On the pricing of pool ride-hailing services in bus lanes

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SHORT SUMMARY

Ride-hailing vehicles contribute to traffic congestion in urban areas, where spatial constraints and uneven multi-modal distribution of infrastructure are a constant problem. In fact, roaming empty vehicles cause additional delays to other concurrent network users without delivering passengers to their destinations. Ride-splitting is one potential solution to counteract the negative impact of ride-hailing on traffic. In this work, we provide a dynamic non-equilibrium modelling framework for ride-splitting, where pool passengers are allowed to use dedicated bus lanes and can potentially travel faster than solo users. The objective is to develop a ride-splitting pricing policy between solo and pool options to encourage trip sharing with the goal of minimizing overall delays in multi-modal networks with bus lanes. Therefore, a Model Predictive Control (MPC) framework is set forward to investigate the price difference between the two ride-hailing alternatives with the objective of reconsidering the space allocation between the available modes. The results show that the proposed strategy is able to adjust the time-dependent fare changes based on the multi-modal demand and speeds in different parts of the network.

Keywords: Model predictive control, multi-modal networks, network delays, public transportation, regulations, ride-splitting services, space allocation.

1. INTRODUCTION

Ride-hailing has quickly established itself as a transportation alternative in its own because of the myriad of benefits it brings. It is a flexible and affordable door-to-door service with relatively low wait times compared to public transit. Despite its advantages, the impact of ride-hailing services on multi-modal demand distribution and traffic congestion is significant at multiple levels. Ride-splitting has the potential to mitigate these impacts by reducing the number of drivers required to achieve the same level of service (Ma, Zheng, & Wolfson, 2015), and cutting down the number of Vehicle Kilometer Travelled (VKT) to serve the same level of demand (Ke, Zheng, Yang, & Ye, 2021). However, the critical mass for pooling is rarely fulfilled, and this is due to the longer travel time that this option entails. The need to regulate ride-hailing services is therefore becoming more substantiated, and much work in this area has examined the efficiency of setting a cap on the fleet size or the maximum empty VKT allowed to be travelled by ride-hailing drivers (Yu, Tang, Max Shen, & Chen, 2020; Schaller, 2018). Other lines of research address the gains from sending empty ride-hailing vehicles to available parking spaces in urban areas in dynamic non-equilibrium settings (Beojone & Geroliminis, 2021), or in static equilibrium frameworks (Li, Qin, Yang, Poolla, & Varaiya, 2020; Xu, Yin, & Zha, 2017). The commonality among these proposed solutions is that they all fall into an enforcement-based regulatory approach. In a previous paper, Fayed, Nilsson, and Geroliminis (2023) demonstrated the potential of an incentive-based policy to reduce the impact of ride-hailing on traffic congestion. By proposing an occupancy-based spatial assignment policy in a multi-modal network with dedicated bus lanes, we showed that it is possible to mitigate the impact of ride-hailing by encouraging users to share their trips. By giving pool drivers the right to utilize the bus lanes, the number of vehicles in the mixed traffic portion of the network is reduced. A similar modal-dependent allocation strategy is used for Autonomous Vehicles (AV) in Lamotte, de Palma, and Geroliminis (2017), or for buses in Geroliminis and Daganzo (2008); Geroliminis, Zheng, and Ampountolas (2014). Nevertheless, a very high number of pool vehicles in bus lanes deteriorates network conditions by causing significant delays for bus users. A pricing strategy is hence needed to encourage pooling when bus lane capacity allows it, and to deter pooling when bus delays are significant. Therefore, the main contribution of this work is to develop an aggregate multi-modal dynamic model that incorporates the proposed allocation policy. This model serves as a basis for establishing a control pricing scheme between the two ride-hailing alternatives in an attempt to contain overall network delays.

2. METHODOLOGY

The following section deals with the dynamic macroscopic modelling of multi-modal networks with bus lanes. Within this scope, we assume that private vehicles and solo ride-hailing users travel in the vehicle network, while buses and pool ride-hailing users travel in the bus network. We begin first by characterizing the aggregate traffic model that we use to determine speeds in both networks. We then use this model to describe the dynamics of private and ride-splitting vehicles, and bus passengers. Finally, we introduce the MPC scheme with the goal of determining the pricing gap between the solo and pool alternatives that minimizes multi-modal user delays. The full list of notations used in this paper is displayed in Table 1.

Network model

In the network under consideration, travellers perform their trips by one of the set of available modes $\mathcal{M}$: private vehicles pv, ride-splitting rs, or buses b, so that $\mathcal{M} := \{pv, b, rs\}$. Therefore, the exogenous and time-dependent hourly demand for each mode is given by $Q_j(k)$ for $j \in \mathcal{M}$ where k is the time-step such that $k \in \mathcal{K} := \{0, ..., k_{max}\}$. Moreover, we assume that the time interval between two consecutive time-steps is equal to Δ. Ride-hailing users choose either a solo trip s in the vehicle network $\mathcal{V}$ or a pool trip p in the bus network $\mathcal{B}$. The fraction of infrastructure allocated to the vehicle network $\mathcal{V}$ is denoted by α where $\alpha \in [0, 1]$. It can be inferred that buses and pool users are allocated a space equal to $1-\alpha$ of the total available network infrastructure. The ride-splitting fleet N is constant, and drivers belong exclusively to one of the following categories at each time-step:

i) idling or dispatching with no passengers inside the vehicle that we denote by n_e,

ii) performing a solo trip in the vehicle network $\mathcal{V}$ which we denote by n_s, and

iii) performing a pool trip in the bus network $\mathcal{B}$ which we denote by n_p.

