Random Utility Maximization model considering the information search process

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SHORT SUMMARY

Choice modelling has been dominated by static representations preferences due to their ease of implementation, transparent economic interpretability, and statistical coherency. Unlike, the Decision Field Theory (DFT) model explicitly includes the attribute scrutiny process within the choice decision, making it more closely related to the behavior that is observed in practice. However, the DFT model lacks the RUM model's microeconomic interpretability and has statistical limitations regarding the identification of the model parameters. This research introduces the "RUM-DFT" model, encompassing ideas from both approaches. Using Monte Carlo simulations and applying the proposed model to a database of real choices, it is first shown that the proposed approach can properly identify the parameters of the deliberation process, replicate the dynamic behavior of the utilities during the deliberation process; and retains full economic interpretability, since the estimated coefficients correspond to marginal indirect utilities when there is perfect knowledge of the information search process.

Keywords: RUM-DFT, Information Search Process, DFT, Cognitive Processing

1. INTRODUCTION

Discrete choice models have been widely proposed, used, and promoted in the literature for several decades. These correspond to a mathematical approach that allows estimations and predictions of the behaviors carried out by economic agents in different areas such as economics, health, marketing, and transportation, among others. Therefore, making a proper representation of the decision-making process, accurately accounting for the true behavioral mechanisms that are behind the choice dynamics, is crucial for making correct inference on the causal relations that are behind choices and for performing accurate forecasting to support informed public policy design. Neglecting the true dynamics that is behind the choice process will result in inconsistent estimators of the model parameters due to endogeneity, because of a model misspecification (Guevara, 2015).

Busemeyer and Townsend (1993) classify choice models according to whether utilities or preferences are dynamically or statically constructed. On the one hand, the static probabilistic models are discrete choice models that ignore the fact that choice probabilities are correlated with decision time and that deliberation time influences choice probability.

Although the different specifications of static choice models are extensive, simple to implement, have low computational cost and a high degree of economic interpretability of the parameters, they are conceptually unrealistic as they do not consider the cognitive process that individuals go through when making a decision; for example, they do not include loss of information, filtration o information search cost. Under classical microeconomic and consumer behavior theory, most models assume that decision-makers evaluate and process information from alternatives in a perfect sense (Swait et al., 2001). However, it has been demonstrated that decision-makers may only focus on specific attributes, acquiring knowledge sequentially to make a final decision in simple and complex public transport route choice tasks (Nova, 2022). Likewise, Noguchi et al.,(2014), Stewart et al.,(2016), Stewart et al.,(2016), Sui et al.,(2020) evidenced, through eye tracking, that this behavior also occurs in simple, risky, and multi-attribute choices.

In contrast, dynamic models define choice probabilities are explicitly affected by the deliberation process, as the amount of time spent making a decision influences the final choice. Probabilities vary over time, as there is a constant acquisition and processing of information (attributes) that are incorporated to update the value of preferences or utilities before the choice is performed. Models that include a cognitive cost in the information search process, such as the Directed Cognition Model (Gabaix 2006) and the Adaptive Path Choice model have been shown to perform better than compensatory models in complex decision contexts (Gao 2011).

The dynamic processing of attributes has been represented by Decision Field Theory (DFT). It was initially designed as a cognitive model to capture the deliberation process in choice making (Busemeyer 1992, Busemeyer 1993). Then, DFT was then extended to a probabilistic-dynamic model that allowed for multiple attributes (Diederich, 1997) and was also generalized to multiple-alternative decision making (Roe et al.,2001). Recent contributions on DFT theory are from Hancock et al.,(2018) who improve the mechanisms that support the DFT model to make it more competitive with traditional discrete choice models. These advances allow for incorporating heterogeneity among and within decision-makers. Furthermore, Hancock et al.,(2021) introduce scale parameters in the basic mechanism of the DFT model, to avoid the requirement of conceptualizing a priori parameter values that may affect model estimation and identification. Finally, Hancock et al.,(2022) extend the model to include data from eye-tracking processes, to capture attribute attention weights more realistically during the deliberation process. All these recent studies show that the DFT model fits the data well and better than conventional static models. However, the DFT model has limitations, such as being based on ad-hoc matrix implementations of the model, identification problems, lack of a robust statistical theoretical framework and an approach compatible with the principle of random utility maximization that makes it impossible to interpret the parameters in a traditional way.

