Dynamic multi-region macroscopic fundamental diagram stochastic user equilibrium: model framework and parameter estimation in a real-life large-scale case study

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SHORT SUMMARY

Multi-region Macroscopic Fundamental Diagram (MFD) traffic equilibrium models have been developed as a more easily calibratable, maintainable, and computationally efficient alternative to traditional link-network traffic assignment models with full disaggregate network representation. In this extended abstract, we discuss the framework of a new dynamic multi-region MFD stochastic user equilibrium model that can base regional path choice on region travel times actually experienced. The model produces continuous outputs, facilitating the development of a maximum likelihood estimation procedure for rigorous statistical estimation of behavioural parameters of underlying regional path choice models. This estimation procedure is operationalised in a first-of-its-kind real-life large-scale multi-region MFD system, with underlying regions and directional superimposing motorway regions. An enormous dataset of GPS records is utilised to calibrate MFD functions and estimate models. Results provide empirical evidence supporting the hypothesis that regional path choices are more realistically based on experienced region travel times rather than instantaneous travel times.

Keywords: dynamic model, macroscopic fundamental diagram, parameter estimation, real-life large-scale multi-region system, stochastic user equilibrium, transportation network modelling

1. INTRODUCTION

Multi-region Macroscopic Fundamental Diagram (MFD) traffic equilibrium models have been developed as an aggregate way of modelling large-scale systems (e.g. national networks). There are several potential advantages of adopting such an aggregated approach over a traditional detailed link-network traffic assignment model. Firstly, the aggregated model should be simpler to calibrate, as it requires data at a coarser grain. For example, the production/speed-accumulation MFD functions require aggregated data over a large area (region), whereas detailed models require link-level calibration, and data may not be available at such a level across the whole area modelled. The above also means the aggregated model should be simpler to maintain as circumstances change. Another advantage is that the aggregated model would be expected to be significantly faster computationally than running the detailed link-network model. This is a particular advantage if many scenarios need to be run, or if the model is embedded as a sub-problem in an overall problem concerned with calibration (e.g. Mariotte et al, 2020), parameter estimation, or optimisation of a design.

Yildirimoglu & Geroliminis (2014) developed the first multi-region MFD traffic equilibrium model, where a Multinomial Logit (MNL) regional path choice model is adopted for traffic flow equilibration along with a stochastic network loading procedure to estimate time-dependent regional trip lengths. Batista & Leclercq (2019) later developed a monte-carlo-simulation-based approach for the traffic equilibrium based on monte carlo simulations of trip length / MFD distributions to account for variability in these attributes. Mariotte et al (2020) try two different approaches for determining the regional path choice: a Deterministic User Equilibrium (DUE) variant, as well as optimising the regional path choices to fit MFD production-point data. Extending these works, Batista et al (2021) developed a heuristic approach for updating the traffic-dependent trip lengths / regional paths during the dynamic traffic assignment, while numerous methods have been proposed for dynamically modelling the transfer of traffic flow at region borders (e.g. Yildirimoglu & Geroliminis, 2014; Mariotte & Leclercq, 2019; Mariotte et al, 2020).

There are two important gaps in the research into multi-region MFD traffic equilibrium modelling.

The *first* gap regards the calibration of behavioural parameters of underlying regional path choice models within the traffic equilibrium. Several studies (e.g. Mariotte et al (2020)) have adopted a DUE approach for the traffic equilibrium, i.e. a model without behavioural parameters. The deficiencies of DUE are, however, well-known; for example, its inability to account for any modelling / driver knowledge uncertainties. Yildirimoglu & Geroliminis (2014) and Batista & Leclercq (2019) adopt Stochastic User Equilibrium (SUE) approaches with behavioural parameters for calibration. To be able to properly capture different travel behaviours or even scales of attributes (e.g. hours - minutes), these parameters need to be estimated according to observed behaviour. However, no such methods have been developed for achieving this.

The *second* gap also regards regional path choice. An assumption that has been made by all approaches thus far, has been that the regional path choices at each departing time interval are based on the current/instantaneous region travel times at that interval. However, the traffic state will inevitably evolve during a driver's journey, so that the traffic state actually experienced in later regions of the regional path may be considerably different from when the driver departed. It appears more realistic that regional path choice will be based on these experienced travel times, assuming driver knowledge.