In addition to the ride-splitting fleet, we denote the number of private vehicles in network $\mathcal{V}$ at time-step $k \in \mathcal{K}$ by $n_{pv}(k)$, and the number of buses in the bus network $\mathcal{B}$ by n_b. Note that the number of buses is assumed to be constant, but the bus occupancy o_b is time-dependent. Accordingly, the total accumulation in the vehicle network $n_{\mathcal{V}}$ at time-step $k \in \mathcal{K}$ is $n_{\mathcal{V}}(k) = n_{pv}(k) + n_e(k) + n_s(k)$ where the idle ride-hailing vehicles travel exclusively in the vehicle network. Similarly, the accumulation in the bus network $n_{\mathcal{B}}$ at time-step $k \in \mathcal{K}$ is $n_{\mathcal{B}}(k) = n_p(k) + n_b$. The fleet size N of ride-hailing vehicles is constant and provided by the platform operator such that $N = n_e(k) + n_s(k) + n_p(k)$ for all $k \in \mathcal{K}$.

Table 1: Notations

Variable name	Description
$\mathcal{M}$	Set of transportation modes where $\mathcal{M} = \{pv, rs, b\}$
$n(k)$	Total vehicle accumulation at time-step k
$P(n(k))$	Total production in the network at time-step k
$v(k)$	Network speed at time-step k
α	Fraction of space allocated to $\mathcal{V}$
$n_{\mathcal{V}}(k)$, $n_{\mathcal{B}}(k)$	Accumulation in $\mathcal{V}$ and $\mathcal{B}$ respectively at time-step k
N	Ride-hailing fleet size
n_{pv}, $n_e(k)$, $n_s(k)$, $n_p(k)$	Number of private, idle, solo and pool ride-hailing vehicles respectively at time-step k
n_b	Number of buses in the bus network
$P_{\mathcal{V}}(n_{\mathcal{V}}(k))$, $P_{\mathcal{B}}(n_{\mathcal{B}}(k))$	Production in $\mathcal{V}$ and $\mathcal{B}$ respectively at time-step k
$P_p\big(n_p(k), n_b\big)$	Production of pool vehicles in $\mathcal{B}$ at time-step k
$P_b\big(n_p(k), n_b\big)$	Production of buses in $\mathcal{B}$ at time-step k
$v_{\mathcal{V}}\big(n_{\mathcal{V}}(k)\big)$, $v_{\mathcal{B}}\big(n_{\mathcal{B}}(k)\big)$	Speed in $\mathcal{V}$ and $\mathcal{B}$ respectively at time-step k
$v_p\big(n_p(k), n_b\big)$	Speed of pool vehicles at time-step k
$v_b\big(n_p(k), n_b\big)$	Speed of buses at time-step k
$Q_i(k)$	Demand for mode $i \in \mathcal{M}$ at time-step k
$c(k)$	Number of ride-hailing customers waiting to be assigned at time-step k
$\beta(k)$	Fraction of ride-hailing requests opting for solo trips at time-step k
$c_s(k)$, $c_p(k)$	Number of ride-hailing customers opting for a solo and a pooled trip respectively at time-step k
$u_s(k)$, $u_p(k)$	Travel time cost for solo and pooled respectively expressed in monetary terms at time-step k
$U_s(k)$, $U_p(k)$	Total cost for a solo and pooled respectively at time-step k
$\tilde{F}_s(k)$, $\tilde{F}_p(k)$	Dynamic fare for pooled and solo trips at time-step k
F_s, F_p	Ride-hailing basic fare for solo and pooled trips at time-step k
N	Ride-hailing fleet size
$o_b(k)$	Actual occupancy per bus at time-step k
$\bar{o}_{pv}$, $\bar{o}_p$	Average private and pool vehicles occupancy respectively
$\bar{v}_b$	Target speed for buses
$\bar{l}_{pv}$, $\bar{l}_s$, $\bar{l}_b$	Average private vehicle, solo, and bus trip lengths respectively
Δl_d, Δl_p	Driver and passenger pooled trip detour distance
Δ	Length of simulation time-step
$M(k)$	Matching rate at time-step k
μ	Mode choice scale parameter
κ	Value of time
a_0, α_e, α_c	Meeting function parameters
$\bar{t}_d$	Bus dwell time at stops
$\bar{s}$	Average spacing between bus stops
$w_{\max}$	Maximum passenger waiting time
$\xi(k)$	Control variable integrated in discrete mode choice at time-step k
$\phi(k)$	Additional controlled fee or discount that pooled vehicle incur at time-step k
$A(k)$	Ride-splitting request abandonment at time-step k

Traffic dynamics

In the following section, we address the macroscopic traffic dynamics that allow us to specify the relationships between demand and accumulation for each mode. Thus, let $P: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ be the total network production, n its accumulation, and v its speed, then we know that for all time-steps, the relation $P(n(k)) = n(k)v(n(k))$ holds, where $\frac{\partial v}{\partial n} \leq 0$. If the entire network space is partitioned into a vehicle and bus network according to the fractional split α, then the production functions in the vehicle network $P_{\mathcal{V}}$ and bus network $P_{\mathcal{B}}$ satisfy the conditions $\alpha P(n(k)) = P_{\mathcal{V}}(\alpha n(k))$ and $\bar{\alpha} P(n(k)) = P_{\mathcal{B}}(\bar{\alpha} n(k))$ respectively where $\bar{\alpha} = 1 - \alpha$ (Ni & Cassidy, 2019; Sirmatel, Tsitsokas, Kouvelas, & Geroliminis, 2021). Similarly, the speed in the vehicle network $v_{\mathcal{V}}$ and in the bus network $v_{\mathcal{B}}$ are given by $v\big(n(k)\big) = v_{\mathcal{V}}\big(\alpha n(k)\big)$ and $v\big(n(k)\big) = v_{\mathcal{B}}\big(\bar{\alpha} n(k)\big)$ respectively. Rewriting the production functions in terms of speeds, we obtain that $P_{\mathcal{V}}(n_{\mathcal{V}}(k)) = n_{\mathcal{V}}(k) v_{\mathcal{V}}\big(n_{\mathcal{V}}(k)\big)$ and $P_{\mathcal{B}}(n_{\mathcal{B}}(k)) = n_{\mathcal{B}}(k) v_{\mathcal{B}}\big(n_{\mathcal{B}}(k)\big)$.

