Beyond links: The power of path incentives in alleviating congestion and emissions in urban networks

Ramin Niroumand*[1], Shaghayegh Vosough[1], Marco Rinaldi[2], Richard Connors[3], and Claudio Roncoli[4]

[1]Postdoctoral Researcher, Department of Built Environment, Aalto University, Finland
[2]Assistant Professor, Department of Transport and Planning, Technical University of Delft, the Netherlands
[3]Research Scientist, Faculty of Science, Technology and Communication, University of Luxembourg, Luxembourg
[4]Associate Professor, Department of Built Environment, Aalto University, Finland

Short summary

This study investigates the potential of link- and path-based incentives to mitigate congestion and reduce emissions in urban transportation networks. Both incentive schemes are formulated as non-linear optimisation problems with complementarity constraints. Mathematically, it is demonstrated that the feasible region of the link-based model is a subset of the feasible region of the path-based model. Consequently, path-based incentives exhibit a higher potential in pushing the user equilibrium flow pattern toward system optimum, compared to link incentives. A column generation-based iterative solution technique, which generates new paths at each iteration, is devised to efficiently solve both optimisation problems. The numerical results in the Sioux Falls network also highlight the superiority of path-based incentives in reducing total travel time and emissions in urban transportation networks.
Keywords: Incentive scheme, System optimum, Traffic assignment, Traffic management.

1 Introduction

Motivation

Providing drivers with sensible route advice is considered a successful traffic management tool, with the potential to reduce congestion (Kaysi, 1993; Fu, 2001; Cheng et al., 2020; Menelaou et al., 2021) and, thereby, improve network efficiency and sustainability (Sunio & Schmöcker, 2017; Andersson et al., 2018), although it may increase individual travel cost (distance and/or time) for some users (van Essen et al., 2016). This implies that some drivers may need to follow routes longer than desirable for the benefit of the community. This situation in which the total social benefit reaches the highest level is called system optimum (SO) and is in contrast to user equilibrium (UE), which aims at achieving the highest individual benefits (Mahmassani & Peeta, 1993). Studies estimated a wide range (5% - 25%) of benefits, in terms of reduced total travel time (TTT), in typical road networks when SO traffic flow is achieved (Peeta & Mahmassani, 1995; Wie et al., 1995; Roughgarden & Tardos, 2002; Boyce & Xiong, 2004). Traditionally, an SO condition prioritises the minimisation of TTT; however, with the recent growth in population and urbanisation, air pollution has emerged as a significant challenge in cities that needs to be addressed.
It should be noted that SO is an ideal situation where a central authority is supposed to dictate routes for all users, causing some users to divert from their (individual) optimal routes, leading to increased (individual) travel times. Thus, a stimulus is needed to encourage such changes in drivers' behaviour. Road pricing (Bergendorff et al., 1997; Yang & Huang, 2004; Zangui et al., 2015; Ren et al., 2020) has been used to push the UE flow pattern toward SO. However, incentivising schemes for voluntary participation (Ettema et al., 2010; Leblanc & Walker, 2013; Sun et al., 2020; Cohen-Blankshtain et al., 2022) have recently gained more popularity due to public dissatisfaction (May et al., 2010) and inequitable welfare distribution across the population (Levinson, 2010; Vosough et al., 2022) resulted from tolling. Due to limited resources, an efficient allocation of incentives within a limited budget is crucial. Yet, optimally assigning incentives to

achieve the highest network efficiency in a complex real traffic network can be challenging due to the optimisation problem being computationally intensive.

Background

Achieving an SO traffic flow, Van Essen et al. (2019) showed that the drivers who comply with the routing advice should take routes slightly longer than the shortest path. Still, a strong stimulus, e.g., an incentive is required to push drivers to take a route that might be significantly worse than their preferred (e.g., faster) route. Vosough & Roncoli (2024), Djavadian et al. (2014), and Klein & Ben-Elia (2018) showed that drivers accept longer routes under incentive strategy, compared to other stimuli for contributing to a more liveable, safer, and less polluted city, while Kröller et al. (2021) showed incentives' positive impact using real-world data. These findings imply that employing incentives can play a vital role in the success of a routing advice system aiming at steering flow toward SO.
One of the first studies investigating the application of link-based incentives to achieve SO has been conducted in two small toy networks with 2 and 4 links (K. Cheng & Jiang, 1994). The study showed that the difference in total travel times between UE and SO was flattened when the demand increased beyond a certain threshold, indicating that the application of incentives may not have economic significance beyond that threshold. This happens because when the entire network gets congested, redistributing the traffic only puts an extra burden on different parts of the network.
Considering path-based incentives, Bie & van Arem (2009) investigated the impact of applying them to designated safe routes on traffic network performance. A logit model was employed to assign traffic to routes based on their generalised costs consisting of travel time, fuel cost, and safety measures minus incentive. Their numerical results indicated that depending on the incentive program setup, the incentive scheme can be beneficial or not. In another study, Ghafelebashi et al. (2023) proposed a path-based personalised incentive chosen from a predetermined set to minimise TTT under various budget limits and user participation levels of the incentive scheme. They showed that the value of saved time was usually larger than the cost of offering incentives, however, for large budget limits the value of saved time might be smaller than the amount spent on incentives. Recently, Luan et al. (2023) conducted a comparison between link- and path-based incentives to analyse their potential to reduce TTT. They formulated single-level optimisation problems to compare the two types of incentives under budget limits and various participation levels of drivers. Their numerical examples in two transportation networks showed that in most cases path-based incentives outperformed link-based incentives, while for a low participation level of drivers, the link incentive reduced TTT more than path incentives. We adopt a similar specification of link and path incentive optimisation problems to compare the performance of these two types of incentives. Nevertheless, our research differs in numerous aspects. First, our proposed solution algorithm computes the shortest paths in each iteration, generating at least 10 paths for each origin-destination (OD) pair, while Luan et al. (2023) enumerated only 3 paths for each OD pair a-priori, resulting in the flow-independent shortest path. Second, we introduce a column generation approach that solves the optimisation problem at each iteration of the algorithm using a solver, while Luan et al. (2023) utilised a customised branch-and-bound algorithm to solve the optimisation problem once. Third, even though the shortest path problem is solved at least 10 times[1] in this research compared to only once in Luan et al. (2023) and a more complex network is employed, our proposed approach significantly outperforms it in terms of computation time. Fourth, rather than only targeting TTT, the objective function in our research accounts for other social goals, including emissions reduction. Finally, we use theory to prove that link incentives cannot outperform path incentives and in their best performance, link-based incentives work as equal as path-based incentives.