Motivated by these two gaps, in this extended abstract we develop a new dynamic multi-region MFD SUE modelling framework and apply it in a real-life large-scale case study. A key feature of the developed model is that regional path choice can be based on region travel times actually experienced. Moreover, the model produces continuous outputs, facilitating the development of a *consistent* maximum likelihood estimation procedure for rigorous statistical estimation of underlying behavioural parameters. To estimate the model parameters consistent with the equilibrium, this means solving a bi-level maximum likelihood problem subject to equilibrium constraints.

The extended abstract is structured as follows. In Section 2 we discuss the framework of the dynamic multi-region MFD SUE model. In Section 3 we detail the likelihood formulation and estimation procedure. In Section 4 we estimate the developed model in a real-life large-scale case study of Zealand, Denmark.

2. THE DYNAMIC MULTI-REGION MACROSCOPIC FUNDAMENTAL DIAGRAM STOCHASTIC USER EQUILIBRIUM MODEL

In this section, we discuss the overall framework of the dynamic multi-region MFD SUE model.

We begin by detailing general multi-region MFD traffic modelling concepts. The study area is partitioned into a set of regions. The traffic conditions in each region are described by a speed-MFD function that maps accumulation (number of vehicles in the region) to average speed of the vehicles in the region. As accumulation increases, average MFD speed decreases. There is a set M of both internal and external Origin-Destination (OD) movements, i.e. trips originating and destinating in the same region and trips originating in one region and destinating in another, respectively. A regional path is defined as a sequence of regions traversed when travelling an OD movement. P_m is the choice set of regional paths for OD movement $m \in M$. The total runtime period of the system is split into an indexed set Ψ of discrete time-slices, each with duration ε. The travel demands d_m^τ for each regional OD movement $m \in M$ departing at a given time-slice $\tau \in \Psi$, are obtained by aggregating travel demands from the underlying network ODs over the time-slice between the OD regions. The travel demand d_m^τ for OD movement m departing at time-slice τ is split among the available regional paths $p \in P_m$ according to a regional path choice model, to give the regional path flows $f_{m,p}^\tau$. For a given accumulation level and thereby average MFD speed in a region at a given moment in time, the travel time of a region when travelling a particular regional path at that moment in time, is obtained by dividing the (regional path and time-slice specific) region length by MFD speed.

The traffic dynamics of the dynamic multi-region MFD SUE model are described by a traffic propagation model utilising features of a Space-Time-Diagram (STD). Due to the word constraints of this extended abstract, we briefly describe the model. The travel demand for each time-slice is assumed to depart uniformly and continuously, and, throughout each time-slice, all drivers are assumed to be experiencing the same speed in a region. Vehicles departing at the beginning and end of each time-slice travelling each regional path are tracked from origin to destination on the STD, based on region travel times, see Fig. 1. Occupied STD areas of regional path flows are then used to calculate accumulation levels, which feedback to determine average vehicle speeds in a region during a time-slice (through the speed-MFD function), and thereby region travel times. The traffic propagation model is thus naturally expressed as a fixed-point problem in terms of region travel times.

