**On fair discounted charging in electric ride-hailing markets with limited budgets**

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## Short summary

Coordinated charging of electric vehicles (EVs) has the potential to provide significant benefits to both electric vehicle owners and the wider community. In fact, intelligent, coordinated charging of large electric fleets, such as the ones operated by ride-hailing companies, could be essential in preventing a collapse of the energy market. We study a scenario in which a central body, e.g., the power-providing company or the government, wants to influence how the EVs of different ride-hailing companies spread among different charging stations by offering discounted prices of charging. Compared to previous works in this domain, we investigate a Stackelberg-based mechanism that takes into account potentially limited discount budgets available to the companies. We propose an iterative method to compute the local Stackelberg equilibrium that guarantees fairness in the sense that we have equal prices of charging for all ride-hailing companies. Finally, we test the proposed method in a simulated case study based on taxi data from the city of Shenzhen.

**Keywords**: Stackelberg game, Electric vehicle charging, Ride-hailing operation

## 1 Introduction

The increased popularity of electric vehicles (EV) and the steep incline in their number Internationa Energy Agency (2021) opened a significant amount of questions in the domain of electric energy management and electric mobility. On one hand, this widespread adoption of electric vehicles has led to a growing need for efficient and reliable charging infrastructure. Combining this with the increasing demand for electricity from EV charging puts the spotlight on the problem of efficient, coordinated charging. On the other hand, combining smart mobility systems and intelligent charging management could pave the way for gaining an opportunity to trade different services to achieve societal optimum, e.g., by providing discounted charging in off-peak hours, it could help improve the stability of the electricity grid by reducing the variability in demand. Moreover, given that ride-hailing companies now constitute the central part of the services offered within a city, it is to be expected that they also electrify their fleets. Hence, the impact of coordinated charging of a large number of vehicles operated by ride-hailing companies can be significant in preventing the collapse of the energy market. Ride-hailing companies already offer access to different service facilities to their drivers so it is not unlikely that they would also offer discounted charging in accordance with the received monetary subsidies aimed at incentivizing the drivers to follow the management's desires. That way, the companies, in collaboration with external financiers, could hope to improve the overall utilization rate of their fleets by increasing the availability of the vehicles or to motivate the drivers to charge in distant areas in an attempt to increase the coverage.

With this in mind, we study a scenario in which a central body, e.g., the power-providing company or the government, has a desire about how the vehicles should spread out among different charging stations in a region where the operators of several ride-hailing companies try to minimize their operational costs, depicted in Figure 1. We assume the charging infrastructure is shared, so the ride-hailing companies are inherently interested in directing their vehicles to different charging stations so as to minimize the queuing time at the stations. Moreover, we assume the central authority has the power to set the prices of charging at different stations and hence, tries to influence how the companies behave in an attempt to attain a personal objective. Since all the agents in the system compete for the resources, this opens the door for a game-theoretic analysis.

There is extensive literature on game theoretic models based on congestion, mean-field, Stackelberg and Inverse Stackelberg games used to solve different problems in the domain of smart mobility systems Basar & Srikant (2002); Brown & Marden (2020); Groot et al. (2012); Laha et al. (2019); Ma et al. (2013); Paccagnan et al. (2016, 2019); Staňková et al. (2011); Tushar et al. (2012); J. Zhang et al. (2018); L. Zhang et al. (2019). This paper, in particular, is a continuation of our previous works Maljkovic et al. (2022a,b), where we analyzed a game-theoretic model of a similar structure as the ones listed in the literature. We did so, however, from the perspective of designing demand-based, feedback pricing policies that guaranteed the exact minimization of the central authority's objective, making our work fall in the category of Inverse Stackleberg games. With this work, however, we aim to address the problem of potentially unfair prices induced by the optimal charging policies proposed in Maljkovic et al. (2022a). Namely, we study a Stackelberg game setup that assumes a game with limited budgets is played between the central authority and the ride-hailing companies. Based on Maljkovic et al. (2023), we propose an iterative method to compute a fixed discount for the ride-hailing companies that aligns well with the individual discount budget constraints. That way, we provide complete fairness in the sense of having the same prices for every company at the expense of being able to guarantee convergence only to the local Stackelberg equilibrium of the game. At the end, we test the proposed method in a simulated case study based on real taxi data from the city of Shenzhen in China.

The paper is outlined as follows: the rest of this section is devoted to introducing some basic notation. In Section 2, we revise the structure of the model with limited budgets and introduce the iterative method for computing the local Stackelberg equilibrium. In the following section, Section 3, we demonstrate the effectiveness of the proposed method in a simulated case study based on the city of Shenzhen. The final section contains the concluding remarks and some ideas for future research.

