Modelling and optimization of flexible users in large-scale ridesharing systems

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Short summary

The performance of ridesharing systems is intricately entwined with user participation. To characterize such interplay, we adopt a repeated multi-player, non-cooperative game approach to model a ridesharing platform and its users' decision-making. Users reveal to the platform their participation preferences over being only riders, only drivers, flexible users, and opt-out based on the utilities of each mode. The platform optimally matches users with different itineraries and participation preferences to maximize social welfare. We analytically establish the existence of equilibria and design an iterative algorithm for the solution. A case study is conducted with real travel demand data in Chicago. The results highlight the effect of users' flexibility regarding mode preferences on system performance. A sensitivity analysis of the participation level of users underscores the effect of economies of scale in such systems, emphasizing the pivotal mode of user participation in system efficiency.
Keywords: Ridesharing, Repeated multi-player non-cooperative game, Linear programming, Random utility model, Mode choice

1 Introduction

Despite the profound benefits of ridesharing in terms of pollution and congestion reduction, participation in ridesharing systems is often relatively low. Taking Switzerland as an example, a country where public transport is generally considered among the most efficient (Michael Page, 2023), statistics of the Federal Statistical Office show that 78% of the households own a car, whereas only 53% own a public transport season ticket. In 2021, the Swiss population traveled around 30 km per person per day, out of which 69% by car (Swiss Federal Statistical Office, 2021). In 2017, the Federal Energy Office estimated the average car occupancy during peak hours to be 1.1 (Swissinfo, 2017). In the US, where public transport is typically much less developed, average car occupancy for commuting trips is 1.2 USDOE (2022). Ridesharing is known to increase vehicle occupancy and reduce congestion. In addition, ridesharing is expected to reduce car ownership in the long term. To this end, we analyze users' participation decisions in a ridesharing system.

User participation is influenced by the cost factors of ridesharing systems. Existing empirical studies have explored factors affecting user participation in ridesharing (Amirkiaee & Evangelopoulos, 2018). Many studies supported cost factors as critical considerations for participation in ridesharing Shaheen et al. (2016); Chen et al. (2017); Wang & Noland (2021); Park et al. (2018); Tahmasseby et al. (2016). Aside from costs, safety concerns, as well as the difficulty of finding rides and social awareness, are major issues preventing the prevailing of ridesharing (Ciari & Axhausen, 2013; Nielsen et al., 2015). Julagasigorn et al. (2021) indicate a growing interest in recent studies on distinct factor analyses for various role preferences, highlighting the necessity to distinguish users by their participation roles.

Conversely, ridesharing costs depend on user participation. As matching decisions usually benefit from economies of scale, the quality of matches depends on the pool size and composition of participants. In turn, users decide to participate in ridesharing based on their perceived costs of ridesharing in comparison to other transportation modes. This highlights the importance of modeling the interaction of system performance and mode/participation choice. Recent analytical studies, e.g., Yao & Bekhor (2023), have modeled such interplay resorting to equilibrium approaches. The efficiency of matching is often hindered by the imbalance of riders and drivers, while the participants' flexibilities can help circumvent this issue (Liu et al., 2020). However, to our best knowledge, such flexibility in role preferences has not been endogenously modeled in the literature yet.

This paper describes a game-theoretic approach to the day-to-day model of ridesharing participation. A game is envisaged between a platform that determines the matching of riders and drivers and users who can choose to participate in the system as riders and/or drivers. Particularly, users can choose to be flexible such that they would accept being matched as both drivers and riders. This is assumed to be a *repeated multi-player non-cooperative game*. Users estimate their expected utility of all feasible modes, and their choices among participation modes are captured by a discrete choice model, in particular, a multinomial logit model. Based on users' choices, the platform determines the matching that maximizes the total utility of all participants. We formulate the decision-making problem of the platform as a Linear Programming (LP) problem. A diagram describing the interaction of these two layers of decision-making is provided in Figure 1. We develop an iterative algorithm to evaluate the optimal and equilibrium solutions in a numerical case study.