The combined limitations of current static and dynamic models motivate the need to create a new theoretical framework. This model should aim to keep the desirable properties of RUM model, whilst also overcoming its limitations with regards to its representation of the choice deliberation process. This could be achieved through the development of a RUM model that reflects cognitive dynamics, including the significant findings regarding the information search process, such as the process typically being breadth-first, that decision-makers revisit attributes more than once and that information is filtered. In this regard, this work introduces a new model `RUM-DFT'. This new model also aims to rectify the DFT model's identification, inference, and parameter interpretation limitations. Likewise, the new model will include parameters that allow the deliberation process to be adequately modelled.

2. FORMULATION RUM-DFT

The RUM-DFT model proposes that, before choosing between alternatives, choice-makers perform a breadth-first information search process to update initially preconceived utilities. This means that individuals make comparisons of alternatives under a particular attribute at each preference updating step of the deliberation process until they make their choice. Specifically, the individual at each step t must choose whether to choose some alternative with the current information $(y^t = 1)$ or whether to perform a new information search to update the utilities $(y^t = 0)$. Suppose the individual decides on the option of choosing with the present utilities. In that case, the process of updating the underlying utilities is finished, and the one that provides the highest utility $[i|y^t]$ is chosen. By contrast, if the individual decides to continue with the information search process, the individual must determine which attribute will be attended to at step t+1 to update future utilities $[k(t+1)|y^t]$.

Hence, in this approach, we define a sequence of attributes attended to prior to the choice that ends with a choice. That is, $h^T = \{k(1), ... k(T)\}$ is defined as the attributes attended at each step t of the deliberation process up to the choice at T. Since, in traditional surveys, information such as eye-tracking data is not recorded, thus the sequence of attended attributes is not known, the set H^T is defined as all possible sequences that the individual can consider until the decision is made. Therefore, the probability of choosing an alternative i given the sequence of attended attributes and the probability of paying attention to an attribute k at step t; given the decision to continue searching for information, can be represented by Eq. (1) and Eq. (2), respectively.

$$P(i|h^T) = P(i|y^T)\Omega(\, y^T|h^T) \quad [1]$$

$$P(k(t)|h^{T-1}) = P(k(t)|1 - y^T)\Omega(\, 1 - y^{T-1}|h^{T-1}) \quad [2]$$

Where $P(i|y^T)$is the probability of choosing alternative i conditional on the decision maker deciding to choose at step T, $P(k(t)|h^{T-1}\,)$ is the probability of attending attribute k at deliberation step t conditional on the attributes attended up to step t-1, $\Omega(\, y^T|h^T)$ is the probability that the individual has decided to choose at step T and $\Omega(\, 1 - y^{T-1}|h^{T-1})$ is that he/she decides to search for information.

The iterative process of searching for information and updating the utilities can be stopped for two main reasons. The first may be due to an external limitation that forces the person to choose an alternative in a maximum deliberation time. The second reason corresponds to the individual reaching their internal limit of preference. Thus, without an external limitation, the individual performs this process until future utilities (U_S^T) do not present a significant change compared to current utilities (U_c^T), modelled through an internal tolerance. Therefore, the probability that the individual has decided to choose, as shown mathematically in Eq. (3), allows the model to capture the difference between the expected value of choosing one of the alternatives at step T and the expected value of continuing to observe an attribute at step T+1.

$$\Omega(\, y^T|h^T) = P(|U_c^T - U_S^T| \leq \delta^T) \quad [3]$$

So far, no assumptions have been made about the functional form of utilities. The following section details the dynamics of the utilities and how the information search process is incorporated in breadth-first during the choice deliberation process up to the point at which a decision is made.