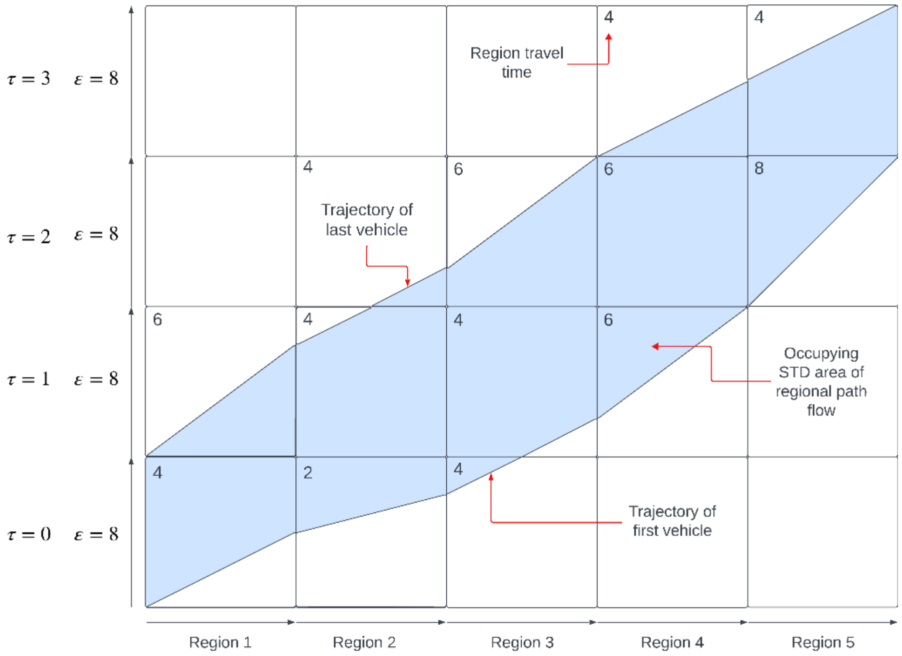

Fig. 1. Example of space-time diagram.

This traffic propagation model is then embedded within a dynamic multi-region MFD SUE model for equilibrating the regional path flows, given equilibrated region travel times from the traffic propagation model. There are two versions of the model.

The *Instantaneous Dynamic* (ID) model bases regional path choice on region travel times at the time-slice of departure. ID multi-region MFD SUE conditions are established as follows: A demand-feasible universal regional path flow vector $\boldsymbol{f}^*$ of all regional path flows departing at all time-slices, is an ID multi-region MFD SUE solution iff:

$$f_{m,p}^{\tau} = d_m^{\tau} Q_{m,p}^{\tau}\big(\boldsymbol{t}^*(\boldsymbol{f})\big), \qquad \forall p \in P_m, \forall m \in M, \forall \tau \in \Psi,$$

where $Q_{m,p}^{\tau}$ is the choice probability of regional path $p \in P_m$ at time-slice $\tau \in \Psi$, determined according to the adopted choice model, given the vector of equilibrated region travel times $\boldsymbol{t}^*$ (from the traffic propagation model), given the regional path flows $\boldsymbol{f}$. Note that in general the travel cost of a region may depend on several variables, however we suppose here that only travel time is considered.

The *Experienced Dynamic* (ED) model bases regional path choice on the average region travel times actually experienced by the flow at the time-slices of travel. ED multi-region MFD SUE conditions are established as follows: A demand-feasible universal regional path flow vector $\boldsymbol{f}^*$ of all regional path flows departing at all time-slices, is an ED multi-region MFD SUE solution iff:

$$f_{m,p}^{\tau} = d_m^{\tau} Q_{m,p}^{\tau}\left(\bar{\boldsymbol{t}}\big(\boldsymbol{t}^*(\boldsymbol{f})\big)\right), \qquad \forall p \in P_m, \forall m \in M, \forall \tau \in \Psi,$$

where $Q_{m,p}^{\tau}$ is the choice probability of regional path $p \in P_m$ at time-slice $\tau \in \Psi$, given the vector of experienced region travel times $\bar{\boldsymbol{t}}$, given $\boldsymbol{t}^*$, given $\boldsymbol{f}$.

The system is inherently full of feedbacks where everything is connected. For example, drivers make regional path decisions based on the region travel times (instantaneous or experienced), but the traffic states (and thereby the region travel times) depend in turn on the regional path decisions. Moreover, the traffic states experienced are not only dependent on their own regional path decisions, but also the regional path decisions of all drivers travelling all regional paths departing at all time-slices. Consequently, an iterative solution method is needed to identify the equilibrium, to iteratively feedback between a Traffic Propagation Stage and Regional Path Flow Updating Stage. Fig. 2 provides a schematic diagram illustrating the general solution method.