Research Contributions

Despite the rich body of literature on incentivising drivers, a key research gap concerns the rationale for selecting either link- or path-based incentives to manage urban traffic. With the emergence of technologies that enable us to track travellers through their journeys and the widespread usage of navigation apps, path-based pricing/incentive has become technically feasible. To the best of the authors' knowledge, little attention has been devoted to assessing the efficiency of link- and path-based incentives and no study has yet shed light on the potential superiority of one incentive type over the other. In this work, we bridge this fundamental gap as follows:

[1] This is to ensure solution stability, as shown in Table 2 of Section 4.

1. We introduce two distinct optimisation problems aimed at maximising the efficiency of the network, i.e., minimising TTT and total emissions, under both link- and path-based incentive schemes within the constraints of a limited budget;
2. We propose an innovative solution algorithm capable of solving both link- and path-based incentive optimisation problems accurately and efficiently in medium-sized transportation networks;
3. We conduct a thorough comparison between link- and path-based incentives, offering valuable insights into their respective performances while concerning various social goals such as minimising TTT and total emissions.

Together, these contributions advance our understanding of incentive-based approaches in traffic management and pave the way for improved urban transportation strategies.

2 Problem Definition and Formulation

In this section, we formulate the path- and link-based problems as two single-level optimisation problems called P1 and P2, respectively, to determine the optimal incentive schemes under budget limitations.

Path-based and Link-based Incentive Optimisation Problems

We represent a transportation network by a graph (V, A), where V is the set of nodes and $A \quad V \quad V$ is the set of links. Let W be the set of OD pairs, and let the travel demand be described by the fixed number of vehicles travelling between w 2 W. Table 1 defines all the parameters and variables used in the formulated optimisation problems.
The single-level optimisation problem for path-based incentives called **P1**, with budget limit, B is formulated as follows:

$$Z1 = \min_{\mathbf{f},\hat{\boldsymbol{y}}} \Sigma_{a2A}(x_a t_a) \tag{1}$$

s.t.

$$\Sigma_{p2P_w} f_w^p = q_w \qquad 8w\ 2\ W \tag{2}$$

$$\Sigma_{a2A}\delta_a^p t_a \quad \hat{y}^p \quad u_w \quad 0 \qquad 8p\ 2\ P_w, \quad w\ 2\ W \tag{3}$$

$$(\Sigma_{a2A}\delta_a^p t_a \quad \hat{y}^p \quad u_w) f_w^p = 0 \qquad 8p\ 2\ P_w, \quad w\ 2\ W \tag{4}$$

$$\Sigma_{w2W}\Sigma_{p2P_w} f_w^p \hat{y}^p \quad B \tag{5}$$

$$x_a = \Sigma_{w2W}\Sigma_{p2P_w}\delta_a^p f_w^p \qquad 8a\ 2\ A \tag{6}$$

$$\boldsymbol{f}, \hat{\boldsymbol{y}}, \boldsymbol{u} \quad 0 \tag{7}$$

The objective function, $Z1$, minimises the network total travel time wrt. path flows and incentives, $\mathbf{f}$ and $\hat{\boldsymbol{y}}$[2]. Constraint 2 guarantees conservation of vehicles[3], Constraints 3 and 4 are the complementarity constraints ensuring Wardrop's first principle with generalised travel times. Constraint 5 imposes the budget limitation, Constraint 6 maps path flows to link flows, and Constraint 7 ensures non-negativity. Function $Z1$, accompanied by Constraints 2, 6, and 7, represents the SO problem in a transportation network under adequate regularity assumptions.
We can similarly formulate the budget-constrained link-based incentive problem **P2** with budget B as follows:

$$Z2 = \min_{\mathbf{f},\boldsymbol{y}} \Sigma_{a2A}(x_a t_a) \tag{8}$$

s.t.

$$\Sigma_{p2P_w} f_w^p = q_w \qquad 8w\ 2\ W \tag{9}$$

[2] It should be noted that for targeting emissions, $Z1$, must be replaced by total emissions produced in the network.

[3] We acknowledge the potential risk of induced car demand associated with incentive schemes. In our proposed method, we do not o er high incentives that could generate revenue for drivers, i.e., negative travel costs. This restraint is guaranteed by Constriant 3. By refraining from assigning high incentives, we can assume that the attraction of travellers from other modes to car trips is prevented, leading to inelastic demand, q_w.