### *Notation*

Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of non-negative reals, and $\mathbb{Z}_+$ the set of non-negative integers. Let $\mathbf{0}_m$ and $\mathbf{1}_m$ denote the all zero and all one vectors of length $m$ respectively, and $\mathbb{I}_m$ the identity matrix of size $m \times m$. For a finite set $\mathcal{A}$, we let $\mathbb{R}^{\mathcal{A}}_{(+)}$ denote the set of (non-negative) real vectors indexed by the elements of $\mathcal{A}$ and $|\mathcal{A}|$ the cardinality of $\mathcal{A}$. Furthermore, for finite sets $\mathcal{A}$, $\mathcal{B}$ and a set of $|\mathcal{B}|$ vectors $x^i \in \mathbb{R}^{\mathcal{A}}_{(+)}$, we define $x := \mathrm{col}\left((x^i)_{i\in\mathcal{B}}\right) \in \mathbb{R}^{|\mathcal{A}||\mathcal{B}|}$ to be their concatenation. For $A \in \mathbb{R}^{n\times n}$, $A \succ 0 (\succeq 0)$ is equivalent to $x^T A x > 0 (\geq 0)$ for all $x \in \mathbb{R}^{n\times n}$. We let $A \otimes B$ denote the Kronecker product between two matrices and for a vector $x \in \mathbb{R}^n$, we let $\mathrm{Diag}(x) \in \mathbb{R}^{n\times n}$ denote a diagonal matrix whose elements on the diagonal correspond to vector $x$. For a differentiable function $f(x) : \mathbb{R}^n \to \mathbb{R}^m$, we let $\mathbf{D}_x f \in \mathbb{R}^{m\times n}$ denote the Jacobian matrix of $f$ defined as $(\mathbf{D}_x f)_{ij} := \frac{\partial f_i}{\partial x_j}$. Finally, for a set-valued mapping $\mathcal{F} : \mathbb{R}^n \rightrightarrows \mathbb{R}^m, \mathrm{gph}(\mathcal{F}) := \{(y, x) \in \mathbb{R}^n \times \mathbb{R}^m \mid x \in \mathcal{F}(y)\}$ denotes its graph.

## 2 Methodology

### *Model*

We begin explaining the proposed methodology by introducing the system model. Let us consider a region where multiple shared charging stations are available for the EV drivers, i.e., let $\mathcal{M}$ be the set of all charging stations such that $|\mathcal{M}| = m$ and $M_j > 0$ denotes the capacity of the charging station $j \in \mathcal{M}$. Let the set of all ride-hailing companies be denoted as $\mathcal{I}$. Let the cardinality of the set of companies be $|\mathcal{I}| = N$ and for every $i \in \mathcal{I}$, let the number $N_i > 0$ represent the number of vehicles that want to recharge. For every company $i \in \mathcal{I}$, let the vector $x^i \in \mathcal{X}_i \subseteq \mathbb{R}^m$ describe the ride-hailing fleet split among charging stations. Namely, let $\left\|x^i\right\|_1 = N_i$ and $x^i_j \geq 0$ denote the number of vehicles to be directed to station $j \in \mathcal{M}$. Moreover, if we define the sets $\mathcal{X} := \prod_{i\in\mathcal{I}} \mathcal{X}_i$ and $\mathcal{X}_{-i} := \prod_{j\in\mathcal{I}\setminus i} \mathcal{X}_j$, then the joint strategy of all followers can be denoted as $x := \mathrm{col}\left((x^i)_{i\in\mathcal{I}}\right) \in \mathcal{X}$ and for every agent $i \in \mathcal{I}$ we can define $x^{-i} := \mathrm{col}\left((x^j)_{j\in\mathcal{I}\setminus i}\right) \in \mathcal{X}_{-i}$.

Let the nominal prices of charging at different stations be encoded in vector $\pi_{\text{base}} \in \mathbb{R}^M$. Furthermore, let us assume that the central authority is interested in determining the optimal discount $\Delta\pi \in \mathbb{R}^M$, such that for every $i \in \mathcal{I}$, the total monetary discount that the company $i$ receives does not exceed a predefined value $B_i \in \mathbb{R}$. Here, $B_i$ represents the limited discount budget of company $i$, which corresponds to the level of external subsidies that the company is entitled to.

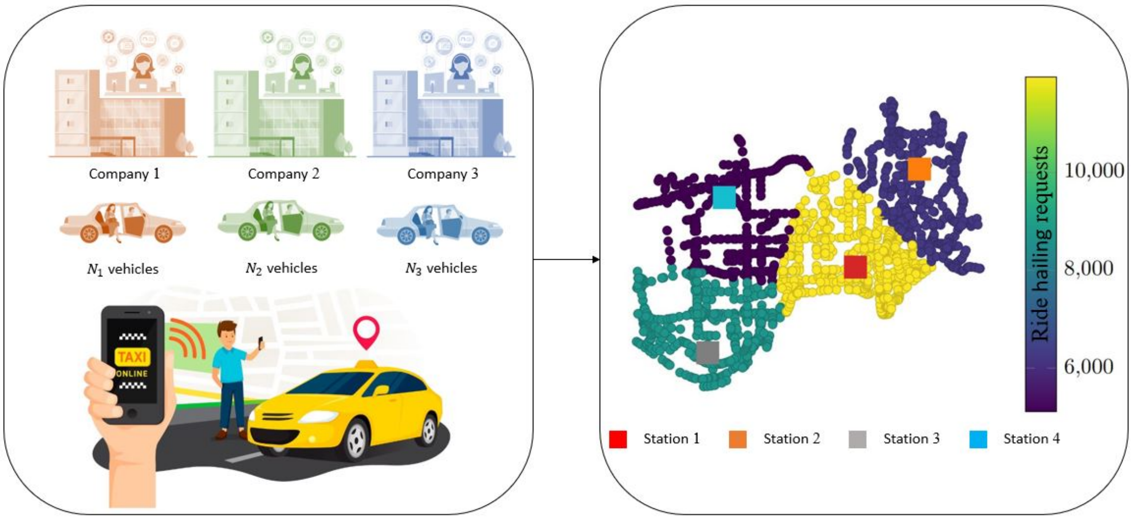


Figure 1: **Ride-hailing market in a region with 4 charging stations** - $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4\}$ and the network topology used in the case study consists of 1858 intersections connected by 2013 road segments, divided in 4 regions based on the Voronoi partitioning of the city with the centroids located at the charging stations.