Utility Functions

The functional form of the utilities in the RUM-DFT model are proposed to capture the evolution of the individuals' preferences in accordance with the information search process. Therefore, the utilities explicitly represent the dynamic aspect, which depends on the attribute k(t) attended to in step \$t\$ of the deliberation process and on past information already considered, that is the utility of the previous instant U_{jn}^{t-1}, given by:

$$U_{jn}^{t} = \alpha \cdot U_{jn}^{t-1} + (1-\alpha) \cdot \beta_{k(t)} \cdot X_{jnk(t)} + \varepsilon_{jn}^{t} \qquad [4]$$

where α is the memory parameter representing the influence of time on past utilities (i.e., forgetting), $\beta_{k(t)}$ is the parameter of the attribute k attended in t, $X_{nik(t)}$ corresponds to the value of attribute k of alternative i for individual n observed in step t, and ε_{ni}^{t} is the error that distributes extreme value I. It is necessary to point out that the random utility U_{jn}^{t} decomposes into two parts. The systematic part containing the information search process of each step t and the random part of the current step.

Calculating probabilities

The conditional probability $\Omega(\, y^{T}|h^{T})$ that the person decides to choose in step T, given the sequence of attributes attended h^{T}, will depend on the difference between the current and future utilities.

$$\omega\left(y^{T}|h^{T}\right) = P(|V_{c}^{T}(h^{T}) + \varepsilon_{c}^{T} - V_{s}^{T}(h^{T}) - \varepsilon_{s}^{T}| \leq \delta^{T}) \quad [5]$$

Where δ is the threshold and in this dynamic, it is assumed that individuals become more intolerant over time, requiring more considerable expected changes to decide to continue searching for information.

Now, the probability of choosing an alternative i conditional on what the individual has decided to choose is like the one from the MNL model, but only considering the attributes attended up to step T:

$$P(i|y^{T}) = \frac{e^{V_{i}^{T}}}{\sum_{j} e^{V_{j}^{T}}} \qquad [6]$$

Finally, the probability of attending to an attribute k is defined. Two possible formulations were considered for the analysis. On the one hand, a Logit model of constants indicating the weight of attention on each attribute in the deliberation process was considered (Φ_k), as shown in Eq. (7). On the other hand, a Logit model was constructed that considers the expected value of the change in overall utilities if the k^{th} attribute is observed in the next step. This aligns with the assumption that people perform breadth-first information search, as shown in Eq. (8).

$$\phi(k|h^{t}) = \frac{e^{\phi_k}}{\sum_{\kappa} e^{\phi_\kappa}} \qquad [7]$$

$$\phi(k|h^{t}) = \frac{e^{\psi_{k(t)}^{t+1}}}{\sum_{\kappa} e^{\psi_{\kappa}^{t+1}}}\,, \quad \psi_{k(t)}^{t+1} = ln\left(\sum_{J} \left|\phi_{k(t)} \cdot x_{jk(t)}\right|\right) \qquad [8]$$

Thus, Eq. (5) and Eq.(6) allow for the construction of the probability of choosing an alternative i if the choice-maker decided to choose at step T under a particular sequence of attended attributes (h^T), as shown in Eq.(9). Similarly, Eqs.(7) and (8) construct the probability that he/she decides to search for information or attend to the k^{th} attribute at step t, given the sequence of attributes up to that step (h^T), as shown in Eq.(10). However, they only model a particular sequence of attended attributes within which the individual could have decided. In general, without knowing the deliberation process, a modeler must integrate or consider all possible sequences of attended attributes H^T, which results in the probability of choosing alternative i being as shown in Eq.(11).