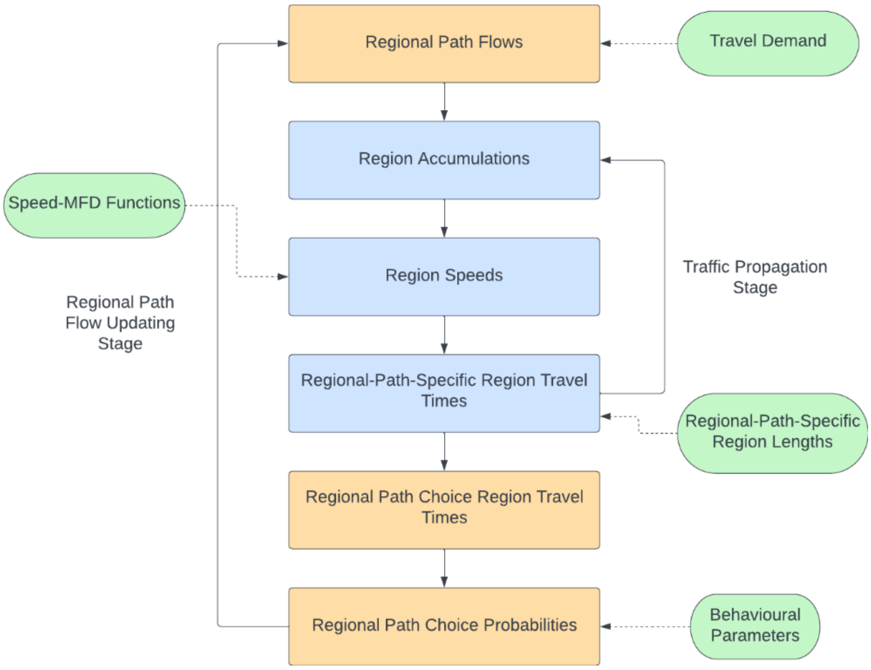

Fig. 2. Schematic diagram illustrating general method for solving the dynamic multi-region MFD SUE model.

3. LIKELIHOOD FORMULATION & ESTIMATION PROCEDURE

Suppose that we have available a set of Z observed regional paths, collected through e.g. GPS units or smart phones. Suppose that details of the multi-region MFD system are known, e.g. the region partitioning, speed-MFD functions, OD movements, travel demands, regional path choice sets, and regional trip lengths are all known. Suppose that the observation data is contained in a vector $\boldsymbol{y}$ of size Z where element z of $\boldsymbol{y}$ details the OD movement, regional path taken, and departing time-slice of the observation.

The Likelihood, L, for an observation data vector $\boldsymbol{y}$ is:

$$L(\boldsymbol{\psi}|\boldsymbol{y}) = \prod_{\tau\in\Psi}\prod_{m\in M}\prod_{p\in P_m}\left(\frac{f_{m,p}^{\tau,*}(\boldsymbol{\psi})}{d_m^{\tau}}\right)^{\phi_{m,p}^{\tau}(\boldsymbol{y})},$$

where $\boldsymbol{\psi}$ is the vector of model parameters from the relevant underlying regional path choice model, $\phi_{m,p}^{\tau}(\boldsymbol{y})$ is the number of observations that take regional path $p \in P_m$ when departing during time-slice τ, and $f_{m,p}^{\tau,*}(\boldsymbol{\psi})$ is the ID/ED multi-region MFD SUE regional path flow solution for regional path $p \in P_m$ given $\boldsymbol{\psi}$. The key feature of any SUE model is that at equilibrium the regional path choice probabilities and regional path flow proportions are equal. $\frac{f_{m,p}^{\tau,*}(\boldsymbol{\psi})}{d_m^{\tau}}$ thus gives the equilibrated choice probability of regional path $p \in P_m$ (of OD movement m) when departing during time-slice τ.

The Log-Likelihood function, LL, to be maximised is:

$$LL(\boldsymbol{\psi}|\boldsymbol{y}) = \ln\left(\prod_{\tau\in\Psi}\prod_{m\in M}\prod_{p\in P_m}\left(\frac{f_{m,p}^{\tau,*}(\boldsymbol{\psi})}{d_m^\tau}\right)^{\phi_{m,p}^\tau(\boldsymbol{y})}\right) = \sum_{\tau\in\Psi}\sum_{m\in M}\sum_{p\in P_m}\phi_{m,p}^\tau(\boldsymbol{y})\ln\left(\frac{f_{m,p}^{\tau,*}(\boldsymbol{\psi})}{d_m^\tau}\right),$$

where $f_{m,p}^{\tau,*}(\boldsymbol{\psi})$ is the ID/ED multi-region MFD SUE regional path flow solution for regional path $p \in P_m$ when departing at time-slice τ given the vector of model parameters $\boldsymbol{\psi}$.