Similar to the cost models introduced in Maljkovic et al. (2022a,b), we assume that based on the announced pricing policy $\pi \in \mathcal{P} \subseteq \mathbb{R}^M$, such that $\pi := \pi_{\text{base}} - \Delta\pi$, the ride-hailing companies choose their strategies in an attempt to minimize personal objective functions $J^i(x^i, x^{-i}, \pi)$ by playing the best response to the other agents' strategies, as illustrated in Figure 2. For every company $i \in \mathcal{I}$, let $\sigma\left(x^{-i}\right) := \sum_{j \in \mathcal{I} \setminus i} x^j$ be the aggregate decisions of the other players and let $\sigma\left(x\right) := \sum_{j \in \mathcal{I}} x^j$. Then, every company operator is interested in minimizing its operational cost under the feasibility constraints imposed by the battery status of its vehicles. Inspired by the objective functions analyzed in Maljkovic et al. (2022a,b); Tushar et al. (2012); Yu et al. (2021); Zavvos et al. (2022), here, we analyze the operator's cost that consists of three terms

$$J^i\left(x^i, \sigma\left(x^{-i}\right), \pi\right) = J_1^i\left(x^i, \sigma\left(x^{-i}\right)\right) + J_2^i\left(x^i\right) + J_3^i\left(x^i, \pi\right) \ ,$$

such that $J_1^i\left(x^i, \sigma\left(x^{-i}\right)\right)$ denotes the expected queuing cost, $J_2^i\left(x^i\right)$ denotes the negative expected revenue and $J_3^i\left(x^i, \pi\right)$ denotes the charging cost.

**Expected queuing cost model** is governed by the cost term of the form:

$$\begin{aligned} J_1^i\left(x^i, \sigma(x^{-i})\right) &= \left(x^i\right)^T Q\left(x^i + \sigma(x^{-i}) - M\right) \\ &= \left(x^i\right)^T Q\left(\sigma(x) - M\right) \ , \end{aligned} \tag{1}$$

where $M \in \mathbb{R}^M$ is the vector of charging station capacities, i.e., $M = \operatorname{col}((M_j)_{j \in \mathcal{M}})$, and $Q = \operatorname{Diag}(q) \in \mathbb{R}^{M \times M}$ is a positive definite scaling matrix such that every element $q_j > 0$ depicts how expensive it is for a vehicle to queue in the region around charging station $j \in \mathcal{M}$. The charging stations located in the city's more busy areas should experience higher queuing costs and hence have a higher corresponding diagonal entry in the $Q$ matrix. Moreover, the more the capacity of the station is exceeded, the higher the cost per vehicle should be, which is directly enabled through the inner product with the vector $\sigma\left(x\right) - M$. To take into account the total queuing cost for the whole fleet, we calculate the inner product between the vector describing the fleet's distribution, i.e., $x^i$, and the incurred cost per vehicle for choosing a particular station, i.e., $Q\left(\sigma(x) - M\right)$.

**The negative expected revenue** is modeled as:

$$J_2^i\left(x^i\right) = \left(e_i^{\text{arr}}\right)^T x^i - \left(e_i^{\text{pro}}\right)^T x^i \, , \tag{2}$$

where $e_i^{\text{arr}} \in \mathbb{R}^M$ is the average cost of a vehicle being unoccupied while traveling to a station and the vector $e_i^{\text{pro}} \in \mathbb{R}^M$ is the expected profit per vehicle, should the vehicle choose to stay in a region around a particular charging station, estimated from historical data.