Table 1: Notation for variables and parameters

Symbol	Definition
A	Set of links
V	Set of nodes
W	Set of all OD pairs
q_w	Travel demand between OD pair w 2 W
P_w	Set of all paths between OD pair w 2 W
t_a	Travel time on link a 2 A
x_a	Vehicle flows on link a 2 A
f_w^p	Vehicle flows on path p 2 P_w between pair w 2 W
y_a	Incentive on link a 2 A
B	Total budget available
$\hat{y}^p$	Incentive on path p 2 P_w between pair w 2 W
u_w	Minimum travel time between OD pair w 2 W
δ_a^p	Link-path incidence matrix

$$\Sigma_{a2A}\delta_a^p(t_a \quad y_a) \quad u_w \quad 0 \qquad 8p\ 2\ P_w,\ \ w\ 2\ W \tag{10}$$

$$(\Sigma_{a2A}\delta_a^p(t_a \quad y_a) \quad u_w)f_w^p = 0 \qquad 8p\ 2\ P_w,\ \ w\ 2\ W \tag{11}$$

$$\Sigma_{a2A}x_a y_a \quad B \tag{12}$$

$$x_a = \Sigma_{w2W}\Sigma_{p2P_w}\delta_a^p f_w^p \qquad 8a\ 2\ A \tag{13}$$

$$\boldsymbol{f}, \boldsymbol{y}, \boldsymbol{u} \quad 0 \tag{14}$$

Similar to problem **P1**, the objective function, $Z2$, minimises the total travel time in the whole network with the path flow, **f**, under link-based incentive, y_a, with Constraint 9- 14 follow the same structure as those of P1.

Differences between Path-based and Link-based Incentive Problems

Theorem: Total travel time obtained by optimally solving **P1** is never higher than the total travel time obtained from optimally solving **P2** under the same budget limit of B.
Proof: Assume that the pair ($\boldsymbol{x}$, $\boldsymbol{y}$) satisfies 9 - 14, i.e., is a feasible pair wrt **P2**. We can show that ($\boldsymbol{x}$, $\boldsymbol{y}$) also satisfies 2 - 7, i.e., that the feasible solution set of **P1** encompasses the feasible solution set of **P2**. Since the two optimisation problems have identical objective functions, **P1** always results in flow patterns with total travel times at most as low as those of **P2**.
Assume incentive of y_a is assigned to link a. All paths (and users) that traverse link a will receive this incentive. Therefore, travelers on path p will receive a link-additive path incentive as $\hat{y}^p = \Sigma_{a2A}\delta_a^p y_a$. We can rewrite Constraint 10 as follows:
$\Sigma_{a2A}\delta_a^p(t_a \quad y_a) \quad u_w = \Sigma_{a2A}\delta_a^p t_a \quad \Sigma_{a2A}\delta_a^p y_a \quad u_w = \Sigma_{a2A}\delta_a^p t_a \quad \hat{y}^p \quad u_w,$ (I)
which results in Constraint 3.
With similar substitutions, we can show that Constraint 11 can be rearranged to result in Constraint 4. Now, we can rewrite Constraint 12 by substituting x_a with its definition, i.e., $x_a = \Sigma_{w2W}\Sigma_{p2P_w}\delta_a^p f_w^p$, as follows:
$\Sigma_{a2A}x_a y_a = \Sigma_{a2A}y_a\Sigma_{w2W}\Sigma_{p2P_w}\delta_a^p f_w^p = \Sigma_{w2W}\Sigma_{p2P_w}\Sigma_{a2A}y_a\delta_a^p f_w^p = \Sigma_{w2W}\Sigma_{p2P_w}\hat{y}^p f_w^p,$ (II)
which is equal to the path incentive budget constraint, i.e., Constraint 5.
Combining the two statements I and II shows that for any pair of ($\boldsymbol{x}$, $\boldsymbol{y}$) that satisfies Constraints 9 - 14, there is a pair ($\boldsymbol{x}$, $\boldsymbol{y}$ =< $\Sigma_{a2A}\delta_a^p y_a$ >) that satisfies constraints 2 - 7. Therefore, the feasible region of the optimisation problem **P1** encompasses the feasible region of the optimisation problem **P2**. Please note that since the incentives collected by drivers do not change, the link flows remain identical for the two pairs.□

3 Solution Algorithm

The incentivised UE problem with a budget limit presented in problems **P1** and **P2** can be solved by employing a centralised approach in simple networks by enumerating all paths where the number of paths is limited. In the case of a large-scale transportation network, the centralised approach is not

useful since it is computationally expensive to enumerate all the paths of the network. Therefore, a column generation approach is developed that is able to generate new paths as needed as the algorithm proceeds. Note that the column generation-based approach in principle can lead to the optimal solution if it iterates long enough to enumerate all the paths of the network. However, it can be stopped when the improvement in two consecutive iterations falls below a certain threshold resulting in a balance between computation time and solution quality.
To solve **P1** and **P2** in a complex network, we propose the following column generation method that considers each path as a column, adds new columns at each iteration, and stops when the minimum iteration number, N, is reached and the relative difference of total travel times in two consecutive iterations falls below a predefined value, ϵ. The general steps of the proposed column generation algorithm are as follows:

1. **Initialisation**:
    1.1. Define values for N and ϵ.
    1.2. For each OD pair $w \in W$, set $P_w = \emptyset$.
    1.3. Set $n = 1$, $\mathbf{y} = 0$, $\mathbf{f} = 0$, $\mathbf{x}^0 = \mathbf{x}(\mathbf{f})$, and $\mathbf{t}^0 = \mathbf{t}(\mathbf{x}^0)$.
2. **Shortest path**: for each OD pair $w \in W$,
    2.1. Find the shortest path p such that $p \notin P_w$, and
    2.2. set $\bar{P}_w = \bar{P}_w \cup p$.
3. **User equilibrium**: solve optimisation problem **P1** or **P2**, and find traffic flows, $\mathbf{f}$, and corresponding incentive values, $\mathbf{y}$.
4. **Updating**: set $\mathbf{x}^n = \mathbf{x}(\mathbf{f})$ and $\mathbf{t}^n = \mathbf{t}(\mathbf{x}^n)$.
5. **Stopping criteria**:
    5.1. Calculate $\bar{\epsilon} = \frac{|\Sigma_{a \in A}(x_a^n t_a^n) - \Sigma_{a \in A}(x_a^{n-1} t_a^{n-1})|}{\Sigma_{a \in A}(x_a^n t_a^n)}$.
    5.2. If $\bar{\epsilon} \leq \epsilon$ and $n \geq N$ stop, otherwise set $n = n + 1$ and go to step 2.

Note that we generate a new path that does not belong to the current active path set at each iteration of the column generation process to prevent getting trapped around a local optimum. However, such a path cannot be found by solving a normal shortest path problem. We, therefore, employ a shortest path algorithm that solves a mixed integer linear problem to minimize the total link travel times while imposing a high penalty for choosing a path between OD pair $w \in W$ that already exists in the current active path set. Due to a lack of space, we refrain from presenting the algorithm.

4 Numerical Experiments

The well-known Sioux Falls (SF) network, shown in Figure 1, is employed to show the ability of the proposed algorithm to solve complex networks, the differences in performance of link- and path-based incentives in a realistic traffic network, and the application of incentive schemes under different social goals including TTT and total emissions. The link volume-delay function in this network follows BPR function (HCM, 2000), $t_a(x_a) = t_a^0(1 + 0.15(\frac{x_a}{C_a})^4)$, where t_a^0 is the free flow travel time in link a and c_a its capacity in vehicles per time unit. The details of the links and OD matrix are sourced from He et al. (2014). This network consists of 24 nodes, 76 links, and 552 OD pairs. We employ the proposed column generation method to solve the two proposed optimisation problems for this network. It is worth noting that incentives and budget are considered in the same unit as travel time, which is in minutes.

Link and path incentives in SF network

As can be seen in Figure 2, for all budget considerations, path incentives consistently outperformed their link-based counterparts. This finding highlights the superiority of the path-based approach in achieving more favorable outcomes.
In order to compare the benefits associated with each incentive scheme, we employ a cost-benefit analysis that can reveal the benefits we gain, i.e., the difference between TTT in a specific case and a benchmark TTT, from the cost that we pay, i.e., the budget allocated to an incentive scheme

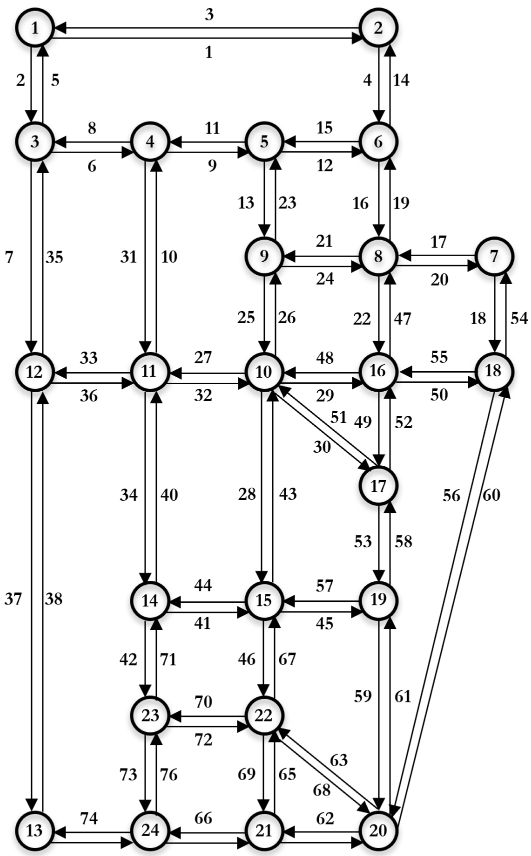


Figure 1: Sioux Falls network

compared to a benchmark situation, which we define as a B/C index. Figure 3 represents the B/C index in the SF network under both link- and path-based incentives where the benefit and cost of each incentive scheme are compared to the previous one with a lower budget, i.e., incremental B/C values. According to this figure, when the budget is small, the benefit of investing in the incentive scheme outweighs its associated extra cost as the B/C values are higher than 1. For budgets over 18,000 in path incentives and 10,000 in link incentives, the benefits gained from a specific incentive scheme, relative to the previous budget limit, are insufficient to justify the costs incurred. This case study indicates that the marginal benefits associated with increasing the incentive budget are getting gradually smaller (until negative).

The convergence behavior of the column generation algorithm

Looking into the convergence of the employed column generation algorithm, we can show the efficiency of the proposed solution method is acceptable. As can be seen in Table 2, under a wide range of budget limits, for both link and path incentives, the algorithm converges after a few iterations with a maximum computation time of 7800 seconds. Still, the proposed algorithm works better for the link incentive problem compared to the path incentive, especially under a small budget limit that converges via 3 steps. However, we enforce the algorithm to iterate 10 times to ensure that the algorithm is not trapped around a local optimum and its solution is stable and consistent across multiple iterations.