To account for the fact that the marginal effects on the speed of buses and pool vehicles are not equivalent, we divide the bus network production into pool vehicle production P_b and a bus production P_b. This is because buses, unlike pool vehicles, frequently stop at stations to allow passengers to board and disembark. We capture this recurrent action by reducing the speed in the bus network $v_{\mathcal{B}}$ by a factor $r(n_b)$ where $r: \mathbb{R}_{\geq 0} \rightarrow (0,1]$ and $\frac{\mathrm{d}r}{\mathrm{d}n_b} < 0$. Consequently, the running speed of the pool vehicles in the bus network is $v_p\big(n_p(k), n_b\big) = v_{\mathcal{B}}\big(n_{\mathcal{B}}(k)\big) r(n_b)$. Taking into account the time that buses spend boarding and alighting passengers, the operational bus speed is

$$v_b\big(n_p(k), n_b\big) = \left(\frac{1}{1 + v_p\big(n_p(k), n_b\big) \frac{\bar{t}_d}{\bar{s}}} \right) v_p\big(n_p(k), n_b\big), \tag{1}$$

where $\bar{t}_d$ and $\bar{s}$ are the average time of buses and the spacing between stops, respectively. Therefore, after defining the individual pool vehicles and bus speeds, the production functions become $P_p\big(n_p(k), n_b\big) = n_p(k) v_p\big(n_p(k), n_b\big)$ and $P_b\big(n_p(k), n_b\big) = n_b(k) v_b\big(n_p(k), n_b\big)$ respectively for all $k \in \mathcal{K}$.

Private vehicles dynamics

According to the modal-dependent spatial allocation policy proposed in this framework, private vehicles utilize the vehicle network $\mathcal{V}$. The change in the accumulation of private vehicles between any two successive time-steps is given by the difference between the exogenous arrival of private vehicle users Q_{pv} and the completion rate of private vehicle trips O_{pv}. The latter is derived based on the accumulation $n_{\mathcal{V}}$ and the network production function $P_{\mathcal{V}}$ such that n_{pv} at time-step k is equal to

$$\begin{aligned} n_{pv}(k) &= n_{pv}(k-1) + \Delta \left[\frac{Q_{pv}(k)}{\bar{o}_{pv}} - O_{pv}(k-1) \right] \\ &= n_{pv}(k-1) + \Delta \left[\frac{Q_{pv}(k)}{\bar{o}_{pv}} - \frac{n_{pv}(k-1)}{n_{\mathcal{V}}(k-1)} \frac{P_{\mathcal{V}}\big(n_{\mathcal{V}}(k-1)\big)}{\bar{l}_{pv}} \right], \quad \forall k \in \mathcal{K} \setminus \{0\}, \end{aligned} \tag{2}$$

where $\bar{l}_{pv}$ is the average trip length of private vehicles and $\bar{o}_{pv}$ is their average occupancy. Note that the accumulation $n_{\mathcal{V}}$ itself depends on the number of private vehicles n_{pv}, but also on the number of solo ride-hailing drivers n_s.

Ride-splitting dynamics

As highlighted earlier, the total ride-splitting demand in this work is exogenous, but the demand split between solo and pool trips is the result of user choice, and is determined endogenously using the total travel cost for each alternative. This cost is the sum of two elements: the alternative-dependent travel fare and the travel time in each network. Accordingly, if $U_s(k)$ and $U_p(k)$ are the disutilities for a solo or pool trip at time-step $k \in \mathcal{K}$ respectively, then their expressions are given by

$$U_s(k) = \tilde{F}_s(k) + \kappa \frac{\bar{l}_s}{v_{\mathcal{V}}\big(n_{\mathcal{V}}(k)\big)}, \tag{3}$$

$$U_p(k) = \tilde{F}_p(k) + \kappa \frac{\bar{l}_s + \Delta l_p}{v_p\big(n_p(k), n_b\big)}, \tag{4}$$

where κ is the value of time, $\bar{l}_s$ is the average trip length for a solo trip, and Δl_p is the additional travel distance that pool users incur to pick up and/or drop off another passenger. The variable $\tilde{F}_s(k)$ represents the fare collected from a solo user and $\tilde{F}_p(k)$ represents the fare collected from a pool user at time-step k. For simplicity, we assume that the solo fare is static such that $\tilde{F}_s(k) = F_s$ for all $k \in \mathcal{K}$ where F_s is the fare set by the platform operator. The pool fare is defined by the expression $\tilde{F}_p = F_p + \phi(k)$ where F_p is the static pool charges collected by the platform, and $\phi(k)$ is the control fare set by the network regulator to steer the system toward its optimum.