**The charging cost** is modeled as:

$$J_3^i\left(x^i, \pi\right) = \pi^T S_i x^i \, , \tag{3}$$

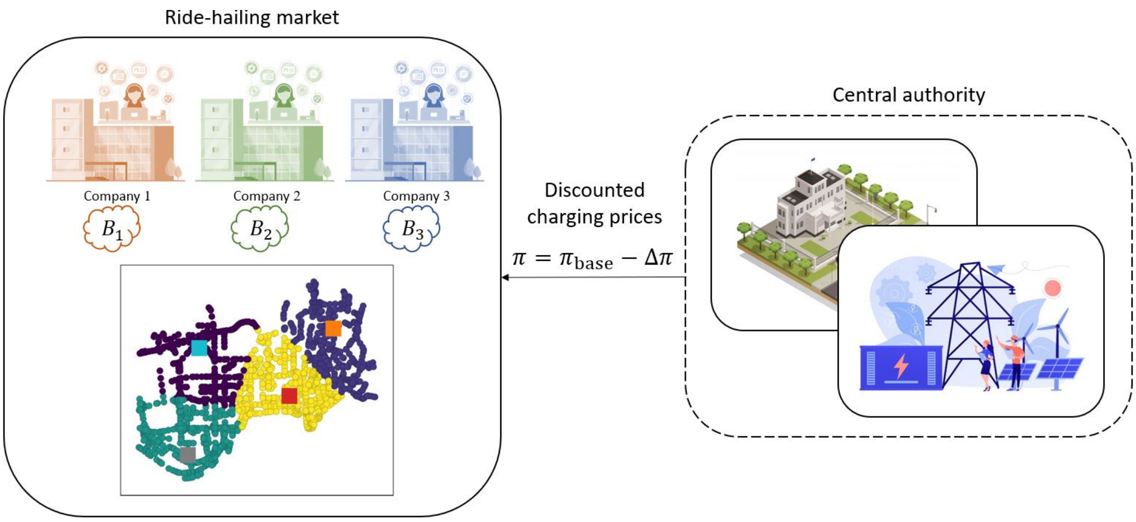


Figure 2: Schematic overview of the interaction between the ride-hailing market and the central authority, i.e., the power-providing company or the government.

where, the matrix $S_i \in \mathbb{R}^{M \times M}$ is diagonal, i.e., $S_i = \text{Diag}\left(d^i\right) \succeq 0$, and every element $d^i_k \in \mathbb{R}_+$ of the vector $d^i \in \mathbb{R}^M_+$ can be interpreted as the expected average charging demand per vehicle when choosing the station $k$. Therefore, the total operational cost of the company can be written in a general form given by:

$$J^i\left(x^i, \sigma\left(x^{-i}\right), \pi\right) = \frac{1}{2}\left(x^i\right)^T P_i x^i + \left(x^i\right)^T Q_i \sigma\left(x^{-i}\right) + r_i^T x^i + \pi^T S_i x^i \,, \tag{4}$$

where $P_i := 2Q$, $Q_i := Q$ and $r_i := e_i^{\text{arr}} - e_i^{\text{pro}}$. Regarding the constraint sets of the ride-hailing companies, it has been shown that for a particularly constructed polytopic constraint, it will always be possible to match every ride-hailing vehicle with exactly one charging station in an attempt to respect the optimal allocation given by the optimal split $x^*$. For every $i \in \mathcal{I}$, the matching constraints in accordance with Maljkovic et al. (2022a) are given by

$$\mathcal{X}_i^m := \left\{x^i \in \mathbb{R}^M \mid A_i x^i = b_i \wedge G_i x^i \leq h_i\right\} . \tag{5}$$

Apart from them, in this paper we also focus on budget constraints. As previously mentioned, for every company $i \in \mathcal{I}$, the total discount budget for the electricity prices that the central authority can provide is $B_i$. Taking into account that the discount is given by $\Delta\pi = \pi_{\text{base}} - \pi$, for a particular pricing strategy $\pi \in \mathcal{P}$, the budget constraint can be described by $\mathcal{X}_i^b\left(\pi\right)$

$$\mathcal{X}_i^b\left(\pi\right) := \left\{x^i \in \mathbb{R}^M \mid \left(\pi_{\text{base}} - \pi\right)^T S_i x^i - B_i \leq 0\right\} , \tag{6}$$

which is also a polytopic constraint in $x^i$. Hence, for a pricing strategy $\pi \in \mathcal{P}$ and for every $i \in \mathcal{I}$, the resulting constraint set is given by $\mathcal{X}_i = \mathcal{X}_i^m \cap \mathcal{X}_i^b\left(\pi\right)$. It is worth mentioning that for particular choices of parameters $S_i$, $\pi_{\text{base}}$ and $B_i$ the set $\mathcal{X}_i^b\left(\pi\right)$ could be empty and, hence, the optimization problem would be infeasible. Therefore, for future analysis, we assume that $\mathcal{X}_i^b\left(\pi\right) \neq \varnothing$.

On the other hand, we assume the central authority is interested in balancing the vehicles so as to minimize the personal objective of the form:

$$\min_{\sigma(x)} J_G(\sigma\left(x\right)) = \min_{\sigma(x)} \frac{1}{2}\sigma\left(x\right)^T A_G \sigma\left(x\right) + b_G^T \sigma\left(x\right) \,, \tag{7}$$

for some diagonal matrix $A_G \succ 0$ and $b_G \in \mathbb{R}^M$. In particular, in this paper, we focus on minimizing a special case of (7) that corresponds to balancing the vehicles so as to match a predefined vehicle distribution given by vector $\mathcal{Z} \in [0,1]^M$ with $\mathbf{1}^T \mathcal{Z} = 1$, i.e., to minimize

$$J_G(\sigma\left(x\right)) = \frac{1}{2}\|\sigma(x) - \mathbf{1}^T n \mathcal{Z}\|_2^2 \,, \tag{8}$$

where $n = \text{col}\left(\left(N_i\right)_{i \in \mathcal{I}}\right)$ is the vector containing the number of vehicles per company that need to be recharged. Having defined the system model, we continue to present the theoretical preliminaries.