This specification, which we name RUM-DFT-SC (A RUM-DFT model explicitly for stated choice scenarios) considers all possible sequences of attended attributes without some maximum deliberation time. Therefore, to estimate this model specification, the maximum deliberation time $T_{\max}$ must be fixed. However, if there is some knowledge about the information search process or decision-makers deliberation, the estimation process can be reduced.

$$P(i|h^T) = \left(\frac{e^{V_i^T}}{\sum_j e^{V_j^T}}\right)\left(\frac{e^{V_s^T+\delta^T}}{e^{V_c^T} + e^{V_s^T+\delta^T}} + \frac{e^{V_c^T+\delta^T}}{e^{V_s^T} + e^{V_c^T+\delta^T}}\right) \quad [10]$$

$$P(k(t)|h^{T-1}) = \left(\frac{e^{\phi_k}}{\sum_\kappa e^{\phi_\kappa}}\right)\left(\frac{e^{V_c^{T-1}}}{e^{V_c^{T-1}} + e^{V_s^{T-1}+\delta^{T-1}}} + \frac{e^{V_s^T}}{e^{V_s^{T-1}} + e^{V_c^{T-1}+\delta^T}}\right) \quad [11]$$

$$P(i) = \sum_{h^T}^{H^t} P(i|h^T) \cdot P(h^T) \quad [12]$$

First, it is possible to have information on the time or the number of steps that individuals perform in the choice process since they can be considered valid proxies on cognitive processes (Horstmann, 2009). From this approach, the likelihood is reduced to the possible sequences of attributes attended up to the maximum time of deliberation found in the database, obtaining the specification RUM-DFT-DT (with deliberation time):

$$P(i) = \sum_{h_l^T \in H^{\tilde{T}}} P(i|h_l^T)P(h_l^{\tilde{T}}) \quad [13]$$

Secondly, the actual sequence of the attributes attended by the respondents can be uncovered using instruments of process data tracking, such as a mouse-tracker, click-tracker or eye-tracker (for example Nova and Guevara (2022)). This data reduces the probability of choice to only one sequence of attributes, resulting in the specification RUM-DFT-IS (with information search process):

$$P(i) = \sum_{h^T}^{H^T} P(i|h^T) \cdot P(h^T) = P(i|\tilde{h}^{T_n}) \quad [14]$$

Where $\tilde{h}^{T_n}$ is the sequence of attended attributes specified for decision maker n with a deliberation time T_n.

3. RESULTS AND DISCUSSION

We analyze the performance of the new model when compared to conventional models in three simulated case studies and one stated preference study. The aim is to test the explanatory power of the RUM-DFT model and to verify whether the parameters are recovered correctly compared to other conventional models.

RUM-DFT-IS

First, 20 simulations and estimates of database A were generated considering the RUM-DFT-IS model, in which the sequence of attributes attended is known. Given this knowledge regarding attribute attendance, it is not necessary to calculate the probabilities of deciding to choose or continue with the information search process at each step t.

Table 1: Average of RUM-DFT model estimates considering the attributes attended.

20 Iterations	RUM-DFT-IS			
(* p<0.05)	Value	Estimate	S.E	$t(\beta)$
β_c	-0.5	-0.492	0.020	0.888
β_{tt}	-2.0	-1.986	0.128	0.771
U_A^0	-0.5	-0.486	0.271	0.655
U_B^0	-0.5	-0.512	0.316	0.757
U_D^0	-1.0	-1.031	0.318	1.237
α	0.6	0.598	0.056	0.757
LL	-4,046.047			
$\bar{\rho}^2$	0.269			
AIC	8,102.1			
BIC	8,134.6			
Time [min]	2.6 (0.2)			

Table 1 shows the average of these estimates and from these results it is possible to see that statistically significant parameters are obtained in most of the cases. Moreover, in all the iterations β_t , β_c and α are statistically significant, the latter being the estimate with the highest efficiency. Therefore, the RUM-DFT-IS model can be applied to process data independent of the generated attributes that define the route choice situation in each iteration. It should be highlighted that this approach makes it possible to incorporate the attended attributes explicitly at each step of the deliberation process into the modelling. It will also be able to deliver estimates of the coefficients of the attributes plus the deliberation process correctly in magnitude, with expected signs and relative importance, allowing the decision-makers to represent the information search process adequately.