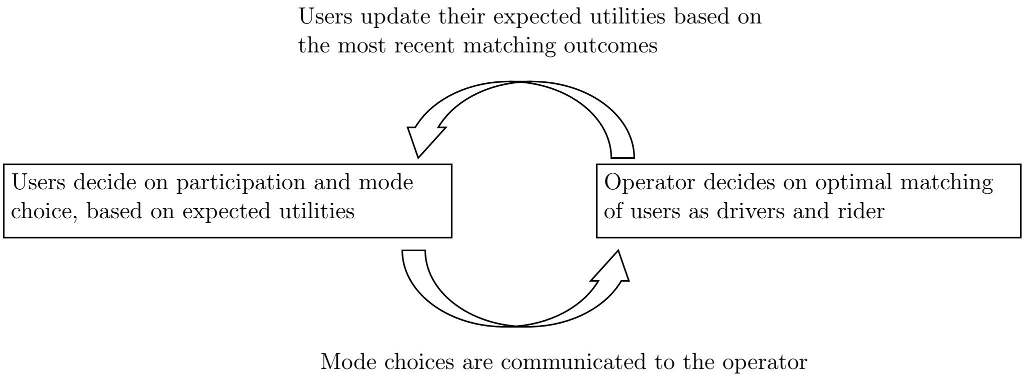

Figure 1: Diagram of the decision-making framework

2 Methodology

In this section, we first characterize the decisions of individual ridesharing users. Then, we delineate the mathematical formulation for modeling the optimal matching decision of the platform. Finally, we discuss the existence of equilibria and the iterative algorithm used to obtain solutions.

Decisions of ridesharing users

Consider a general road network where heterogeneous users with different itineraries reveal their travel preferences to be enrolled in the system. In this regard, we consider a set J of classes among users registered in the ridesharing platform. Users in the same class $j \in J$ are homogeneous regarding origin, destination, scheduling preference, and car ownership. Our setting can be generalized to cases with any other type of heterogeneity by adjusting set J.

Each user chooses a participation mode m from the set $M = \{\text{opt-out}, \text{rider}, \text{driver}, \text{flexible}\}$. Here, "flexible" refers to the case where users want to participate in ridesharing but are flexible in their mode as either riders or drivers. In class $j \in J$, we denote the number of users as n_j and the fractions of users who enrolled as opt-out, riders, drivers, and flexible users as $p_j^{\text{opt-out}}$, p_j^{rider}, p_j^{driver}, and p_j^{flexible}, respectively. Obviously, if users in class j do not own a car, the corresponding proportions p_j^{driver} and p_j^{flexible} are forced to zero. We consider four types of matches for a combination of user modes, $\mathcal{A} = \{\text{II}, \text{FI}, \text{IF}, \text{FF}\}$, as follows: 1) an inflexible rider matches to an inflexible driver, 2) a flexible rider matches to an inflexible driver, 3) an inflexible rider matches to a flexible driver, 4) a flexible rider matches to a flexible driver. We use decision variables x_{ij}^a to define the number of matched riders of class i to drivers of class j that are matched according to type a.

The matching outcomes depend on the preferred mode choice and might bring different utilities to users. Opting-out users will never be matched and thus leave the system driving alone or choosing another transportation mode, such as public transit. We denote the utility of leaving the system for users from class $j \in J$ as u_j^{out}, which is exogenous and independent of the choices of other users. Then the expected utility of opting out would be

$$\mathbb{E}u_j^{\text{opt-out}} = u_j^{\text{out}}. \tag{1}$$

Meanwhile, inflexible riders and drivers have stochastic matching outcomes and utility. The possible matching outcomes of enrolled riders (m = rider) are further affected by the decisions of other users and the matching decisions of the platform. Suppose that the platform decides that the number of riders from class $i \in J$ that match drivers from class $j \in J$ according to type $a \in \mathcal{A}$ is x_{ij}^a. We characterize the probabilities and utilities of different matching outcomes as follows.
(1) Being matched with a driver in class $j \in J$. Assuming that matching outcomes of users from the same group are independent, the probability $P_i^{\text{rider}}(ij)$ of an inflexible rider from class i being matched with a driver in class j is

$$P_i^{\text{rider}}(ij) = \frac{x_{ij}^{\text{II}} + x_{ij}^{\text{IF}}}{p_i^{\text{rider}} n_i}. \tag{2}$$