***Theoretical preliminaries***

The central authority and the ride-hailing companies admit a single-leader, multiple-follower Stackelberg game with the leader being the central authority. Upon the announcement of the leader's strategy, the aggregative game between the ride-hailing companies is described by

$$G_0(\pi) := \left\{ \min_{x^i \in \mathcal{X}_i} J^i\left(x^i, \sigma\left(x^{-i}\right), \pi\right), \forall i \in \mathcal{I} \right\}, \tag{9}$$

whose Nash equilibrium $x^*$ is given in the definition below:

**Definition 1 (Nash equilibrium)** *For any pricing strategy $\pi \in \mathcal{P}$ of the central authority, a joint strategy $x^* \in \mathcal{X}$ is a Nash equilibrium of the game $G_0$, if for all $i \in \mathcal{I}$ and all $x^i \in \mathcal{X}_i$ holds*

$$J^i\left(x^{i*}, x^{-i*}, \pi\right) \leq J^i\left(x^i, x^{-i*}, \pi\right).$$

We focus our attention on the subset of general Nash equilibria given by Definition 1, called the Variational Nash equilibria (v-NE), because different methods in the literature facilitate their decentralized computation Grammatico et al. (2016). Based on the theory of variational inequalities Harker & Pang (1990), if we define a map $F : \mathcal{X} \times \mathcal{P} \to \mathbb{R}^{NM}$ as

$$F(x, \pi) := \mathrm{col}\left(\left(\nabla_{x^i} J^i\left(x^i, x^{-i}, \pi\right)\right)_{i \in \mathcal{I}}\right),$$

then the set of v-NE of the game $G_0(\pi)$ is given by $\mathcal{V}_0(\pi)$:

$$\mathcal{V}_0(\pi) := \left\{ x \in \mathcal{X} \mid (y - x)^T F(x, \pi) \geq 0, \ \forall y \in \mathcal{X} \right\}.$$

With this in mind, we now proceed to state the existence and uniqueness result for $G_0(\pi)$.

**Proposition 1** *For any $\pi \in \mathcal{P}$, let the game $G_0(\pi)$ between the ride-hailing companies be defined as in* (9)*. Moreover, for every company $i \in \mathcal{I}$, let the constraint sets $\mathcal{X}_i$ be defined as $\mathcal{X}_i = \mathcal{X}_i^m \cap \mathcal{X}_i^b(\pi)$, with $X_i^m$ defined in* (5) *and $\mathcal{X}_i^b(\pi)$ defined in* (6)*. If the company operator's objective is defined by* (1), (2), (3) *and* (4)*, then the game $G_0(\pi)$ admits a unique v-NE joint strategy $x^* \in \mathcal{X}$.*

**Proof** Since $P_i \succ 0$ for every $i \in \mathcal{I}$, the agents' cost functions are convex in $x^i$. For $\mathcal{X}_i$ defined as $\mathcal{X}_i = \mathcal{X}_i^m \cap \mathcal{X}_i^b(\pi)$, with $X_i^m$ as in (5) and $\mathcal{X}_i^b(\pi)$ as in (6), based on (Rosen, 1965, T.1), there exists a Nash equilibrium of the game $G_0(\pi)$. A sufficient condition for the uniqueness of the Nash equilibrium is that the operator $F(x, \pi)$ be strictly monotone in $x$ (Facchinei & Pang, 2007, Ch.2). The pseudo gradient can be written as $F(x, \pi) = F_1 x + F_2$, such that $F_1 = \mathbb{I}_N \otimes Q + \mathbf{1}_N \mathbf{1}_N^T \otimes Q$ and $F_2 = \mathrm{col}\left((r_i + S_i \pi)_{i \in \mathcal{I}}\right)$. To show that $F(x, \pi)$ is strictly monotone, it suffices to prove that $F_1 \succ 0$ Bauschke & Combettes (2017). This is true as for any $x \in \mathcal{X}$, it holds that $x^T F_1 x = \sum_{i \in \mathcal{I}} \left(x^i\right)^T Q x^i + \left(\sum_{i \in \mathcal{I}} x^i\right)^T Q \left(\sum_{i \in \mathcal{I}} x^i\right) > 0$.

Since the unique v-NE can be computed using the Picard-Banach fixed point iteration Berinde (2004), we can now formally define the Stackelberg game played between the central authority and the ride-hailing market. Moreover, we can introduce the notion of the local Stackelerg equilibrium (l-SE) that we wish to compute for this hierarchical game structure.

**Definition 2 (Stackelberg game)** *Let the game between the ride-hailing companies be defined as in Proposition 1 and the central authority's objective be defined as* (8)*. Then the Stackelberg game is defined by a bi-level optimization problem*

$$G_L := \left\{ \begin{array}{c} \min_{\pi \in \mathcal{P}} J^L(x^*, \pi) = \frac{1}{2} \left\| \sigma(x^*) - \mathbf{1}^T n \mathcal{Z} \right\|_2^2 \\ s.t. \ x^* \in \mathcal{V}_0(\pi) \end{array} \right\}. \tag{10}$$

In general, there could exist multiple Srtackelberg equilibria that solve the game given by (10). Therefore, we shift our focus towards computing the local Stackleberg equilibria given by the following definition and previously analyzed in Fabiani et al. (2022).