Standard MLE procedures can be used to estimate the parameters of the ID and ED models for a given multi-region MFD setup. Using a standard iterative estimation procedure, model parameters can be found that maximise the Log-Likelihood function as formulated above for a given set of data. A key distinguishing feature though from standard MLE procedures is that for each parameter setting tested, ID/ED multi-region MFD SUE must be re-solved to update the regional path flows. This makes the estimates consistent with the equilibrium model.

4. CASE STUDY

The model is estimated in a real-life large-scale case study. A multi-region MFD system is calibrated for the large-scale area of Zealand, Denmark. There are both underlying urban and rural areas and a superimposing motorway network that is treated separately. The underlying regions , displayed in Fig. 3, were partitioned manually according to logic, local understanding, and trial-and-error, similar to as done in Mariotte et al (2020). After this, the superimposed motorway regions were then partitioned according to the underlying region, i.e. using the region borders of the urban/rural regions to also partition the motorway regions. Moreover, in order to better capture homogeneous traffic conditions, the motorway regions were partitioned further by motorway name, direction, and upon intersection with other motorways, see a demonstration in Fig. 4. This all resulted in a total of 135 regions: 22 urban regions, 17 rural regions, and 96 motorway regions.

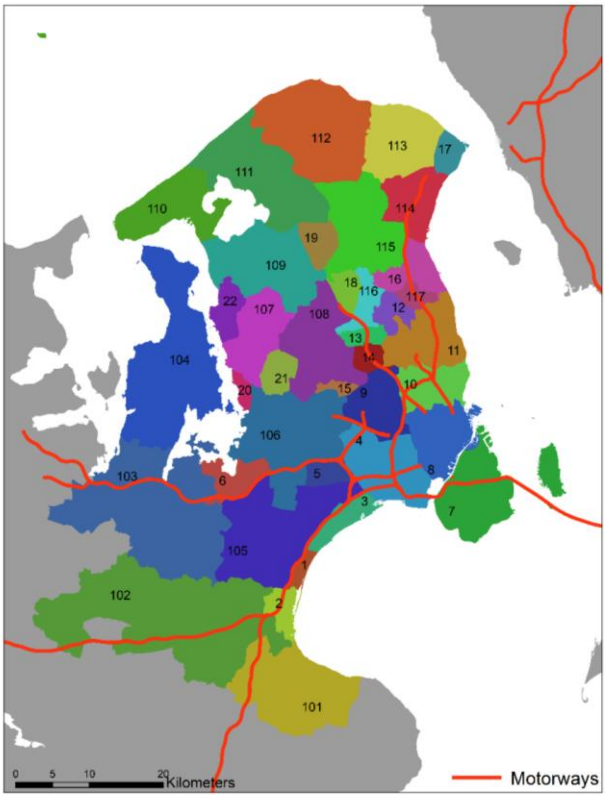

Fig. 3. Underlying region partitioning and overlaying motorway network of the real-life large-scale case study of Zealand, Denmark.