The utility of such an outcome is determined by the gained utility from the match and the internal transfer scheme between drivers and riders. For a match between a rider from $i \in J$ and a driver from $j \in J$, we assume that their joint utility is a constant u_{ij}, and if they choose to opt-out, the individual utilities for them would be u_i^{out} and u_j^{out}, respectively. Then the gained utility g_{ij} is

$$g_{ij} = u_{ij} - u_i^{\text{out}} - u_j^{\text{out}}. \tag{3}$$

Under the internal transfer scheme, the gained utility g_{ij} be split across riders and driver as fractions ϕ^{rider} and $\phi^{\text{driver}} = 1 - \phi^{\text{rider}}$. Then the utilities of the rider and the driver, u_i^{rider} and u_j^{driver}, are given by

$$u_i^{\text{rider}} = u_i^{\text{out}} + \phi^{\text{rider}} g_{ij}, \tag{4}$$
$$u_j^{\text{driver}} = u_j^{\text{out}} + \phi^{\text{driver}} g_{ij}. \tag{5}$$

(2)Being unmatched. The number of unmatched enrolled drivers from class i is $p_i^{\text{rider}} n_i - \sum_{j \in J}(x_{ij}^1 + x_{ij}^3)$, such that the probability $P_i^{\text{rider}}(0)$ of an enrolled rider not being matched is

$$P_i^{\text{rider}}(0) = 1 - \sum_{j \in J} \frac{x_{ij}^{\text{II}} + x_{ij}^{\text{IF}}}{p_i^{\text{rider}} n_i}. \tag{6}$$

Similar to the outcome of choosing opt-out, unmatched inflexible riders would leave the system. However, we assume that the utility would be lower than that of choosing opt-out in the beginning to capture the impact of the risk of being unmatched on users' willingness to be enrolled as a rider. In this regard, we incorporate a fixed discomfort penalty δ of being unmatched such that the utility u_i^{rider} of an unmatched inflexible rider is

$$u_i^{\text{rider}} = u_i^{\text{out}} - \delta. \tag{7}$$

Given the utilities and probabilities of being matched with different drivers and being unmatched, the expected utility for an inflexible rider from class i is

$$\mathbb{E}u_i^{\text{rider}} = u_i^{\text{out}} - P_i^{\text{rider}}(0)\delta + \sum_{j \in J} P_i^{\text{rider}}(ij)\phi^{\text{rider}} g_{ij}. \tag{8}$$

Similarly, the expected utility for an inflexible driver from class j is

$$\mathbb{E}u_j^{\text{driver}} = u_j^{\text{out}} - P_j^{\text{driver}}(0)\delta + \sum_{i \in J} P_j^{\text{driver}}(ij)\phi^{\text{driver}} g_{ij}. \tag{9}$$

where $P_j^{\text{driver}}(ij)$ is the probability of an inflexible driver from class j being matched with a rider in class i, i.e.,

$$P_j^{\text{driver}}(ij) = \frac{x_{ij}^{\text{II}} + x_{ij}^{\text{FI}}}{p_j^{\text{driver}} n_j}. \tag{10}$$

The probabilities $P_i^{\text{flexible}}(ij)$ and $P_i^{\text{flexible}}(ji)$ of a flexible user from class i being matched with a driver and a rider in class j are respectively given by

$$P_i^{\text{flexible}}(ij) = \frac{x_{ij}^{\text{FI}} + x_{ij}^{\text{FF}}}{p_i^{\text{flexible}} n_i}. \tag{11}$$

$$P_i^{\text{flexible}}(ji) = \frac{x_{ji}^{\text{IF}} + x_{ji}^{\text{FF}}}{p_i^{\text{flexible}} n_i}. \tag{12}$$

The probability of a flexible user from class i not being matched is

$$P_i^{\text{flexible}}(0) = 1 - \sum_{j \in J} \frac{x_{ij}^{\text{FI}} + x_{ij}^{\text{FF}} + x_{ji}^{\text{FF}} + x_{ji}^{\text{IF}}}{p_i^{\text{flexible}} n_i}. \tag{13}$$

Then, the expected utility of flexible users is given by:

$$\mathbb{E}u_i^{\text{flexible}} = u_i^{\text{out}} + \sum_{j \in J} \left[\phi^{\text{rider}} g_{ij} P_i^{\text{flexible}}(ij) + \phi^{\text{driver}} g_{ji} P_i^{\text{flexible}}(ji)\right] - \delta P_i^{\text{flexible}}(0) \tag{14}$$

Given the expected utilities defined in Equations (1)- (14), the choices among the three modes are characterized in a multinomial logit manner. The probability of each user from any group $j \in J$ choosing mode $m \in M$ is given by

$$p_j^m = \frac{e^{\frac{\mathbb{E}u_j^m}{\lambda}}}{\sum_{m' \in M} e^{\frac{\mathbb{E}u_j^{m'}}{\lambda}}}, \forall j \in J, \forall m \in M. \tag{15}$$

where λ is a tuning parameter. We assume that users' mode preferences are independent of each other.