**Definition 3 (Local Stackelberg equilibrium)** *Let $G_L$ be a game as in Definition 2. A pair $(\hat{x}^*, \hat{\pi}) \in gph(\mathcal{V}_0) \cap (\mathcal{X} \times \mathcal{P})$ is a local Stackelberg equilibrium of $G_L$ if there exist open neighborhoods $\Omega_{\hat{x}^*}$, $\Omega_{\hat{\pi}}$ of $\hat{x}^*$ and $\hat{\pi}$ respectively, such that*

$$J^L(\hat{x}^*, \hat{\pi}) \leq \inf_{(x^*, \pi) \in gph(\mathcal{V}_0) \cap \Omega} J^L(x^*, \pi),$$

*where $\Omega := \Omega_{\hat{x}^*} \times (\mathcal{P} \cap \Omega_{\hat{\pi}})$.*

In the following section, we will present a bi-level, iterative method for computing the local Stackelberg equilibria.

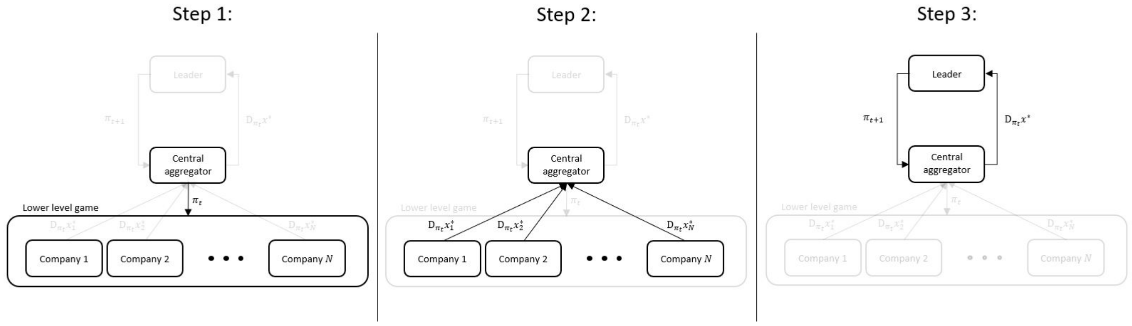


Figure 3: Overview of the three-step iterative procedure used for calculating the local Stackelberg equilibrium.

***Computing the l-SE***

To compute the local Stackelberg equilibria, we build on top of the standard iterative structure used for finding the v-NE of the aggregative game played between the ride-hailing companies. As previously mentioned, based on Proposition 1, we can utilize the Picard-Banach iteration to find the v-NE of $G_0(\pi)$ for a particular $\pi \in \mathcal{P}$. Based on the aggregative structure of the operator's cost, standard methods leverage the entity typically referred to as the 'central aggregator' to transmit the information about $\sigma(x)$ to all the agents in the game during the procedure.

To compute the l-SE, we will perform a gradient-based, iterative procedure that requires communication just between the 'central aggreagator' and the central authority. Namely, we aim to update the central authority's pricing strategy according to

$$\pi_{t+1}\left(\pi_t, s\right) := \Pi_{\mathcal{P}}\left[\pi_t - s \left.\frac{\mathrm{d} J^L\left(x^*\left(\pi\right), \pi\right)}{\mathrm{d}\pi}\right|_{\pi=\pi_t}\right], \tag{11}$$

where $\Pi$ denotes the projection operator, $\pi_t$ is the current value of the central authority's pricing policy and $s$ is the step size carefully determined according to the Armijo step-size rule Bertsekas (1999). If we restrict ourselves to compact and convex pricing spaces $\mathcal{P} \subseteq \mathbb{R}^M$, then the complexity of each update step defined by (11) boils down to estimating how the Nash equilibrium of $G_0(\pi)$ reacts to any change in $\pi$, i.e., calculating the gradient

$$\frac{\mathrm{d} J^L\left(\cdot\right)}{\mathrm{d}\pi} = \frac{\partial J^L\left(\cdot\right)}{\partial \pi} + \sum_{i \in \mathcal{I}} \mathbf{D}_\pi^T x^{i*} \frac{\partial J^L\left(\cdot\right)}{\partial x^{i*}}. \tag{12}$$

Here, it is of paramount importance to show that the Jacobians $\mathbf{D}_\pi^T x^{i*}$ are well defined. To do so, we take into account that the unique v-NE, $x^* \in \mathcal{X}$, pre-computed for the current value of $\pi_t$, has to satisfy the KKT optimality conditions of the best-response optimization problem given by (9) for each ride-hailing company. For every $i \in \mathcal{I}$, let us describe the constraint set $\mathcal{X}_i := \mathcal{X}_i(\pi)$ as

$$\mathcal{X}_i(\pi) = \left\{ x^i \in \mathbb{R}^M \mid \left[\begin{array}{c} G_i \\ \left(\pi_{\text{base}} - \pi\right) S_i \end{array}\right] x^i \leq \left[\begin{array}{c} h_i \\ B_i \end{array}\right] \right\} = \left\{ x^i \in \mathbb{R}^M \mid \Gamma_i x^i \leq \delta_i \right\}. \tag{13}$$

