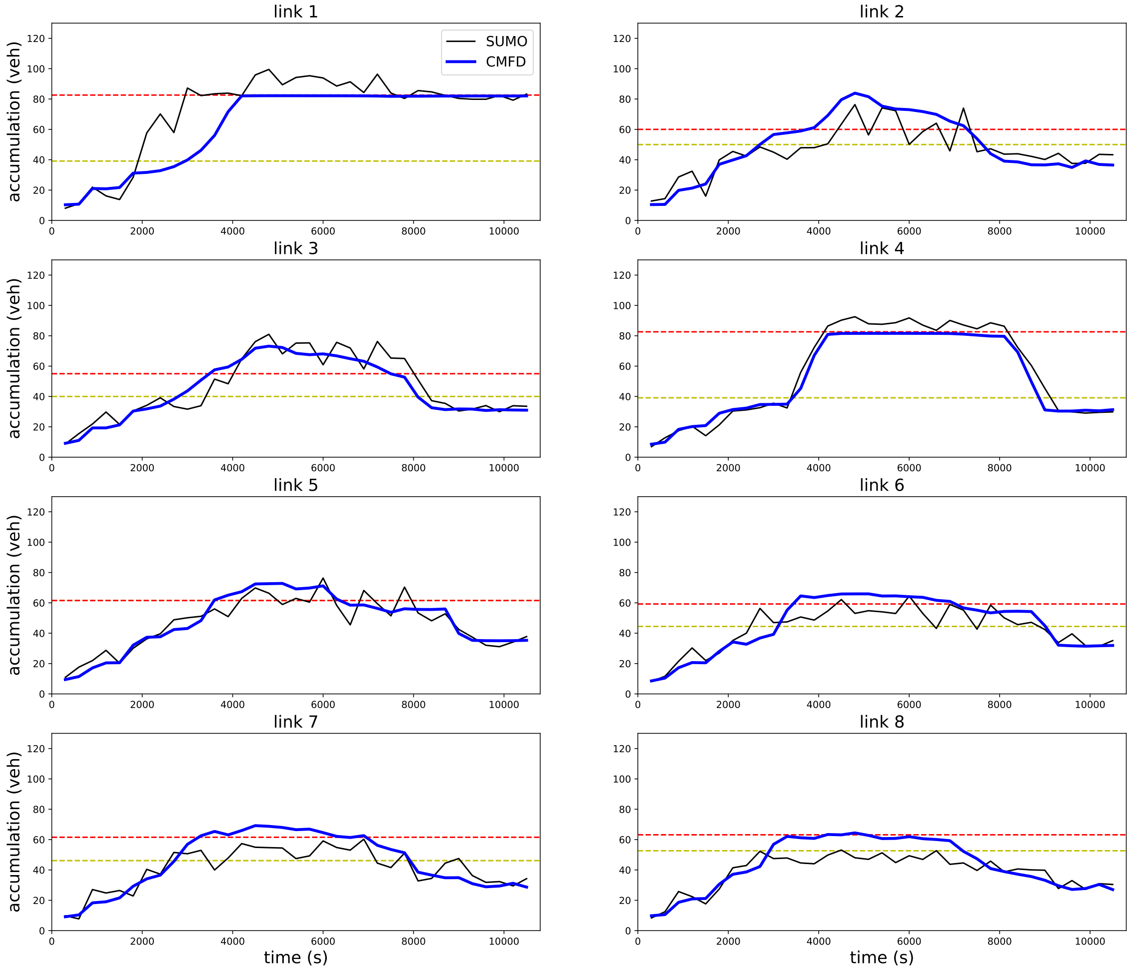

Figure 7: Accumulation evolution on each link of the heterogeneous corridor (yellow dashed line: critical accumulation, red dashed line: oversaturated accumulation)

In this paper, only the inflow and mean speed of the main OD are analyzed. It is also worth investigating the delay experienced by other ODs over the simulation period.

5 CONCLUSIONS

This paper proposes a new modeling paradigm to simulate urban traffic dynamics at the link-level based on CMFD. It is able to track the traffic performance on each link within the network over the simulation period with a concise model formulation. In terms of computation efficiency, the model is considered an alternative which lies between microscopic traffic simulation tools and typical macroscopic link-level traffic modeling methods. By using the trip-based modeling approach, it does not require the simulation of vehicle movements in detail using microscopic car-following models. However, the event-based resolution may result in longer computation time than other discrete-time models require in a congested network with a large amount of circulating vehicles. On the other hand, as such a simulation scheme can prevent the violation of FIFO at intersections in a congested network with spillback phenomena, it is speculated that the proposed model may yield better accuracy compared to other multi-commodity flow modeling approaches particularly when there are multiple ODs in the network.

The simulation results in the two corridor test cases show a satisfactory degree of compliance with the SUMO microsimulation outcome. The modeling accuracy is high for a homogeneous corridor, while it degrades when the corridor has a relatively heterogeneous layout due to the approximation of link-level traffic dynamics based on CMFD. Sensitivity of the model performance against corridor layout heterogeneity is a research question to be explored. Furthermore, it can also be compared with the multi-commodity version of other link-level urban traffic models, such as the extended SaF (Tsitsokas et al., 2021), to understand the improvement in modeling accuracy and the potential influence on computational efficiency.

Most importantly, the use of CMFD to describe link-level traffic dynamics allows convenient adaptation for changes in lane configuration on the road space of the corridor. Hence, the model can be easily integrated into traffic problems pertaining to corridor-level decision-making, such as the multi-modal urban road space allocation strategy. Future work will extend the model application to the network-scale. An urban network can first be divided into multiple corridors, each of which consists of road links with the same lane configuration. In reality, this often applies to road sections with the same street name. The route choice behavior for each OD considering the travel time affected by traffic congestion dynamics can then be incorporated.

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