Minimising total emissions

In this section, the objective function of problem **P1**, $Z1$, is turned to minimise emissions using path incentives in the SF network. We assumed emissions are proxied by CO2, and the CO2 rate per vehicle kilometer traveled (veh-km) in link a is computed by Equation 15 introduced by Dimitriou et al. (2009).

$$E_a = 72.73 + 33.98 \quad 10^2/s_a + 23.26 \quad 10 \quad {}^3 s_a^2, \tag{15}$$

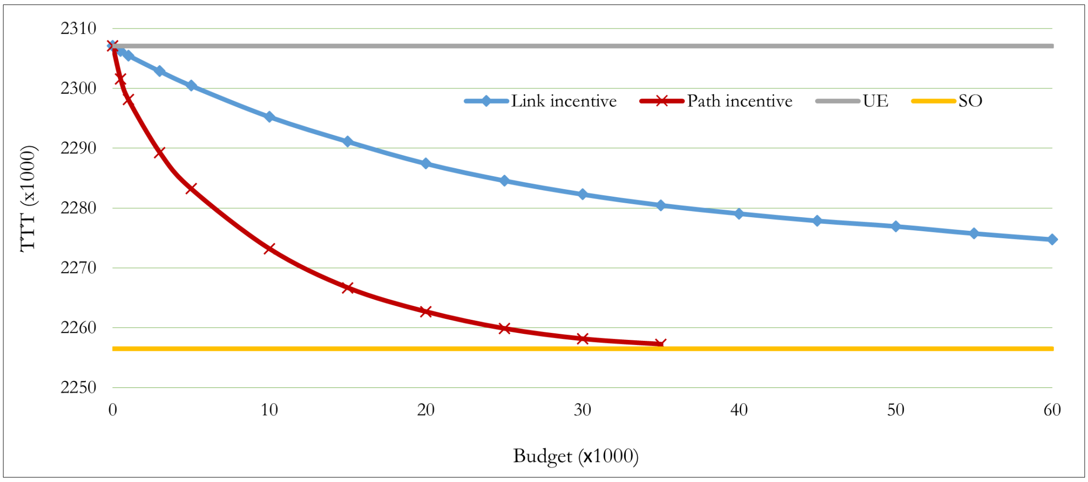


Figure 2: Total travel time of link- and path-based incentive schemes with various budget limits in Sioux Falls network

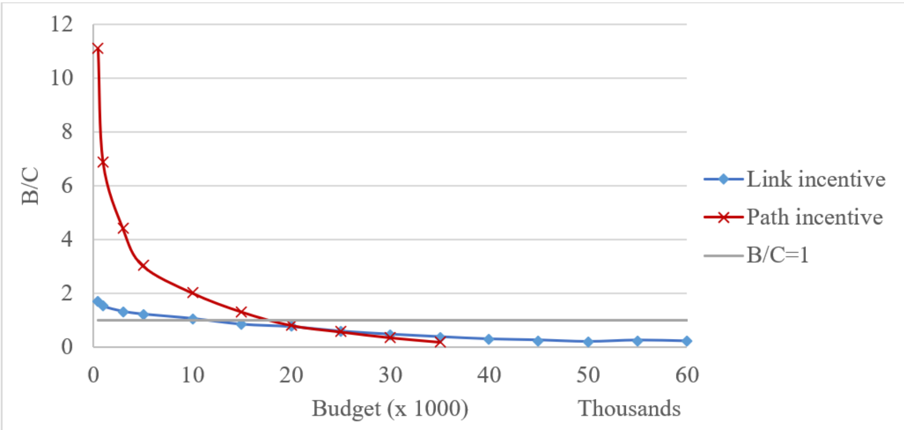


Figure 3: Cost-benefit analyses for the link- and path-based incentive schemes in Sioux Falls network

where s_a is the average speed of link a in km/hr and E_a is in g/veh-km. This function used in macroscopic problems, has a U-shaped dependence on vehicle speed, as shown in Figure 4 with respective minima at 42 km/hr. (Kickhöfer & Nagel, 2016).
Using Equation 15, we also computed the total CO2 in the SF network when the objective is to minimise TTT, as shown in Figure 5(a). Then, Figure 5(b) illustrates the total CO2 and TTT in the SF network when the objective function in problem **P1** is $Z1 = \min_{\mathfrak{f}} \Sigma_{a2A}(x_a E_a l_a)$, where l_a is the length of link a.
It can be seen that targeting only TTT leads to an increase in total CO2 emitted in the network. On the contrary, targeting only emissions brings about increased TTT. This is due to the fact that the minimum CO2 occurs when the average speed on a link is 42 km/h while for minimising TTT, the higher the average speed the better. Figure 6 illustrates how an incentive scheme changes the average speed to direct the traffic flow from UE toward SO by redistributing the traffic subject to the budget limit. According to this figure, targeting TTT minimisation smooths vehicle statistics and increases the average speed in most of the links but not necessarily in the direction of reducing emissions. For instance, the average speeds on links between nodes 11 and 14, and 21 and 24 have increased while the emissions also fall in a higher range according to Figure 4. However, in the case of targeting CO2 minimisation, the changes in the average speeds occurred only to reduce the CO2 emissions, such as decreased average speed of links between nodes 11 and 12.