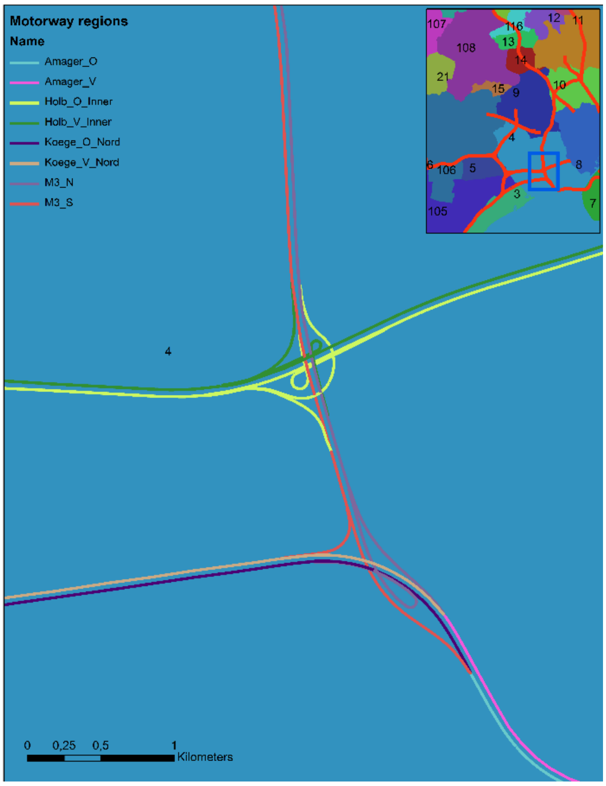

Fig. 4. Demonstration of superimposed motorway region partitioning.

Speed-MFD functions were calibrated for each region using two datasets. The first consisted of an extensive set of link-map-matched GPS points of cars and the second of loop detector vehicle counts. Upon manual inspection of the data, exponential speed-MFD functions were calibrated for the urban/rural regions, and piecewise-exponential speed-MFD functions were calibrated for the motorway regions. Fig. 5A-B show examples of the former and latter, respectively.

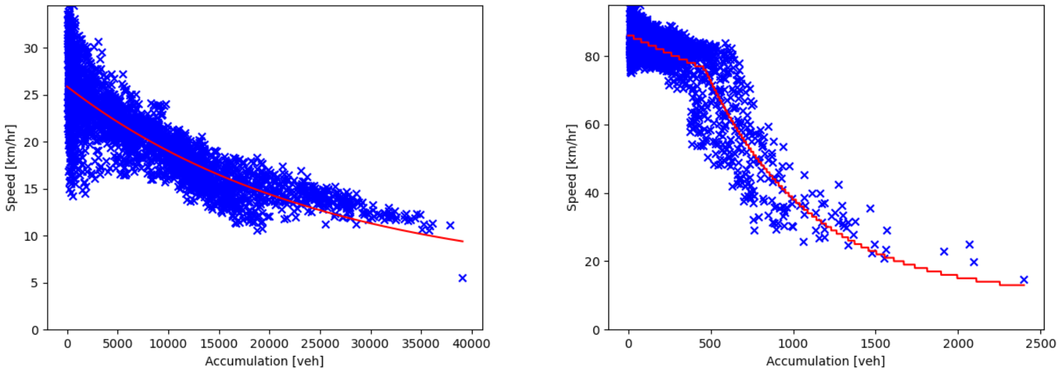

Fig. 5. Examples of calibrated speed-MFD functions. A: Exponential function for urban region in central Copenhagen. B: Piecewise-exponential function for motorway region.

The travel demands for each regional OD movement were obtained by aggregating intra-zonal travel demands from an underlying network model. Fig. 6 displays the total travel demand at each hour of the day, showing the morning and evening peaks. Regional path choice sets and time-of-day-dependent regional trip lengths were obtained by generating intra-zonal shortest path routes on the underlying network based on time-of-day-dependent congested link travel times. Fig. 7A displays the distribution of the generated regional path choice set sizes and Fig. 7B displays the distribution of the regional path lengths.

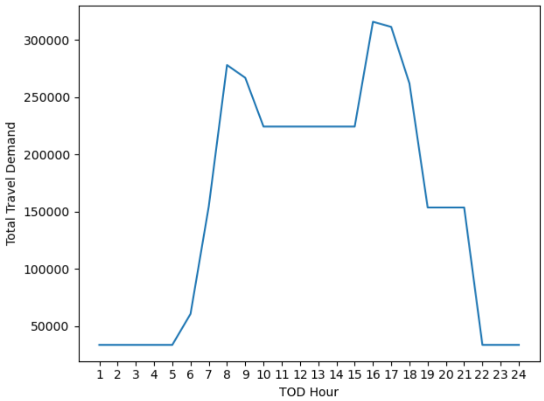

Fig. 6. Total travel demand at each hour of the day.