Decisions of the Operator

The operator matches potential drivers and riders in order to maximize the total welfare of users. We macroscopically model the operator's decisions as an LP. The operator knows a priori the possible utilities of users before matching. We model the problem using a continuous number of drivers and riders such that the problem can be modeled as an LP. We formulate the problem as follows:

$$\max_{\mathbf{x}} \quad U(\mathbf{x}) = \sum_{i \in J} \sum_{j \in J} \sum_{a \in \mathcal{A}} u_{ij} x_{ij}^a + \sum_{j \in J} u_j^{\text{opt-out}} \Big(n_j - \sum_{i \in J} \sum_{a \in \mathcal{A}} x_{ij}^a - \sum_{i \in J} \sum_{a \in \mathcal{A}} x_{ji}^a \Big) \tag{16}$$

$$\text{s.t.} \quad \sum_{j \in J} \sum_{a \in \mathcal{A}} x_{ij}^a \le (p_i^{\text{rider}} + p_i^{\text{flexible}}) n_i, \forall i \in J \tag{17}$$

$$\sum_{i \in J} \sum_{a \in \mathcal{A}} x_{ij}^a \le (p_j^{\text{driver}} + p_j^{\text{flexible}}) n_j, \forall j \in J \tag{18}$$

$$\sum_{i \in J} (x_{ij}^{\text{IF}} + x_{ij}^{\text{FF}} + x_{ji}^{\text{FF}} + x_{ji}^{\text{FI}}) \le p_j^{\text{flexible}} n_j, \forall j \in J \tag{19}$$

$$\sum_{i \in J} \sum_{a \in \mathcal{A}} (x_{ij}^a + x_{ji}^a) \le (p_j^{\text{rider}} + p_j^{\text{driver}} + p_j^{\text{flexible}}) n_j, \forall j \in J \tag{20}$$

$$x_{ij}^a \ge 0, \forall i, j \in J, a \in \mathcal{A} \tag{21}$$

The objective (16) is the utility of matched and unmatched users. Constraints (17) and (18) ensure that the number of potential riders and drivers are not exceeded, respectively. Constraints (19) ensures that every agent is used at most once.

Here, utilities depend on the travel time, scheduling displacement, and values of time for each travel mode. Let $dist(j)$ be the distance between the origin and destination of class j. Let $detour^{inter}(i,j)$ and $detour^{intra}(i,j)$ be the inter-class and intra-class detour (in time), respectively, a driver in class j makes to pick up a rider in class i. The former is the additional time needed for a driver to reach the pickup and dropoff region of the matched rider. The latter is the time needed for a driver inside a region to pick up the rider, which depends on the number of participating drivers and riders from each class and is defined as:

$$detour^{\text{intra}}(i,j) = \frac{Q}{(p_i^{\text{rider}} + p_i^{\text{flexible}})n_i(p_j^{\text{driver}} + p_i^{\text{flexible}})n_j} \tag{22}$$

where Q is a case-dependent tuning parameter, $\alpha^{\text{opt-out}}$ and α^{pool} are the values of time when traveling alone and sharing rides, s is the subsidy for ridesharing. To allow for scheduling preferences, t_j^* is the desired arrival time of class j. The unit costs of early and late arrival are denoted by β and γ, respectively, with $\beta < \gamma$. Solo drivers arrive exactly at their desired arrival time. Users sharing a rider determine the departure time that maximizes their joint utility. By de Palma et al. (2022), in our setting, two ridesharing users i and j maximize their joint utility if they arrive at $\min(t_i^*, t_j^*)$. In this case, one user arrives exactly on time while the other arrives early. Then the utilities are formulated as follows:

$$u_j^{\text{opt-out}} = -\alpha^{\text{opt-out}} dist(j) \tag{23}$$

$$u_{ij} = s - \alpha^{\text{pool}} dist(i) - \alpha^{\text{opt-out}}(dist(j) + detour^{\text{inter}}(i,j) - dist(i)) \tag{24}$$

$$- \beta[\max(t_i^*, t_j^*) - \min(t_i^*, t_j^*)] - \alpha^{\text{opt-out}} detour^{\text{intra}}(i,j) \tag{25}$$