If the choice of ride-hailing users is the outcome of a binary logit choice model, then the fraction of solo trips that we denote by β is

$$\beta(k) = \frac{\exp\big(-\mu U_s(k)\big)}{\exp\big(-\mu U_s(k)\big) + \exp\big(-\mu U_p(k)\big)}, \tag{5}$$

where $\mu > 0$ is the binary logit scale parameter. Rewriting the disutilities for solo and pool that we denote by u_s and u_p respectively in terms of static fare only, we obtain the following

$$u_s(k) = F_s + \kappa \frac{\bar{l}_s}{v_{\mathcal{V}}\big(n_{\mathcal{V}}(k)\big)}, \tag{6}$$

$$u_p(k) = F_p + \kappa \frac{\bar{l}_s + \Delta l_p}{v_p\big(n_p(k), n_b\big)}. \tag{7}$$

Therefore, the expression for β as function of u_s and u_p is

$$\beta(k) = \frac{\exp\big(-\mu u_s(k)\big)}{\exp\big(-\mu u_s(k)\big) + \underbrace{\exp\big(-\mu \phi(k)\big)}_{\xi(k)} \exp\big(-\mu u_p(k)\big)}, \tag{8}$$

where $\xi(k) \in (0, +\infty)$ is an auxiliary variable that paves the way for a more pragmatic implementation of the MPC framework. From the auxiliary variable $\xi(k)$, it is straightforward to derive the control price $\phi(k)$ by using the expression

$$\phi(k) = \frac{\log(\xi(k))}{-\mu}. \tag{9}$$

Once the choice of users is determined, it is straightforward to divide the waiting passengers into two categories. If $c(k)$ is the number of requests waiting to be assigned at time k, then the number of requests that choose to ride solo is $c_s(k) = \beta(k)c(k)$ and those that choose to pool is $c_p(k) = (1 - \beta(k))c(k)$. These passengers are matched to idling vehicles according to the following bilateral meeting rate M, so that

$$M(k) = a_0 n_e(k)^{\alpha_e} \left(c_s(k) + \frac{1}{2} c_p(k)\right)^{\alpha_c}, \tag{10}$$

where $a_0 > 0$, $\alpha_e > 0$, and $\alpha_c > 0$ are tthe parameters of the Cobb-Douglas meeting function in (10). Note that a factor $\frac{1}{2}$ is added to c_p to indicate that pool waiting passengers are assigned to a single unoccupied vehicle.

Once all these elements are defined, it becomes possible to determine the dynamics of the ride-splitting market. We first start with idle vehicles, so that the value of n_e at time-step k is given by

$$\begin{aligned} n_e(k) &= n_e(k-1) + \Delta[O_s(k-1) + O_p(k-1) - M(k-1)] \\ &= n_e(k-1) + \Delta\left[\frac{n_s(k-1)}{n_{\mathcal{V}}(k-1)}\frac{P_{\mathcal{V}}\big(n_{\mathcal{V}}(k-1)\big)}{\bar{l}_s} + \frac{P_p\big(n_p(k-1), n_b\big)}{\bar{l}_s + \Delta l_d} - M(k-1)\right], \forall k \in \mathcal{K} \setminus \{0\}, \end{aligned} \tag{11}$$

where O_s and O_p are the completion rates for a solo and a pool trip respectively, $\bar{l}_s$ is the average solo trip length, and Δl_d is the driver detour, i.e., the extra distance that a driver travels to deliver a pool trip. Note that all drivers who complete their trips become idling again, and thus represent the inflow for the idling vehicle category. In contrast, the outflow for this specific category consists of every vehicle that has been matched, which is derived from $M(k)$.

Moving to the solo vehicle category, the number of solo vehicles at every time-step is

$$n_s(k) = n_s(k-1) + \Delta\left[\beta(k-1)M(k-1) - \frac{n_s(k-1)}{n_{\mathcal{V}}(k-1)}\frac{P_{\mathcal{V}}\big(n_{\mathcal{V}}(k-1)\big)}{\bar{l}_s}\right], \quad \forall k \in \mathcal{K} \setminus \{0\}. \tag{12}$$

Since only a fraction β of the total ride-hailing requests choose a solo trip, the inflow for the solo vehicle category is determined by a portion β of the total matching rate, and the outflow is computed using vehicle network production $P_{\mathcal{V}}$.

In a similar manner, we calculate the change in the number of pooling vehicles n_p by taking the difference between the number of drivers matched to a pool trip and the number of pool drivers that completed their trips. Thus, the expression of n_p is given by

$$n_p(k) = n_p(k-1) + \Delta\left[\big(1 - \beta(k-1)\big)M(k-1) - \frac{P_p\big(n_p(k-1), n_b\big)}{\bar{l}_s + \Delta l_d}\right], \quad \forall k \in \mathcal{K} \setminus \{0\}. \tag{13}$$

The trip completion rate for the pool vehicle category in computed using the pool vehicle production P_p, which itself is a function of the time-dependent number of pool vehicles n_p and the static number of buses n_b.

Finally, the dynamics of waiting passengers remain to be defined. Knowing that the demand for ride-splitting at time-step k is given by $Q_{rs}(k)$, the number of waiting passengers $c(k)$ is

$$c(k) = c(k-1) + \Delta[Q_{rs}(k) + \big(\beta(k-1) - 2\big)M(k-1)] - A(k), \quad \forall k \in \mathcal{K} \setminus \{0\}. \tag{14}$$

where $A(k)$ is the number of abandoning requests that are not served within reasonable waiting times. We point out here that the calculation of the passenger outflow takes into account that each pool trips results in two passengers leaving the queue of waiting requests. The number of abandonments $A(k)$ is computed as follows

$$A(k) = \max\left(c(k-1) - \frac{1}{k-1}\sum_{\tilde{k}=1}^{k-1} M(\tilde{k}-1)w_{\max}, 0\right), \tag{15}$$

where $w_{\max}$ is a measure of the maximum wait tolerance of ride-hailing requests. This equation is an approximation of the number of waiting requests when the wait tolerance is set to $w_{\max}$. It computes the number of requests that leave the platform due to a poor level of service.