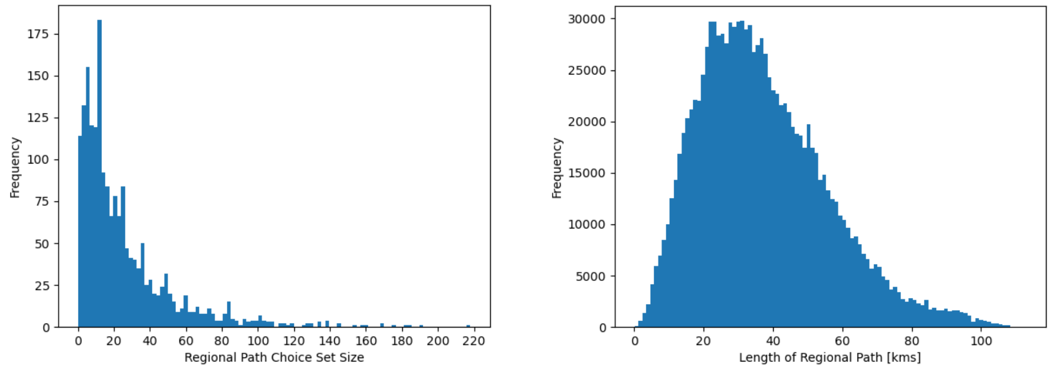

Fig. 7. A: Distribution of the generated regional path choice set sizes. B: Distribution of the regional path lengths.

Using the estimation procedure introduced in Section 3, the ID and ED models were estimated utilising a large dataset of 997,121 regional path observations, tracked by GPS. A MNL model was used to determine the regional path choices, which has a Logit scaling parameter θ that scales sensitivity to instantaneous or experienced region travel time. Table 1 displays the estimation results. As shown, the estimated θ parameters are similar between ID and ED, but the ED model provides a considerable improvement in Log-Likelihood.

Model	$\hat{\theta}$	LL
ID	0.2729	-2585785
ED	0.2701	-2578786

Table 1. Parameter estimates and Likelihood values for estimation of instantaneous and experienced dynamic multi-region MFD SUE models in real-life case study.

From the calibrated ED model, Fig. 8 plots the average time it takes to travel from region 6 (Roskilde) to region 11 (DTU) throughout the day. These travel times align with personal experiences from the authors.

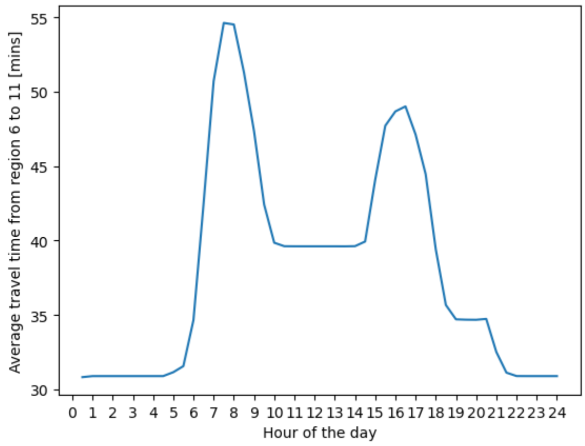

Fig. 8. Average travel time from region 6 to region 11 throughout the day, from calibrated ED model.

5. CONCLUSIONS

This extended abstract has established the framework of a new dynamic multi-region MFD SUE model. Unlike existing approaches, the regional path choices can be based on region travel times actually experienced. The above features create a challenge for solving the model, but an iterative solution method is developed. Given the well-behaved continuity property of the model, a MLE procedure is proposed for estimating behavioural parameters of underlying regional path choice models. This estimation procedure was operationalised in a first-of-its-kind real-life large-scale multi-region MFD system to estimate the Logit scaling parameter of an MNL regional path choice model, within ID and ED multi-region MFD SUE. Results found ID and ED estimated similar parameter values, but that the ED model provided considerably better fit to the data, thus providing empirical evidence to support the hypothesis that drivers more realistically base their route choice decisions on experienced region travel times rather an instantaneous travel times. Future research includes estimating more complex regional path choice models and considering additional travel cost attributes. Moreover, accounting for capacities of the network, for example at region borders, such as done by Mariotte & Leclercq (2019), Mariotte et al (2020). And, making the regional trip lengths consistent with the equilibrium, such as done by Yildirimoglu & Geroliminis (2014), Batista et al (2021).

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