Equilibrium conditions

Now we define the equilibrium of the interaction between the operator's decisions and the stochastic decision-making behavior of users. We define a *ridesharing strategy* for each class $j \in J$ as a quadruple $\mathbf{p}_j = (p_j^m)_{m \in M}$. The *ridesharing strategy* $\mathbf{p}_j$ is determined through the utilities of all options via the defined multinomial logit model. The matching of drivers and riders is determined via the defined LP problem. This is a fixed-point problem where the decisions are interrelated. According to Theorem 1, there always exists a solution to this problem. The proof is omitted here and relegated to the full version of the paper.

Theorem 1. *The multi-player problem defined by the operator and user decisions always admits a solution.*

Iterative algorithm

To evaluate the practical convergence of our methods to an equilibrium, we design an algorithm that iteratively obtains and updates the decisions of the operator and the individual users. The general structure of the algorithm is described in Algorithm 1. This iterative process is repeated until no changes to the optimal matching $\mathbf{x}$ and the mode choice quadruple $\mathbf{p}_j$ are observed.

Algorithm 1: Iterative algorithm

Input: A set of users J, with utility values $u_j^{\text{opt-out}}$ and joint utility functions u_{ij}
Initialize $\mathbf{p}_j$ as random variables on [0,1] such that $\sum_{m \in M} p_j^m = 1$ for all $j \in J$.
while *Optimal matching or mode choice changes* **do**
  Solve (16) - (21)
  Update the expected utility v_j^m for every $j \in J$ and $m \in M$
  Update every agent's mode choice decisions $\mathbf{p}_j$ according to the choice model
end
return Equilibrium matching decisions and mode choices

To foster convergence of the algorithm, we apply a moving average update rule for mode choice probabilities $\mathbf{p}_j$. Let $\mathbf{p}_j^{(k)}$ be the value for $\mathbf{p}_j$ in iteration k. The moving average is updated as follows:

$$\mathbf{p}_j^{(k)} = \rho \mathbf{p}_j^{(k-1)} + (1-\rho)\mathbf{p}_j \tag{26}$$

The parameter ρ is tuned based on the specific characteristics of the system.

3 Results

In this section, we evaluate the effect of mode choice, participation, and flexibility on the performance of a ridesharing system. We evaluate the convergence of the iterative algorithm, the effect of geographical properties of users on their mode choice, and the presence of economies of scale.

Case study

We evaluate the results of the developed problem, as well as our iterative algorithm, for a case study of the city of Chicago, USA. We use data provided by the City of Chicago (2010) to establish 77 nodes based on the communities in the city. We consider a fully connected graph, where the distances between nodes are obtained as Euclidian distances between the geographical centers of the communities. The origin-destination data of users is based on the use of ride-hailing vehicles, provided by City of Chicago (2022). The historical data has been used to construct demand rates of origin-destination pairs. In turn, this has been used to randomly generate instances of users. The distributions of origins and destination are displayed in Figure 2.

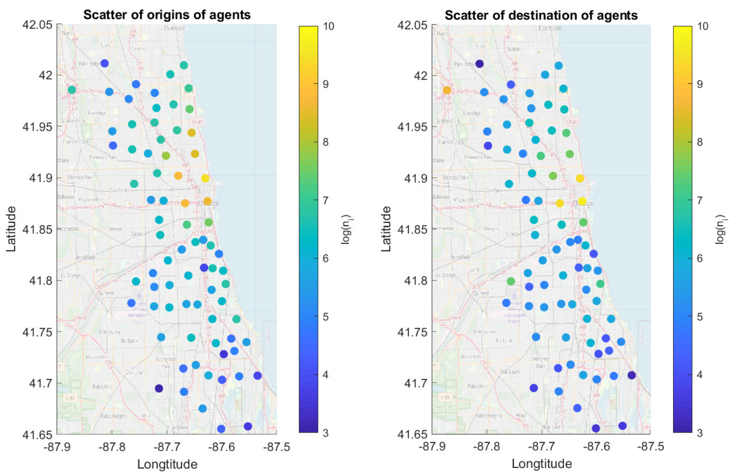

Figure 2: Distribution of origins and destinations in Chicago (log scale)