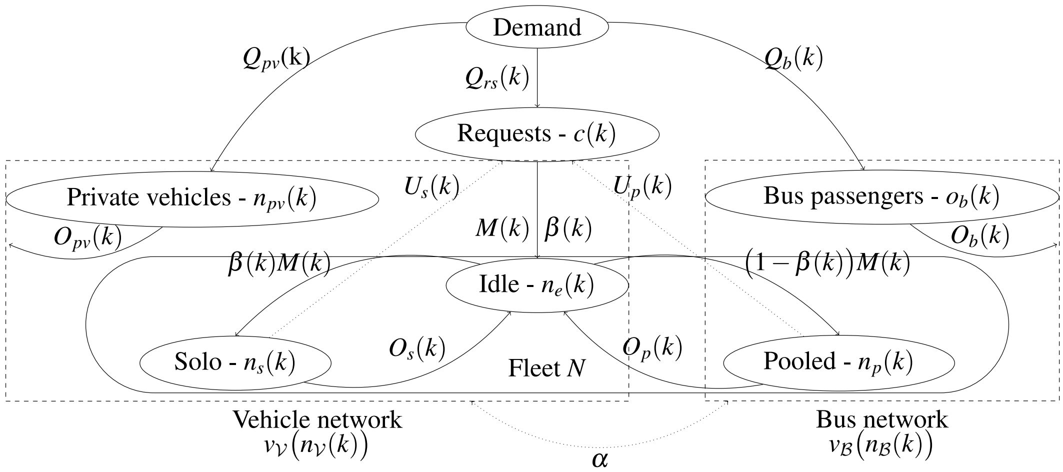

Figure 1: Modelling framework of the proposed dynamic and modal-dependent space allocation policy

Bus dynamics

Moving to the bus dynamics, we assume in this framework that the number of buses circulating in the bus network $\mathcal{B}$ is constant. This number is computed using

$$n_b = \frac{\bar{Q}_b \bar{l}_b}{\bar{o}_b \bar{v}_b}, \tag{16}$$

such that $\bar{Q}_b$ is the average expected bus demand per hour, $\bar{l}_b$ is the average bus trip length, $\bar{o}_b$ is the target bus occupancy, and $\bar{v}_b$ is the expected bus operating speed. Assuming that n_b remains constant in time, we track the bus dynamics by following the variation of the average bus occupancy o_b, which at time k, is given by

$$o_b(k) = o_b(k-1) + \Delta \frac{1}{n_b}\left[Q_b(k) - \frac{P_b\big(n_p(k-1), n_b\big)}{\bar{l}_b} o_b(k-1)\right], \quad \forall k \in \mathcal{K} \setminus \{0\}. \tag{17}$$

We assume in (17) that bus demand $Q_b(k)$ is uniformly distributed over the available bus fleet. The trip completion rate is computed using the bus production function P_b, and is converted to passenger trips by multiplying the completion rate by the average bus occupancy.

Figure 1 summarizes the full network model including the dynamics that we previously described.

Model predictive control

The modelling framework proposed captures the full impact on the multi-modal commuters when pool users are allowed to use the bus lanes. In Fayed et al. (2023), it was shown that the overall network situation deteriorates when the number of pool vehicles in the bus lanes is relatively high, causing additional delays for buses. Therefore, a pricing scheme is needed to encourage or discourage ride-hailing users to utilize the bus lanes depending on the speeds in the vehicle network $\mathcal{V}$ and in the bus network $\mathcal{B}$. To provide such a pricing scheme, we integrate the network dynamics into a MPC framework, and use the latter to determine the fare control variable $\phi(k)$ that minimizes the total Passenger Hour Travelled (PHT) such that PHT at time k is equal to the sum of the individual PHT of each commuter category in the network

$$\text{PHT}(k) = \Delta[n_{pv}(k)\bar{o}_{pv} + n_b(k)o_b(k) + n_s(k) + n_p(k)\bar{o}_p].$$

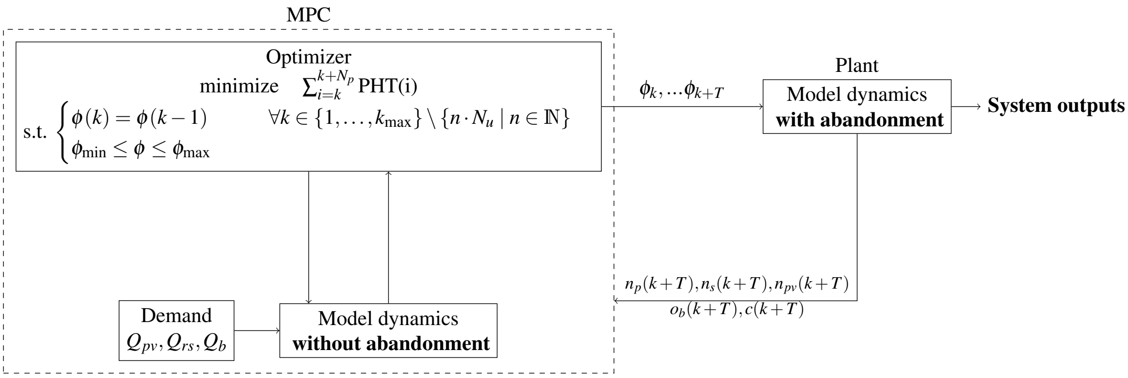

Figure 2: MPC framework with distinct plant and MPC dynamics

The variable $\bar{o}_p$ is the average occupancy of the pool vehicles, where $\bar{o}_p \in [1,2]$. The objective of MPC is to solve the following minimization problem

$$\begin{aligned} \underset{\{\phi(k)\}_{k\in\mathcal{K}}}{\text{minimize}} \quad & \sum_{k\in\mathcal{K}} \text{PHT}(k) \\ \text{subject to} \quad & (1)-(17) \\ & \phi(k) = \phi(k-1)\,, \qquad \forall k \in \{1,\dots,k_{\max}\} \setminus \{n \cdot N_u \mid n \in \mathbb{N}\} \\ & \phi_{\min} < \phi(k) < \phi_{\max}\,, \quad \forall k \in \{1,\dots,k_{\max}\} \end{aligned} \tag{18}$$

where the second constraint in (18) ensures that the control variable is updated only every $N_u \in \mathbb{N}$ time-steps. The third constraint sets a minimum and maximum range for the control variable.