Moreover, let us partition $\Gamma_i$ and $\delta_i$ into active and inactive inequality constraints described by $\overline{\Gamma}_i$, $\underline{\Gamma}_i$, $\overline{\delta}_i$ and $\underline{\delta}_i$ such that

$$\overline{\Gamma}_i x^{i*} = \overline{\delta}_i \text{ and } \underline{\Gamma}_i x^{i*} < \underline{\delta}_i. \tag{14}$$

Then, applying the Implict Function theorem Dontchev & Rockafellar (2009) to the KKT mapping of an equivalent best-response optimization problem with partitioned constraints described by (14), directly yields

$$\mathbf{D}_\pi x^{i*} = -\left[\begin{array}{ccc} \frac{\partial^2}{\partial x_i \partial x_i} J^i & \underline{\Gamma}_i{}^T & \bar{A}_i{}^T \\ \mathbf{0} & \operatorname{Diag}\left(\underline{\Gamma}_i x^{i*} - \underline{\delta}_i\right) & \mathbf{0} \\ \bar{A}_i & \mathbf{0} & \mathbf{0} \end{array}\right]^{-1} \left[\begin{array}{c} S_i \\ \mathbf{0} \\ \mathbf{0} \end{array}\right], \tag{15}$$

where $\overline{A}_i$ is full row-rank and obtained by removing redundant constraints from $A_{\text{total}}^T = \left[A_i^T, \overline{\Gamma}_i^T\right]$. The schematic representation of the three-step procedure is presented in Figure 3. Finally, we can summarize the convergence results in the following proposition.

**Proposition 2** *Let the Stackelberg game between the central authority and the ride-hailing companies be defined as* (10). *Moreover, let the update step of the central authority's pricing strategy be defined by equations* (11), (12), (13) *and* (15) *and the step size $s > 0$ in* (11) *be chosen according to Armijo step-size rule. Then, for $\mathcal{P}$ compact and convex, the following convergence result holds:*

$$\lim_{t \to +\infty} \left[ J^L(\cdot, \pi_{t+1}) - J^L(\cdot, \pi_t) \right] = 0 \,,$$

**Proof** The proof follows directly from applying the Implicit function theorem Dontchev & Rockafellar (2009) and applying the properties of the Armijo step-size rule Bertsekas (1999).

In the following section we illustrate the performance of our algorithm in a simulated case study based on taxi data from the city of Shenzhen.

## 3 Results and discussion

### *Case study*

We begin this section by introducing the case study that was previously analyzed in Maljkovic et al. (2022a). We consider 3 ride-hailing companies $\mathcal{I} = \{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3\}$ with fleet sizes given by $N_{\text{fleet}} = [450, 400, 350]^T$ that operate in the Shenzhen region with 4 public charging stations $\mathcal{M} = \{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4\}$. The stations are described by the vector of their capacities $M = [15, 60, 35, 50]^T$ and are located in parts of Shenzhen with different demands for ride-hailing services as shown in the color-coded map depicted in Figure 1. We consider a 3 hour long simulation that represents one of the two peak-hour periods during the day. New passengers constantly arrive in the system and either increase the number of private vehicles in the system or request a ride-hailing vehicle to be assigned to them. The demand profile represents the real taxi demand that we assume is now served by the ride-hailing companies Beojone & Geroliminis (2021). The congested conditions in the city are represented by modeling the space mean speed of the vehicles as a decreasing function of the total vehicle accumulation $n_{\text{v}}$ in the region and according to the network Macroscopic Fundamental Diagram (MFD) Geroliminis & Daganzo (2008) obtained from Ji et al. (2014). Under the assumption of homogeneous congestion in the city, the MFD of the region is given by:

$$v_{\text{space}}(n_{\text{v}}) = \begin{cases} 36 \exp\left(-\frac{29 n_{\text{v}}}{60000}\right), & \text{if } \frac{n_{\text{v}}}{1000} \leq 36 \\ 6.31 - 0.28\left(\frac{n_{\text{v}}}{1000} - 36\right), & \text{if } 36 < \frac{n_{\text{v}}}{1000} \leq 60 \\ 0, & \text{if } \frac{n_{\text{v}}}{1000} > 60 \end{cases} .$$

To prevent the ride-hailing vehicles from flocking in the busiest parts of the city, the desired distribution of the ride-hailing vehicles $\mathcal{Z}$ is formed so as to match the spatial distribution of the ride-hailing service requests. To approximate this distribution, the city region is divided into 4 cells according to the Voronoi Kang (2008) partitioning of the map. The charging stations are chosen as the centroids of the Voronoi cells, the number of vehicles per company that want to recharge after a 3 hour simulation is given by $n = [194, 181, 157]$, and $\mathcal{Z}$ is chosen to correspond to the total number of requests in each cell. For the analyzed case study, this results in obtaining $\mathcal{Z}$ such that $\mathbf{1}^T n \mathcal{Z} = [198, 103, 144, 87]$ and we set $\mathcal{P} := [p_{\min}, p_{\max}]^4$, such that $p_{\min} = 0.0$ and $p_{\max} = 5.0$. All the remaining parameters in the simulation are kept identical as in Maljkovic et al. (2022a).