We assume car ownership among the users is equal to 75% and users are divided into three groups by desired arrival time (7:30, 8:00, and 8:30). The value of time for commuting individually ($\alpha^{\text{opt-out}}$) is set to 6.8 [$/h] and the value of time for ridesharing (α^{pool}) is set to 7.8 [$/h]. Earliness is penalized by β equal to 3.00 [$/h]. A base subsidy of 1.00$ is used per matched couple. The gained utility is split evenly across rider and driver, which means that $\phi^{\text{rider}} = \phi^{\text{driver}} = 0.5$. The default value of λ is set to 0.5, Q is set to 4.0, and ρ is set to 0.5. The iterative algorithm has an iteration limit of 10.

Convergence of iterative algorithm

We evaluate the convergence of the iterative algorithm considering the following values of $\lambda = [0.1, 0.55, 1.0]$ to evaluate the effect of this behavioral parameter on convergence. We compare the objective value $U(x)$ and the norm of the difference between the $\mathbf{p}$ vectors of the current and the previous iterations. The results are presented in Figure 3.

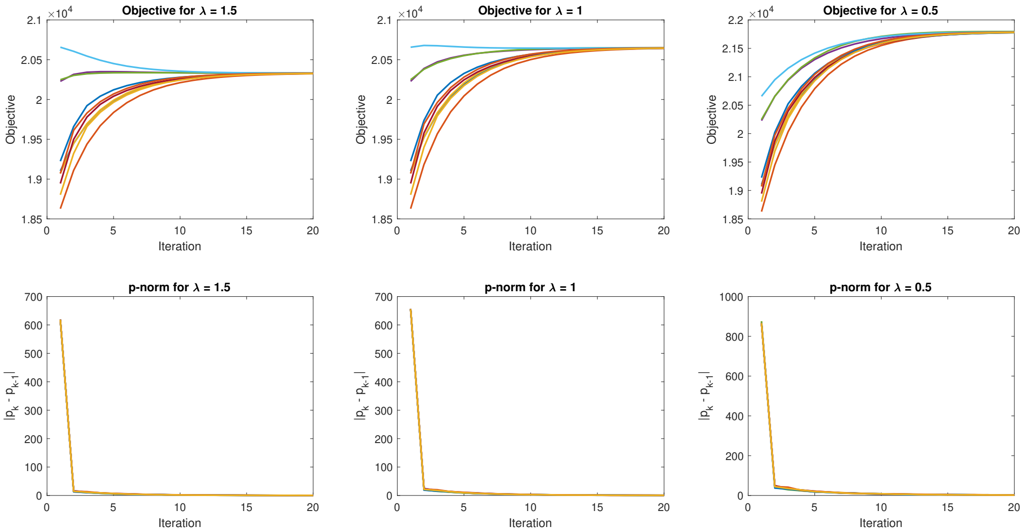

Figure 3: Convergence of algorithm

We observe that the algorithm converges to a stationary solution within a couple of iterations. The algorithm takes more iterations to converge when the value for λ becomes smaller. Smaller values of λ indicate that users are more sensitive to utility and more likely to choose the option with the highest utility. This can, therefore, amplify the differences between iterations, making convergence more difficult. This also implies that for smaller values of λ, larger values of ρ combined with higher iteration limits can be used to smoothen the progress and improve convergence.

Network analysis

We analyze the influence of geographical features on the mode choice and match of a user by looking at node-specific results. Figure 4 displays the proportion of drivers and riders for users with an origin (left) and destination (right) at each node. Figure 5 displays the percentage of users that are succesfully matched. The results show that users with an origin and/or destination closer to the city center are more likely to be riders than drivers. Conversely, users with an origin and/or destination further from the city center are more likely to be drivers than riders. The reason for this is that in the system, a driver picks up the rider on their way. Therefore, it is usually favorable for the agent with the longest itinerary to perform the pickup.

Similarly, users with an origin and/or destination closer to the city center are more likely to be matched, whereas those further from the city center are less likely to be matched. This suggests that subsidies need to be targeted to those individuals with origins and/or destinations further away from the city center.