The minimization framework presented in (18) is straightforward if we disregard abandonment from (14). Nevertheless, including abandonment in our framework introduces an additional computational complexity since $A(k)$ is calculated based on prior matching rates. To circumvent this, we adapt the MPC framework according to the approach displayed in Figure 5. In this scope, we neglect the abandonment in the MPC dynamics and solve the optimization problem to obtain the values of the control fare for a prediction horizon $N_p \in \mathbb{N}$. However, we extract only the control variables up to an update horizon, such that $T \in \mathbb{N}$ and $T < N_p$, and incorporate them into the actual plant dynamics with abandonment to estimate the new state variables. The state variables are then fed as inputs to the MPC dynamics and the optimizer is relaunched.

3. RESULTS AND DISCUSSION

In this section, we will numerically show the performance of the proposed control strategy and quantify the influence of abandonment on the model dynamics. To do so, we consider a network which production function is given by $P(n) = A_0 n^3 + B_0 n^2 + C_0 n$, such that $A_0 = 5.74 \cdot 10^{-9}$, $B_0 = -1.02 \cdot 10^{-3}$, and $C_0 = 36$ for $n \in [0, 58536]$. Assuming the fraction of the total space allocated to the vehicle network α is equal to 0.8, it becomes straightforward to derive the production functions $P_{\mathcal{V}}$ and $P_{\mathcal{B}}$. The function capturing the marginal effect of buses on traffic is given by $r(n_b)$ such that $r(n_b) = e^{-6.5\cdot 10^{-4} n_b}$. This function, in addition to $\bar{s} = 0.8$ km and $\bar{t}_d = 30$ s, are used to compute the bus speed v_b.

Next, we list the different constant values used in the modelling framework. The average private vehicle and solo trip lengths are set to be equal such that $\bar{l}_{pv} = \bar{l}_s = 3.86$ km. The trip length by bus is generally larger, and therefore $\bar{l}_b$ is set to be equal to $1.4\bar{l}_{pv}$. The values of the driver and passenger detour are $\Delta l_d = 0.7\bar{l}_s$ and $\Delta l_p = 0.15\bar{l}_s$, and the average occupancies for private vehicles $\bar{o}_{pv}$, pool vehicles $\bar{o}_p$, and bus $\bar{o}_b$ are equal to 1.2, 1.5, and 20 respectively. The average design bus speed $\bar{v}_b$ used to compute the number of buses n_b is 18 km/hr The a_0, α_e, and α_c parameters for

Table 2: PHT for different simulation frameworks with no abandonment

Scenario	β	N_p	T	PHT [pax.km/hr]
Benchmark	logit	-	-	171300
Pool trips only	0	-	-	182037
Solo trips only	1	-	-	175798
MPC	logit	3600	3600	168437
MPC	logit	900	450	168729

Table 3: PHT for different simulation frameworks with abandonment

Scenario	β	N_p	T	PHT [pax.km/hr]	Abandonment
Benchmark	logit	-	-	170007	8590
Pool trips only	0	-	-	177376	20463
Solo trips only	1	-	-	174952	21626
MPC	logit	3600	3600	167635	10271
MPC	logit	900	450	167938	13961

the Cobb-Douglas matching function are 0.025, 0.93, and 0.98 respectively. With respect to the computation of the mode choice between solo and pool, we set the scale parameter $\mu = 1$ and the value of time $\kappa = 30$ CHF/hr. The constant solo and pool trip fare F_s and F_p are equal to 5 and 4 CHF respectively. The ride-hailing fleet size N is equal to 3500 and the discretized time interval Δ is equal to 6 s.

The aggregate simulation that we advance spans over a 6-hour period covering the afternoon on-peak in-between two off-peak periods. The demand profiles for private vehicles, ride-splitting services, and buses are shown in Figure 3. To start with, we test our controller for a scenario where abandonment is always set to 0, and we show the results in Table 2. We compare different simulation settings, including a benchmark scenario with no external intervention and an MPC framework with different prediction and update horizon settings, with $N_u = 180$. The same simulations are then repeated when the plant (but not the dynamics in the MPC) has abandonment rates as given by (15), and the results are displayed in Table 3. Logically, the values of PHT are lower for scenarios with abandonment mainly because the travel time of abandoning requests is not accounted for in the computation of PHT. Moreover, irrespective of the abandonment settings, the MPC with a full prediction horizon returns the lowest PHT, and decreasing the prediction horizon slightly deteriorates the results. Finally, for instances with abandonment, even if the MPC framework reduces delays, the increase in abandonment relative to the benchmark scenario does not allow a fair comparison basis. Therefore, designing an MPC that accommodates abandonment in its dynamic framework is a research area that we plan to investigate in the future.

Lastly, we plot the variations of the main model variables for the benchmark scenario with no abandonment in Figure 4, and for the MPC framework with abandonment and a prediction horizon of 900 in Figure 5. Compared to a scenario with no control, i.e.,$\phi(k) = 0$ for all $k \in \mathcal{K}$, the MPC imposes high charges for pooling to prevent the deterioration of the conditions in bus lanes, and to improve the overall network situation. During on-peak periods however, we observe that the charges decrease, and the control variable ϕ assumes negative values to further encourage pooling as the speed in the vehicle network $v_{\mathcal{V}}$ drops significantly. The aim in these particular time spots is to alleviate congestion in the vehicle network.