### *System performance*

For the Picard-Banach fixed point iteration procedure used to compute the v-NE before each update step of the central authority's pricing strategy, we used $k_{\text{v-NE}} = 5000$ iterations whereas for the iterative procedure between the central authority and the 'central aggregator' we used $k_{\text{l-SE}} = 350$ iterations. The evolution of the achieved total vehicle accumulations and the central authority's objective are shown in Figure 4. For the given number of iterations, the system manages to achieve perfect matching with respect to desired vehicle distribution.

This is further supported by the plot on the right-hand side of Figure 4, which shows that the objective function converges to the global minimum value of 0. Finally, we can investigate the trend in the evolution of the actual discount budget used for each of the ride-hailing companies by looking at Figure 5. The evolution of the discount budget used is presented for the base price of $\pi_{\text{base}} =$

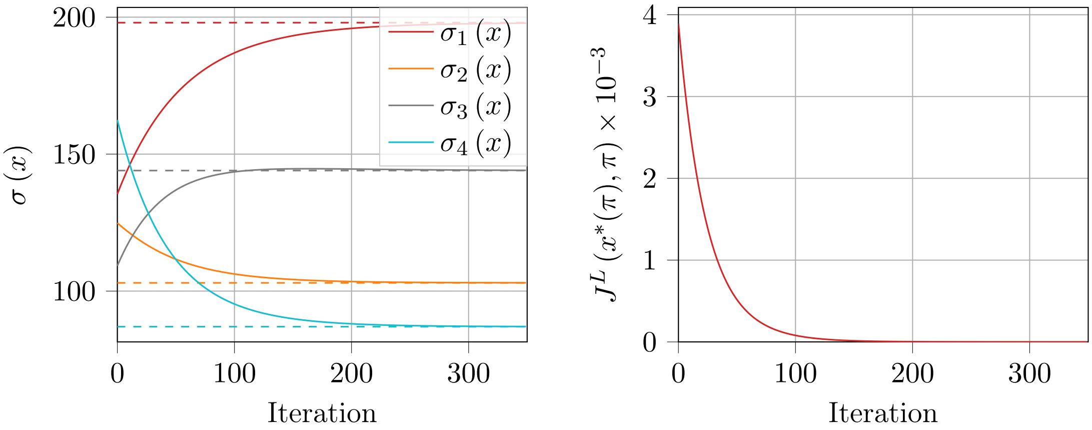


Figure 4: The left plot shows the evolution of the total vehicle accumulation $\sigma(x)$, whereas the right plot shows the evolution of the central authority's objective during the procedure.

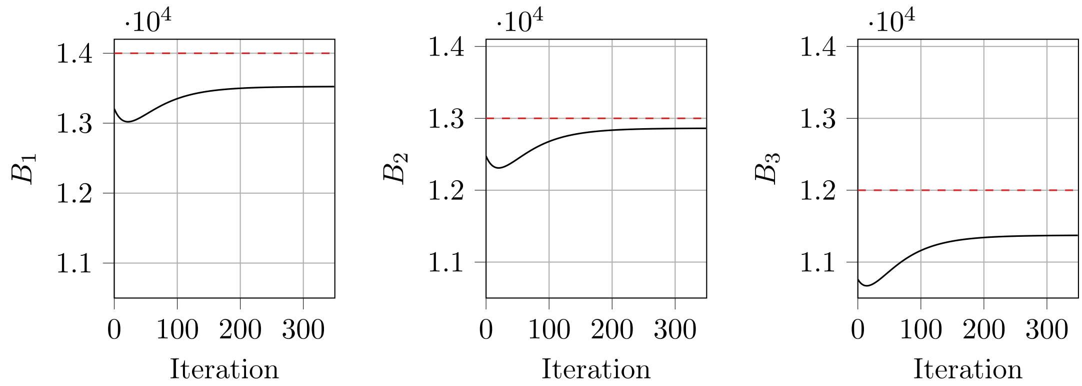


Figure 5: The three plots show the evolution of the value of the used discount budget. The black lines represent the achieved values whereas the red line represents the maximum possible value of the discount budget.

$[5.0, 3.0, 5.0, 3.0]$ and the attained charging discount is given by $\Delta\pi = [1.560, 0.809, 2.179, 1.440]$. Note that in this case, the system was able to recover a solution that matches the global minimum of the central authority's objective. However, achieving such a local Stackelberg equilibrium is not always necessarily possible. In fact, for some cases, it could be that the chosen initial value of the pricing policy $\pi_{\text{init}}$ largely determines which discount policy the algorithm converges to. Therefore, we plan to investigate in the future how different initial values of the pricing policy influence the result of the iterative procedure.

## 4 Conclusions

In this paper, we presented an iterative framework for computing a pricing strategy corresponding to a local Stackelberg equilibrium in a pricing game with one leader, i.e., the central authority and multiple followers, i.e., the ride-hailing companies, where the ride-hailing companies are constrained by fixed, a priori defined, discount budgets. We provided theoretical convergence guarantees and demonstrated the performance of the system in a simulated case study based on taxi data from the city of Shenzhen. However, what remains a promising research direction for the future is the question of how to choose the initial conditions in order to converge to a global optimum of the central authority's objective whenever such an optimum exists. Moreover, we aim to increase the complexity of the model in an attempt to better describe the reality.

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