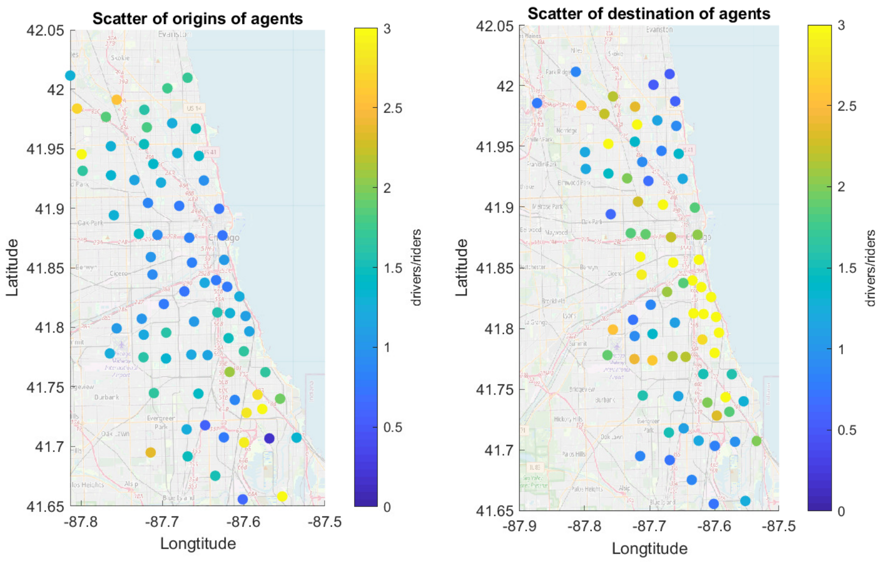

Figure 4: Proportion of drivers over riders in every region

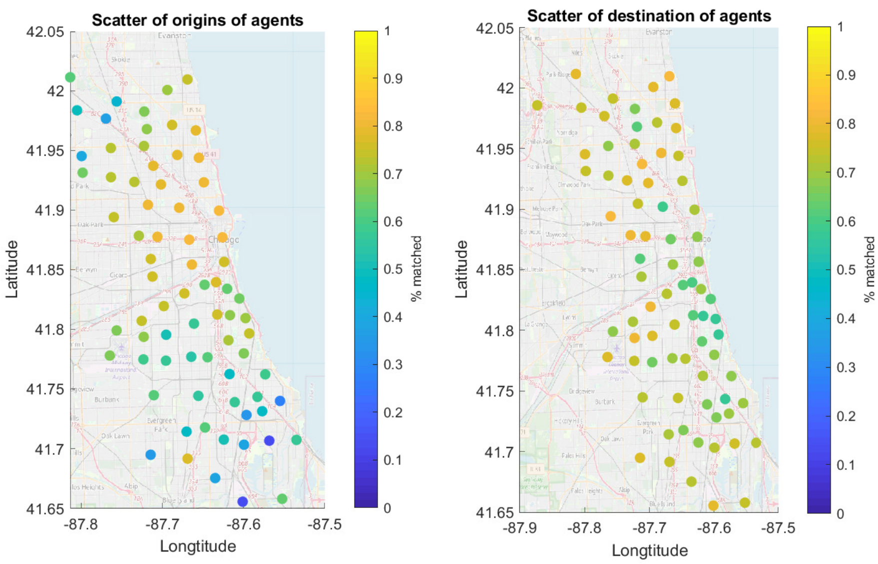

Figure 5: Proportion of matched users in every region

Effect of participation levels

To study the effect of participation on the observed results, we study various demand levels. We let d be the fraction of the total number of considered users that are interested in participating in the ridesharing scheme. The previous sections considered a base level of $d = 0.3$. We plot the mode choice probabilities in Figure 6. The corresponding operator-assigned modes are displayed in Figure 7.

The results indicate that a higher demand level decreases the probability of users opting out or being flexible participants while it increases their probability of being a driver (especially downtown $\rightarrow$ suburb) and a rider (especially downtown $\rightarrow$ downtown). The number of matches increases substantially, especially for users going from downtown to the suburb or from the suburb to downtown. For the others, the effect is less substantial. In the absence of flexible users (i.e., when p_j^{flexible} is forced to 0), the number of unmatched users is between 5% and 15% higher than when users are allowed to be flexible.