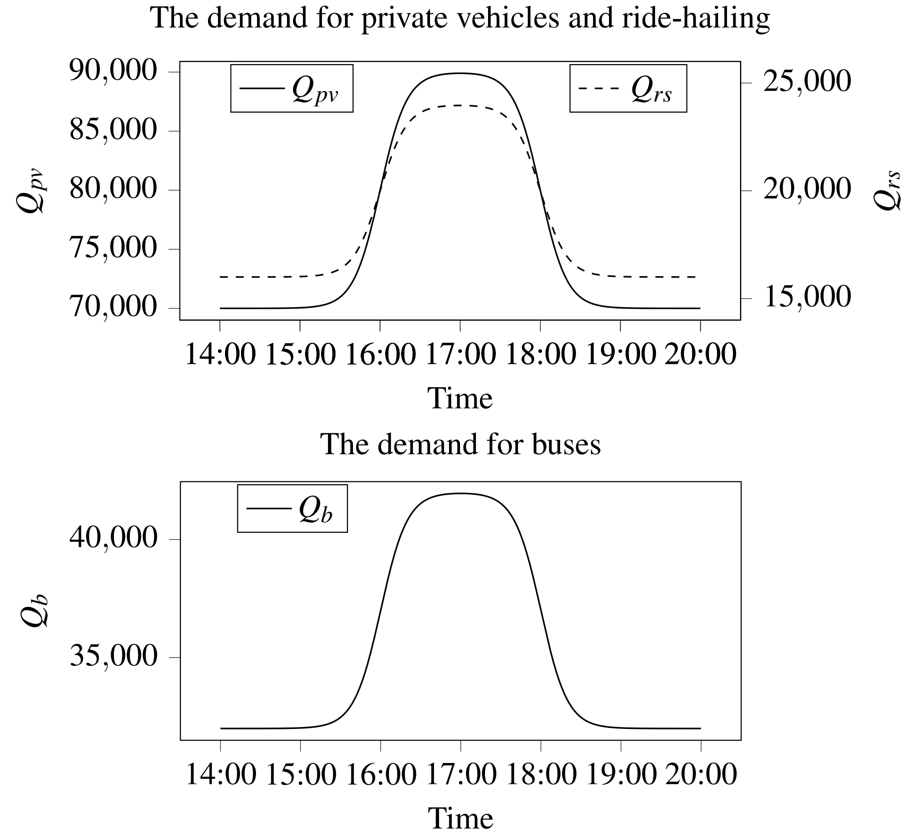

Figure 3: Demand profiles for Q_{pv}, Q_{rs} and Q_b

4. CONCLUSIONS

In this work, we designed a dynamic space and occupancy-dependent allocation policy for multi-modal networks with ride-splitting services. We then build upon this model to construct a pricing control strategy that encourages or discourages pooling in bus lanes according to the overall user delays. Using an MPC framework, we solve the minimization problem, and determine what is the additional fare or discount that ride-splitting users should incur to improve the total network situation. By comparing many different scenarios, we demonstrate that our control scheme is indeed capable of reducing total Passenger Hour Travelled in scenarios with or without ride-hailing request abandonment. Future research direction however will focus on a more reasonable integration of abandoning requests within the MPC framework through reintroducing them to the system as bus or private vehicle users.

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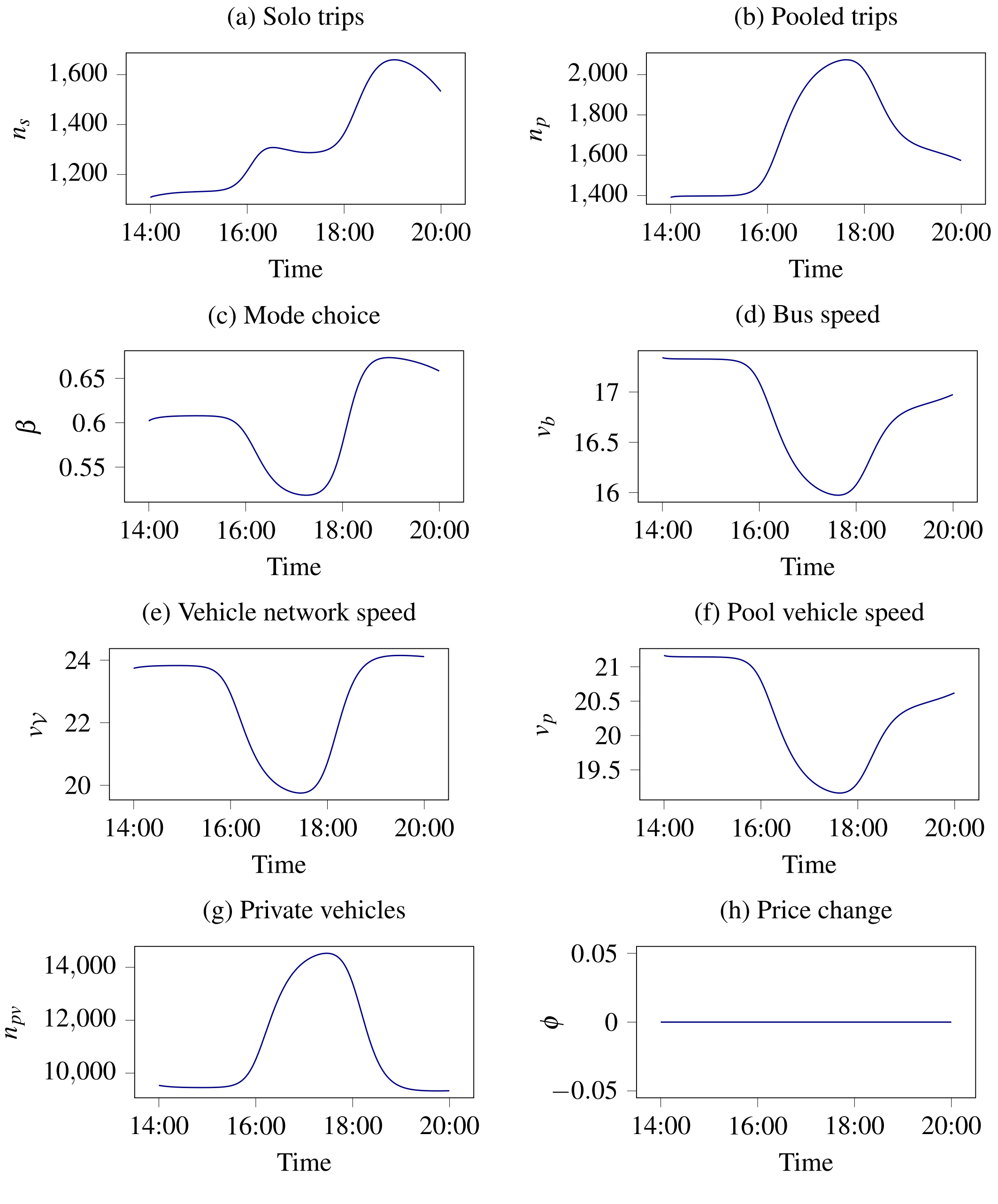