RUM-DFT-DT

The second specification considers that the deliberation time of the respondents is known, but not the sequence of attributes attended. This analysis compares the proposed model with a DFT model that includes the total number of fixations in the attention weights. At this level, it is reasonable to compare these approaches as they both use fixations in an aggregated form to represent deliberation time. To estimate these models, we simulated a number of fixations for each choice task for all decision-makers.

Table 2 shows the estimation results of the RUM-DFT-DT model. It is worth mentioning that, like the previous specification, it is not feasible to estimate the parameters of the deliberation process, but the memory factor can be known. Based on the results, the estimated parameters are close to the real value with which they were generated, being the memory factor (α) and the coefficient associated to the travel time (β_t) the most efficient ones.

Table 2: Estimation results of RUM-DFT and scaled DFT model including the aggregate fixations of the deliberation process.

1 Iteration (* $p < 0.05$)	Value	RUM-DFT-DT	DFT-Scaled-DT1	DFT-Scaled-DT2
β_c	-0.5	-0.510*		0.411*
β_t	-2.0	-2.046*		1.000
γ_c			-1.069*	
γ_t			0.000	
U_A^0/P_A^0	-0.5	-0.772*	1.429	0.930
U_B^0/P_B^0	-0.5	-0.211*	2.337*	1.418
U_D^0/P_D^0	-1.0	1.258*	-1.514	-0.885
α	0.6	0.670*		
ϕ_1			0.781	0.105
ϕ_2			-0.016	-0.106
σ_ε			6.311*	4.560*
τ			1.875*	0.876
α_f			0.418*	0.496*
LL		-4,153.369	-4,205.220	-4,206.560
$\overline{\rho}^2$		0.250	0.240	0.240
AIC		8,318.738	8,428.440	8,431.120
BIC		8,356.502	8,485.086	8,487.766
Time [min]		21.3	4.1	3.3

RUM-DFT-SC

The third application corresponds to a case in which neither the sequence of attributes attended, nor the deliberation time, is known by the researcher. This is the case with most SP and RP data sources.

Table 3 shows the estimation results of the RUM-DFT-SC, DFT-scaled-1, DFT-scaled-2 (not including fixations) and MNL model. It is observed that the proposed model presents the best log-likelihood and lower values in the AIC and BIC information criteria than the rest of the approaches. The cost and travel time parameters (attention weights in DFT) differ significantly from zero in all models. However, only the RUM-DFT-SC model delivers close values, in magnitude and sign, to the true ones. The alternative-specific constants cannot be recovered correctly in most cases. Therefore, testing the proposed model with the methodological improvement mentioned in the previous section is necessary.

Table 3: Estimation results of RUM-DFT-SC and scaled DFT model.

Parameters (* $p<0.05$)	Value	RUM-DFT-SC T_{max}=8 steps	DFT-Scaled-1	DFT-Scaled-2	MNL
β_c	-0.5	-0.476*		0.000	-0.295*
β_t	-2.0	-1.959*		0.844*	-0.732*
γ_c			1.000		
γ_t			0.838*		
U_A^0/P_A^0	-0.5	-1.779	-0.469	-0.746	0.000
U_B^0/P_B^0	-0.5	0.444	-0.596	-2.904	-0.132*
U_D^0/P_D^0	-1.0	-3.223	0.000	-1.283	-0.047
δ	0.001	0.093			
ϕ_1			25.545*	2.557	
ϕ_2			-0.114*	-0.016	
σ_ε			6.881*	7.887*	
τ			2.780*	2.969*	
LL		-4,221.788	-4,385.14	-4,386.42	-4,379.93
$\bar{\rho}^2$		0.238	0.209	0.208	0.210
AIC		8,455.58	8,786.27	8,788.84	8,769.87
BIC		8,493.34	8,836.62	8,839.2	8,801.34
Time [min]		41.2	4.2	3.5	0.2