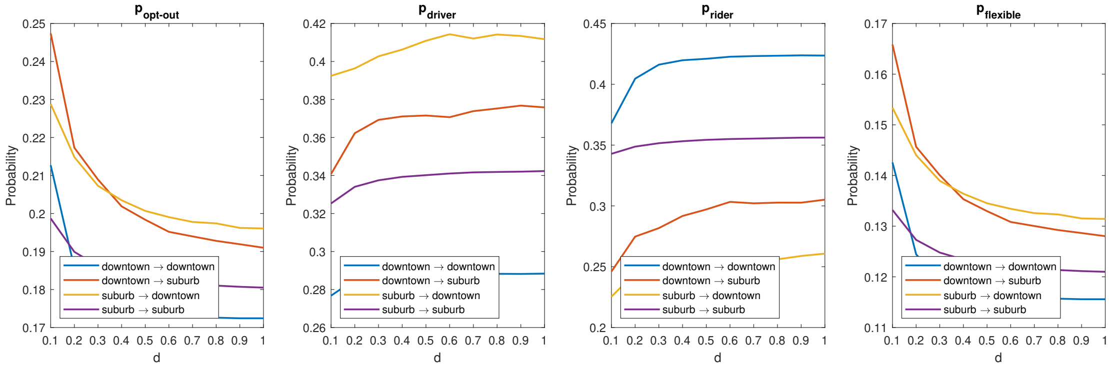

Figure 6: Probabilities of mode choice for different demand values.

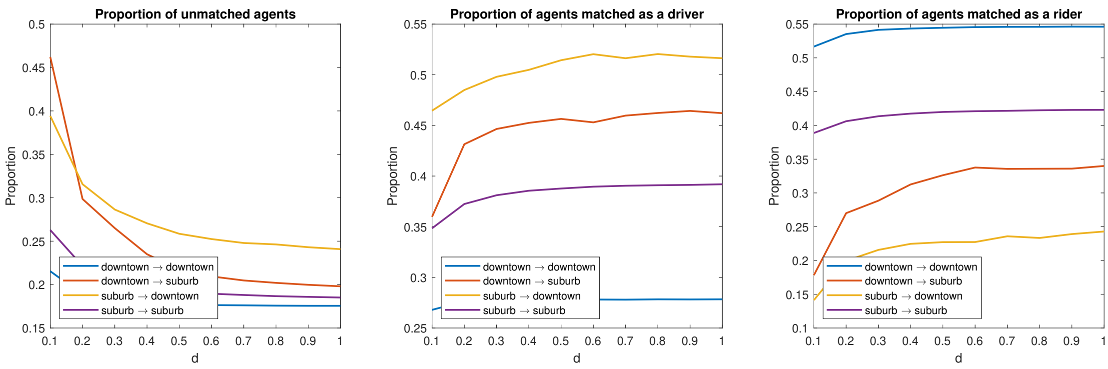

Figure 7: Matches for different demand values.

4 Conclusion

In this paper, we studied the participation of drivers and riders in a ridesharing system with flexible mode choices. We formulated the problem as a repeated multi-player non-cooperative game where the decisions of the operator were modeled as an LP, and the decisions of the users were modeled through a random utility model. The central operator decides on the optimal matching of drivers and riders, and users determine whether to participate in the ridesharing system and in what mode they wish to participate. To find the equilibrium solution to this multi-player problem, we developed an iterative approach that updates the expectations and decisions of users based on previous observations.

Our theoretical results show that an equilibrium solution to this problem exists but is not necessarily unique. Our numerical results are evaluated on a case study of the city of Chicago. Our iterative algorithm tends to converge to an equilibrium solution within ten iterations. We observe that users with an origin and destination in the downtown area have a lower probability of being unmatched and are more likely to be matched as a rider than as a driver. We have also shown that the considered ridesharing system benefits heavily from economies of scale. As the participation of users in the system increases, the percentage of matched users increases, and the user equilibrium solution gets closer to the system-optimal solution. The results also suggest that allowing users to be flexible in their mode choice improves the performance of the ridesharing system and the utility of the users.

The results obtained in this paper can be useful in designing subsidy schemes that improve the performance of the ridesharing system and can be used to identify a critical mass for participation in the ridesharing service. These areas are marked as important directions for future research.

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