Figure 4: Time-dependent model variables for the benchmark scenario with no request abandonment

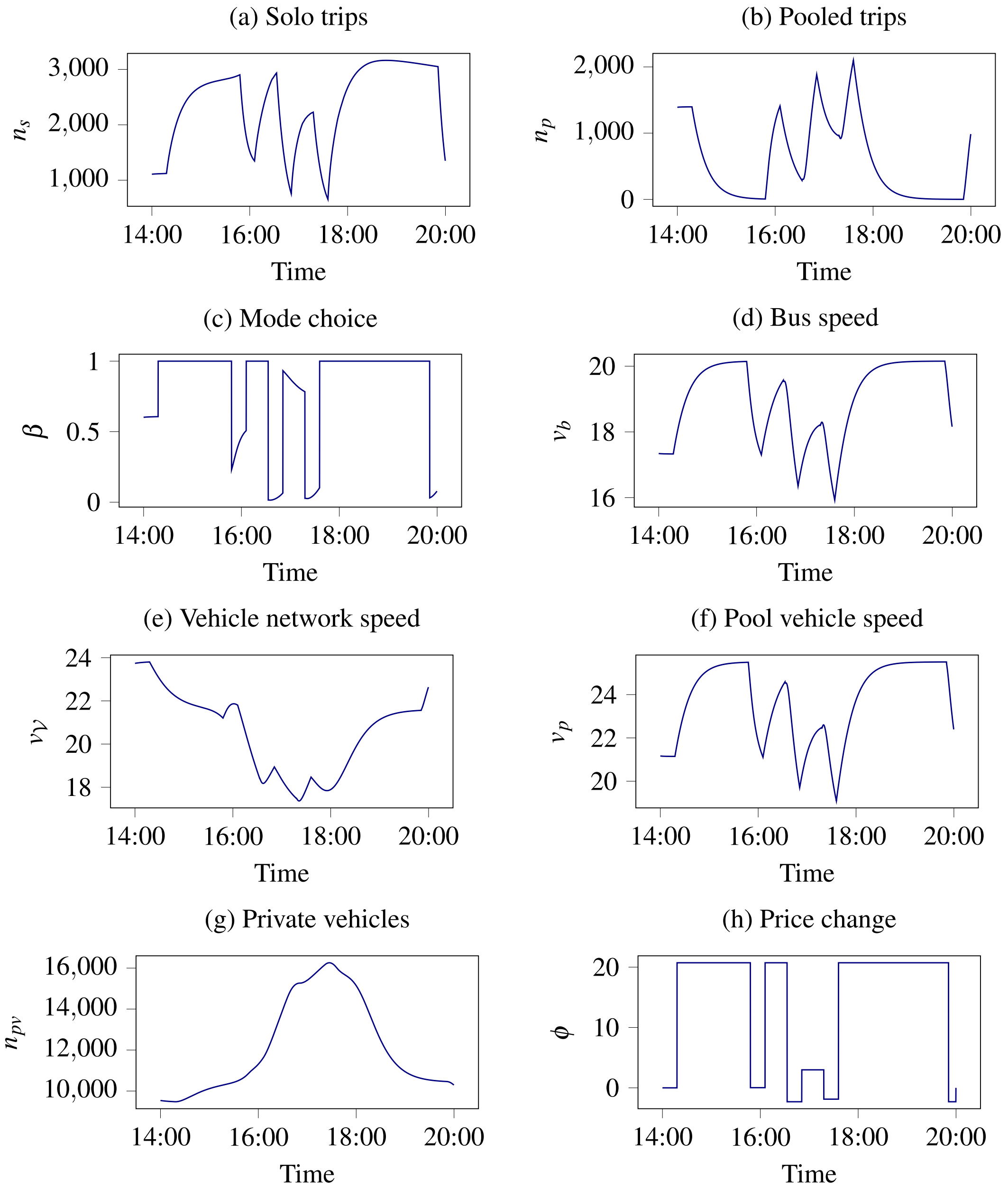

Figure 5: Time-dependent model variables for the MPC scenario with abandonment, $N_p = 900$**, and** $T = 450$

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