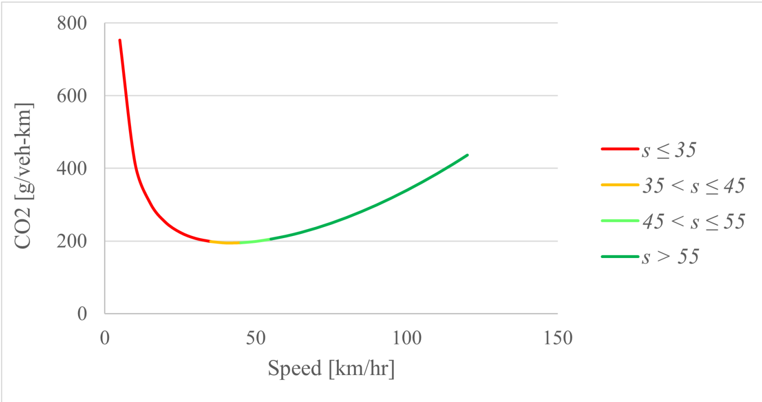
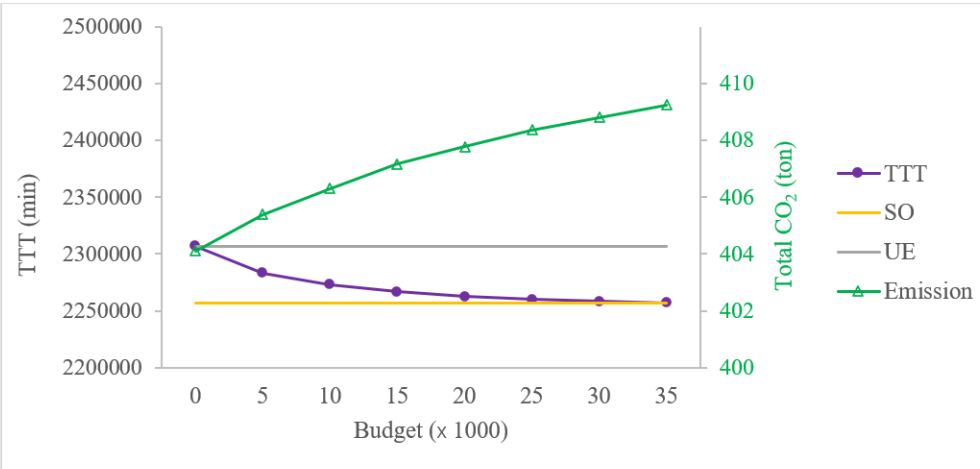
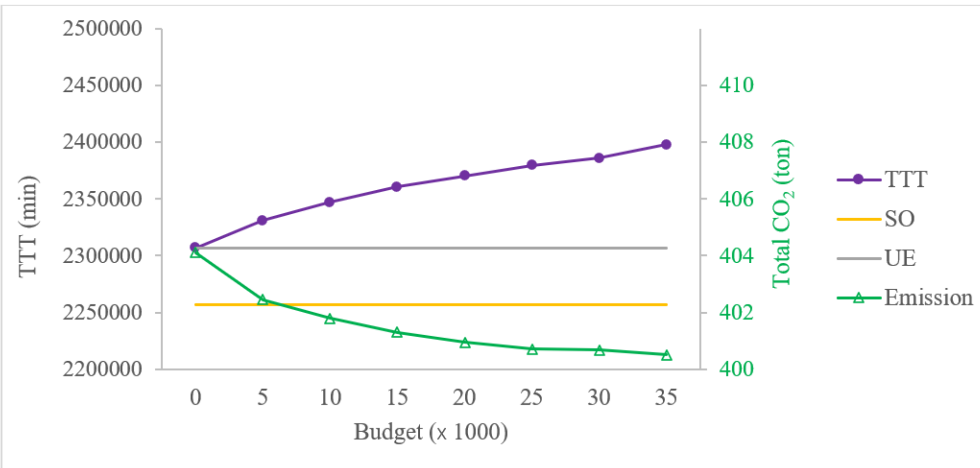


Figure 4: Emissions-speed curves for CO2, where s is the average travel speed in km/hr