Empirical application to SwissMetro Data

This section compares the RUM-DFT-SC against the DFT-Scaled, RUM, C RRM, mu RRM and P RRM using the SwissMetro database (Bierlaire et al.,2001). The DFT model shown corresponds to the one that estimates the attention weights (Hancock et al.,2018). However, the memory value is fixed to 0, the sensitivity to 0, and the error term to 1 since these values are generally insignificant when there is no information on the deliberation of the respondents. The RUM approach corresponds to a Multinomial Logit, and the last three are variants of the RRM model in which the depth of regret is incorporated (van cranenburgh et al., 2015).

The results shown in Table 4 demonstrate that the models that include parameters that model the deliberation process, both the RUM-DFT model and the DFT approach, have a better performance than the rest, in terms of log-likelihood, AIC and BIC information criteria. However, only from the proposed model is it possible to make an economic interpretation of the coefficients of the attributes, with the tolerance, memory and attention weight also allowing for interpretation of the information search process.

Table 4: Model estimates applied to real SwissMetro data.

Parameters (* p¡0.05)	RUM-DFT-SC T_{max}=7 steps	RUM-DFT-SC T_{max}=10 steps	RUM MNL	DFT	C RMM[1]	μ RRM[1]	P RRM[1]
β_c	-3.745*	-3.657*	-1.150*		0.010*	0.010*	0.010*
β_t	-4.665*	-4.649*	-1.270*		0.010*	0.010*	0.010*
γ_c				-0.867*			
γ_t				-1.251*			
U^0_{Train}	-0.902*	-0.904*	-1.168	-1.120*	N.R.	N.R.	N.R.
U^0_{Car}	-0.010	-0.013	0.250	-0.076*	N.R.	N.R.	N.R.
δ	0.273*	0.259*					
α	0.959^2	0.931^2					
ϕ_{time}	0.467*	0.482*					
ϕ_1				0.000			
ϕ_2				0.000			
σ_ε				1.000			
τ				0.564*			
LL	-4,233.941	-4,233.918	-4,382.500	-4,277.170	-4,539.672	-4,373.356	-4,418.252
$\overline{\rho}^2$	0.312	0.312	0.305	0.305	0.263	0.290	0.283
AIC	8,479.882	8,479.820	8,772.712	8,564.340	9,087.265	8756.452	8844.482
BIC	8,519.673	8,519.611	8,799.626	8,597.499	9,113.618	8,790.094	8,870.835
Time [min]	$59.7^2 + 8.3^3$	$301.9^2 + 3.4^3$	0.03	7.03	N.R.	N.R.	N.R.

[1]Estimates from (Belgiawan et al., 2017).
[2]Estimated using DEoptim.
[3]Estimated using OptimParallel setting α.

4. CONCLUSIONS

This work meets the initially stated general objective since it was possible to incorporate the characteristics of the deliberation process implicit in the public transport route choices as methodological improvements in the development of the Random Utility Maximization model considering the information search process (RUM-DFT)

The results of the RUM-DFT model are promising for the first simulated case study, as this specification recovers the parameters, and the utilities behave as expected. This is mainly because this approach avoids the integration in the space of all possible sequences of attributes, which considerably increases its dimension at each additional step (K^t). For the other simulated cases, the model provides attribute parameters close to the real values. However, this is not the case for the initially preconceived utilities.

On the other hand, the RUM-DFT model was estimated using the SwissMetro database. Signs, magnitudes and significance of parameters, goodness-of-fit indicators and estimation time were compared with the classical models. In general, the models incorporating the assumptions supported in this paper obtain the best fit indicators. Moreover, the RUM-DFT model specification, which in this case does not include information about the deliberation process, can still significantly estimate the attribute, tolerance and recall coefficients with reasonable values, successfully outperforming the DFT and MNL approaches.

REFERENCES

Bierlaire, M., Axhausen, K., Abay, G., 2001. The acceptance of modal innovation: The case of swissmetro, in: Swiss transport research conference.

Busemeyer, J.R., Townsend, J.T., 1992. Fundamental derivations from decision field theory. Mathematical Social Sciences 23, 255–282.

Busemeyer, J.R., Townsend, J.T., 1993. Decision field theory: a dynamic-cognitive approach to decision making in an uncertain environment. Psychological review 100, 432.

Chorus, C., 2012. Random regret minimization: an overview of model properties and empirical evidence. Transport reviews 32, 75–92.

Chorus, C.G., 2010. A new model of random regret minimization. European Journal of Transport and Infrastructure Research 10.

Chorus, C.G., Arentze, T.A., Timmermans, H.J., 2008. A random regret-minimization model of travel choice. Transportation Research Part B: Methodological 42, 1–18.

Diederich, A., 1997. Dynamic stochastic models for decision making under time constraints. Journal of Mathematical Psychology 41, 260–274.

Gabaix, X., Laibson, D., Moloche, G., Weinberg, S., 2006. Costly information acquisition: Experimental analysis of a boundedly rational model. American Economic Review 96, 1043–1068.

Gao, S., Frejinger, E., Ben-Akiva, M., 2011. Cognitive cost in route choice with real-time information: An exploratory analysis. Procedia-Social and Behavioral Sciences 17, 136–149.

Guevara, C.A., 2015. Critical assessment of five methods to correct for endogeneity in discrete-choice models. Transportation Research Part A: Policy and Practice 82, 240–254.

Hancock, T.O., Hess, S., Choudhury, C.F., 2018. Decision field theory: Improvements to current methodology and comparisons with standard choice modelling techniques. Transportation Research Part B: Methodological 107, 18–40.

Hancock, T.O., Hess, S., Choudhury, C.F., 2022. Secret in their eyes': Incorporating eye-tracking and stress indicator data into travel behaviour models Working paper University of Leeds.

Hancock, T.O., Hess, S., Marley, A.A., Choudhury, C.F., 2021. An accumulation of preference: two alternative dynamic models for under- standing transport choices. Transportation Research Part B: Methodological 149, 250–282.

Horstmann, N., Ahlgrimm, A., Gl ̈ockner, A., 2009. How distinct are intuition and deliberation? an eye-tracking analysis of instruction-induced decision modes. An Eye-Tracking Analysis of Instruction-Induced Decision Modes (April 1, 2009). MPI Collective Goods Preprint.

Noguchi, T., Stewart, N., 2014. In the attraction, compromise, and similarity effects, alternatives are repeatedly compared in pairs on single dimensions. Cognition 132, 44–56.

Nova, G., Guevara, C.A., 2022. In depth, breadth-first or both? characterising the information search process in a public transport sp experiment Working paper University of Chile.

Roe, R.M., Busemeyer, J.R., Townsend, J.T., 2001. Multialternative decision field theory: A dynamic connectionst model of decision making. Psychological review 108, 370.

Stewart, N., Gächter, S., Noguchi, T., Mullett, T.L., 2016a. Eye movements in strategic choice. Journal of behavioral decision making 29, 137–156.

Stewart, N., Hermens, F., Matthews, W.J., 2016b. Eye movements in risky choice. Journal of Behavioral Decision Making 29, 116–136.

Sui, X.Y., Liu, H.Z., Rao, L.L., 2020. The timing of gaze-contingent decision prompts influences risky choice. Cognition 195, 104077.

Swait, J., Adamowicz, W., 2001. The influence of task complexity on consumer choice: a latent class model of decision strategy switching. Journal of Consumer Research 28, 135–148.

van Cranenburgh, S., Guevara, C.A., Chorus, C.G., 2015. New insights on random regret minimization models. Transportation Research Part A: Policy and Practice 74, 91–109.