(a) The objective is to minimize TTT

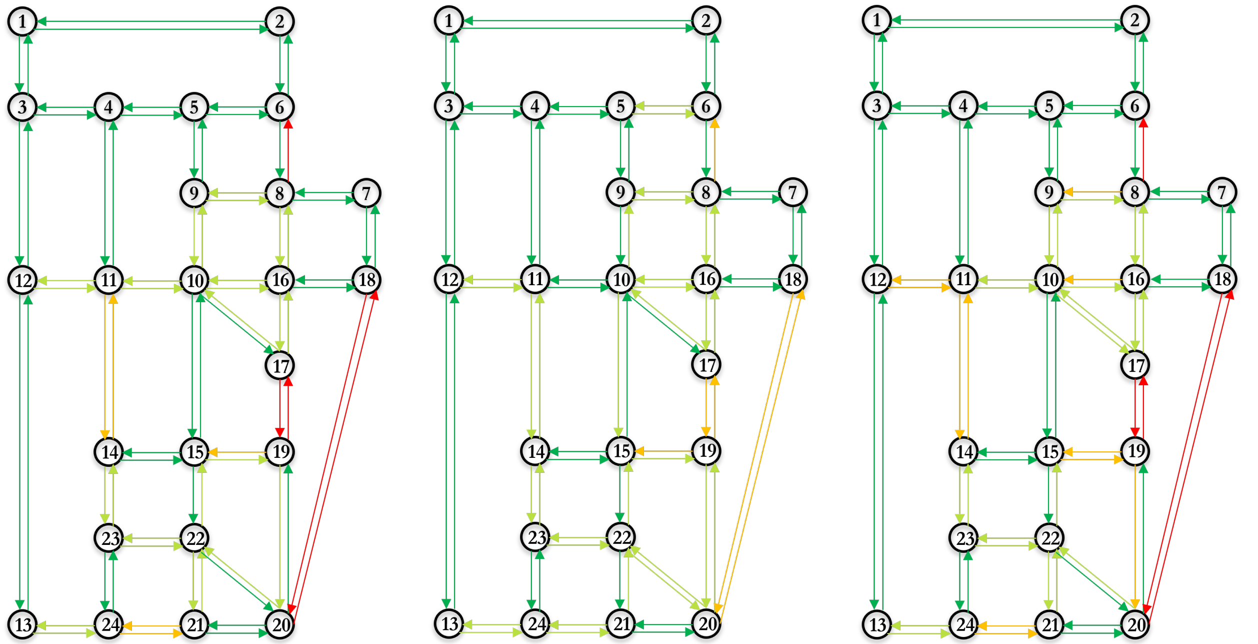


(b) The objective is to minimize CO2

Figure 5: Total emissions and travel time in the SF network under path-based incentives with different objective functions

Table 2: The error term, $\bar{\epsilon}$, computed in each iteration of the proposed column generation algorithm

Budget (x 1000)	5		30	
Iteration	Link incentive	Path incentive	Link incentive	Path incentive
1	1	1	1	1
2	0.742	0.756	0.762	0.783
3	0.006	0.005	0.008	0.006
4	0	1.12E-14	3.32E-04	1.60E-04
5	0	0	0	3.57E-14
6	0	0	0	6.00E-05
7	0	0	0	6.69E-15
8	0	0	0	7.52E-15
9	0	0	0	0
10	0	0	0	6.27E-16



(a) UE condition when the budget is zero

(b) Path-based incentive scheme with a budget of 30,000 to reduce TTT

(c) Path-based incentive scheme with a budget of 30,000 to reduce emissions

Figure 6: The changes in link speeds before and after implementing an incentive scheme, where dark green arrows represent $s > 50$, light green is $45 < s$ 50, orange illustrates $35 < s$ 45, and red arrows are $s < 35$ km/hr.

5 Conclusions

In this paper, path- and link-based incentives are used to push the user equilibrium flow pattern toward the system optimum in order to minimise total travel time and emissions in the network. Wardrop's first principle is applied to the generalised travel time, which is defined as travel time minus link or path incentives. Both incentivising schemes are formulated as non-linear optimisation problems with complementarity constraints and the objective of minimising total travel time or total emission. The mathematical properties of the two models reveal that the feasible region of the path-based optimisation problem encompasses that of the link-based problem. Since generating all paths for a real-sized network is computationally expensive, an iterative column-generation-based solution technique is proposed that generates a new path between each origin-destination pair at each iteration. The results of the two optimisation problems for the Sioux Falls network demonstrate their effectiveness in reducing total travel time and emission across the network under different budget limits. Notably, the reduction rate under the path-based incentive is higher than that under its counterpart link-based scheme.

This study assumes that all drivers will accept the provided incentivised routes. Additionally, it simplifies the optimisation problems by overlooking the elastic nature of travel demand and the impacts of new technologies, such as connected and automated vehicles. Relaxing these assumptions in future studies would yield more realistic results.

Acknowledgements

This research is partially funded by the FinEst Twins Center of Excellence (H2020 Grant 856602), the Academy of Finland projects AiforLEssAuto (no. 347200) and ALCOSTO (no. 349327), and EU-funded research project ACUMEN (no. 101103808).

References

Andersson, A., Hiselius, L. W., & Adell, E. (2018). Promoting sustainable travel behaviour through the use of smartphone applications: A review and development of a conceptual model. *Travel behaviour and society*, *11*, 52–61.

Bergendorff, P., Hearn, D. W., & Ramana, M. V. (1997). Congestion toll pricing of traffic networks. In *Network optimization* (pp. 51–71). Springer.

Bie, J., & van Arem, B. (2009). A route-based incentive structure for traffic safety enhancement. In *Proc. 16th its world congress.*

Boyce, D., & Xiong, Q. (2004). User-optimal and system-optimal route choices for a large road network. *Review of Network Economics*, *3*(4).

Cheng, Pang, M.-S., & Pavlou, P. A. (2020). Mitigating traffic congestion: The role of intelligent transportation systems. *Information Systems Research*, *31*(3), 653–674.

Cheng, K., & Jiang, X. (1994). Economic incentives to achieve convergence of user and system optima. In *Proceedings of vnis'94-1994 vehicle navigation and information systems conference* (pp. 513–518).

Cohen-Blankshtain, G., Bar-Gera, H., & Shiftan, Y. (2022). Congestion pricing and positive incentives: conceptual analysis and empirical findings from israel. *Transportation*, 1–27.

Dimitriou, L., Kaltsounis, A., & Stathopoulos, A. (2009). Introducing transportation-related carbon footprint considerations in optimal urban road infrastructure management. *International Journal of Energy and Environment*, *3*(3), 103–111.

Djavadian, S., Hoogendoorn, R. G., Van Arerm, B., & Chow, J. Y. (2014). Empirical evaluation of drivers' route choice behavioral responses to social navigation. *Transportation research record*, *2423*(1), 52–60.

Ettema, D., Knockaert, J., & Verhoef, E. (2010). Using incentives as traffic management tool: empirical results of the" peak avoidance" experiment. *Transportation Letters*, *2*(1), 39–51.

Fu, L. (2001). An adaptive routing algorithm for in-vehicle route guidance systems with real-time information. *Transportation Research Part B: Methodological*, *35*(8), 749–765.

Ghafelebashi, A., Razaviyayn, M., & Dessouky, M. (2023). Congestion reduction via personalized incentives. *Transportation Research Part C: Emerging Technologies*, *152*, 104153.

HCM. (2000). Highway capacity manual. *Washington, DC*, *2*(1).

He, F., Yin, Y., & Lawphongpanich, S. (2014). Network equilibrium models with battery electric vehicles. *Transportation Research Part B: Methodological*, *67*, 306–319.

Kaysi, I. (1993). *An integrated approach to vehicle routing and congestion prediction for real-time driver guidance* (Vol. 1408).

Kickhöfer, B., & Nagel, K. (2016). Towards high-resolution first-best air pollution tolls: An evaluation of regulatory policies and a discussion on long-term user reactions. *Networks and Spatial Economics*, *16*, 175–198.

Klein, I., & Ben-Elia, E. (2018). Emergence of cooperative route-choice: A model and experiment of compliance with system-optimal atis. *Transportation research part F: tra c psychology and behaviour*, *59*, 348–364.

Kröller, A., Hüffner, F., Kosma, Ł., Kröller, K., & Zeni, M. (2021). Driver expectations toward strategic routing. *Transportation research record*, *2675*(11), 44–53.

Leblanc, R., & Walker, J. L. (2013). Which is the biggest carrot? comparing nontraditional incentives for demand management. In *Proceedings of the transportation research board 92nd annual meeting.*

Levinson, D. (2010). Equity effects of road pricing: A review. *Transport Reviews*, *30*(1), 33–57.

Luan, M., Waller, S. T., & Rey, D. (2023). A non-additive path-based reward credit scheme for traffic congestion management. *Transportation Research Part E: Logistics and Transportation Review*, *179*, 103291.

Mahmassani, H. S., & Peeta, S. (1993). Network performance under system optimal and user equilibrium dynamic assignments: Implications for advanced traveler information systems. *Transportation Research Record*, *1408*, 83.

May, A. D., Koh, A., Blackledge, D., & Fioretto, M. (2010). Overcoming the barriers to implementing urban road user charging schemes. *European Transport Research Review*, *2*(1), 53–68.

Menelaou, C., Timotheou, S., Kolios, P., & Panayiotou, C. G. (2021). Joint route guidance and demand management for real-time control of multi-regional traffic networks. *IEEE Transactions on Intelligent Transportation Systems*, *23*(7), 8302–8315.

Peeta, S., & Mahmassani, H. S. (1995). System optimal and user equilibrium time-dependent traffic assignment in congested networks. *Annals of Operations Research*, *60*(1), 81–113.

Ren, T., Huang, H.-J., Liu, T.-L., & Nie, Y. M. (2020). Some analytical results on spatial price differentiation in first–best congestion pricing schemes. *Transportation Research Part C: Emerging Technologies*, *114*, 425–445.

Roughgarden, T., & Tardos, É. (2002). How bad is selfish routing? *Journal of the ACM (JACM)*, *49*(2), 236–259.

Sun, J., Wu, J., Xiao, F., Tian, Y., & Xu, X. (2020). Managing bottleneck congestion with incentives. *Transportation research part B: methodological*, *134*, 143–166.

Sunio, V., & Schmöcker, J.-D. (2017). Can we promote sustainable travel behavior through mobile apps? evaluation and review of evidence. *International journal of sustainable transportation*, *11*(8), 553–566.

Van Essen, M., Eikenbroek, O., Thomas, T., & Van Berkum, E. (2019). Travelers' compliance with social routing advice: Impacts on road network performance and equity. *IEEE Transactions on Intelligent Transportation Systems*, *21*(3), 1180–1190.

van Essen, M., Thomas, T., van Berkum, E., & Chorus, C. (2016). From user equilibrium to system optimum: a literature review on the role of travel information, bounded rationality and non-selfish behaviour at the network and individual levels. *Transport reviews*, *36*(4), 527–548.

Vosough, S., de Palma, A., & Lindsey, R. (2022). Pricing vehicle emissions and congestion externalities using a dynamic traffic network simulator. *Transportation Research Part A: Policy and Practice*, *161*, 1–24.

Vosough, S., & Roncoli, C. (2024). Achieving social routing via navigation apps: User acceptance of travel time sacrifice. *Transport Policy*. doi: https://doi.org/10.1016/j.tranpol.2024.01.026

Wie, B.-W., Tobin, R. L., Bernstein, D., & Friesz, T. L. (1995). A comparison of system optimum and user equilibrium dynamic traffic assignments with schedule delays. *Transportation Research Part C: Emerging Technologies*, *3*(6), 389–411.

Yang, H., & Huang, H.-J. (2004). The multi-class, multi-criteria traffic network equilibrium and systems optimum problem. *Transportation Research Part B: Methodological*, *38*(1), 1–15.

Zangui, M., Aashtiani, H. Z., Lawphongpanich, S., & Yin, Y. (2015). Path-differentiated pricing in congestion mitigation. *Transportation Research Part B: Methodological*, *80*, 202–219.