Welfare impacts of remote and flexible working policies in the bottleneck model

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Short summary

This study investigates the effects of remote and flexible working styles on traffic congestion. We first formulate an integrated equilibrium model simultaneously considering the working style, official work start time, and departure time choice of workers via an extension of the bottleneck model. Subsequently, we derive the equivalent optimization problem of the equilibrium problem as linear programming (LP) and demonstrate that we can obtain an analytical solution to the LP. This analytical solution enables us to assess the effects of remote and flexible working on social surplus and queueing loss. By comparing various situations, we show that implementing remote and flexible work causes higher queueing loss with an equal social surplus than implementing only remote work. Finally, we propose an integrated road management scheme that includes dynamic pricing to prevent this paradoxical phenomenon and efficiently implements remote and flexible working.
Keywords: remote working, flexible working, working style choice, departure time choice, bottleneck model

1 Introduction

New working styles, which are different from the conventional style of commuting to an office at a designated time, have become widespread, mainly owing to the COVID-19 pandemic. These new working styles include teleworking, staggered work hours, and flexible work. Each working style not only decreases the number of opportunities available to workers to contact each other in the office but also reduces or disperses the commuting demand during peak-periods. Therefore, promoting these working styles could potentially reduce commute-related congestion. However, many companies and workers have adopted these ways of working independently of the aim of the urban transportation system, which is to relieve traffic congestion. Under these circumstances, whether remote work and staggered work hours contribute to reducing congestion and their effect on road congestion are not well understood.

Many studies examined the relationship between traffic congestion and working style based on Vickrey's bottleneck model. Mun & Yonekawa, 2006; Fosgerau & Small, 2017; Takayama, 2015 formulated peak-period congestion models based on the bottleneck model and developed models describing the choice of firms and workers to adopt fixed or flexible schedules. In addition, many analyses of bottleneck models that consider the heterogeneity of preferred arrival times can be interpreted as modeling flexible or staggered work hours (e.g., Hendrickson & Kocur, 1981; Lindsey, 2004; Lindsey et al., 2019). Zhang et al. (2005) studied the trade-off between teleworking and office working, and they considered the elastic travel demand by incorporating teleworking as an alternative. Gubins & Verhoef (2011) analyzed the welfare effects of teleworking on road traffic congestion in the context of Vickrey's model. These studies highlight the positive effects of flexible and remote working on traffic congestion and social welfare. However, these results are based on analyses that consider only one type of working style. To develop effective congestion reduction policies for the current situation where multiple new working styles are widespread, we need to investigate how the relationship between these working styles affects traffic congestion.

This study investigates the effects of remote and flexible work on traffic congestion. We first formulate an integrated equilibrium model that accounts for flexible and remote work simultaneously via an extension of the bottleneck model. Subsequently, we derive the equivalent optimization problem of the equilibrium problem as linear programming (LP) and demonstrate that we obtain an analytical solution to the LP. This analytical solution enables us to assess the impacts of remote

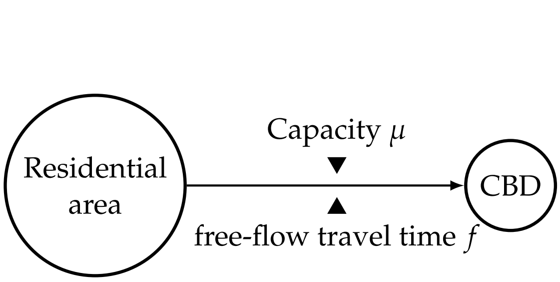

Figure 1: Network.

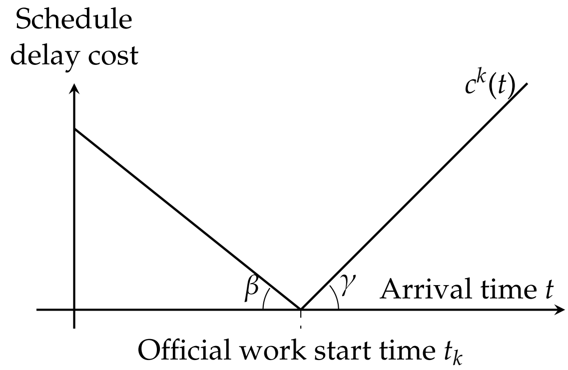

Figure 2: Schedule delay cost function.

and flexible work on social surplus and queueing loss. We present the following facts by comparing four scenarios: - no policies, remote working, flexible working, and both remote and flexible working:

- Implementing remote work causes lower queueing loss and higher social surplus than implementing remote work.
- Implementing flexible work causes lower queueing loss and higher social surplus than not implementing flexible work.
- Implementing remote and flexible work causes higher queueing loss with equal social surplus than implementing only remote work.

The third fact describes a paradoxical phenomenon in which the simultaneous introduction of flexible and remote working may increase queueing losses. This phenomenon may be interesting and important for road management.

The remainder of this paper is structured as follows: Section 2 introduces the integrated equilibrium model and derives its equivalent optimization problem. Section 3 presents an analytical approach to solving the problem and the effects of remote and flexible working arrangements on traffic congestion. Finally, Section 4 concludes the study and discusses future studies.

2 Model

Consider a city that consists of a central business district (CBD) and residential area connected by a freeway (Figure 1). This freeway has a single bottleneck with capacity μ, and its free-flow travel time is denoted by f. If the arrival rates of the workers at the bottleneck exceed bottleneck capacity, a queue develops. To model queueing congestion, we use first-in-first-out (FIFO) and a point queue in which vehicles have no physical length, as in standard bottleneck models.

All workers reside in the residential area. The workers are treated as a continuum, and the total mass Q is a given constant. The firm, located in the CBD, offers two working styles for workers: office and remote work. In addition, the firm allows K official work start (OWS) times $\{t^1, ..., t^K\}$ for office workers. Each worker can choose between office and remote work. If they choose the former, they must commute and choose the OWS time and the actual departure/arrival time. If they choose the latter, they work from home without commuting to the office.

The trip costs for each commuter (i.e., office worker) are assumed additively separable into free-flow travel, queueing delay, and schedule delay costs. The schedule delay cost is defined as the difference between the actual arrival times and OWS time at the office. We assume piecewise linearity in the schedule delay cost function, which is expressed as follows (Figure 2):

$$c_k(t) \equiv \begin{cases} \beta(t_k - t) & \text{if} \quad t_k < t \\ \gamma(t - t_k) & \text{if} \quad t_k \geq t \end{cases} \qquad \forall k \in \mathcal{K}, \quad \forall t \in \mathcal{T}. \tag{1}$$

The trip cost of a commuter whose destination arrival time is t and OWS time is t^k is defined as follows:

$$C_k(t) \equiv c_k(t) + \alpha(w(t) + f) \qquad \forall k \in \mathcal{K}, \quad \forall t \in \mathcal{T}. \tag{2}$$

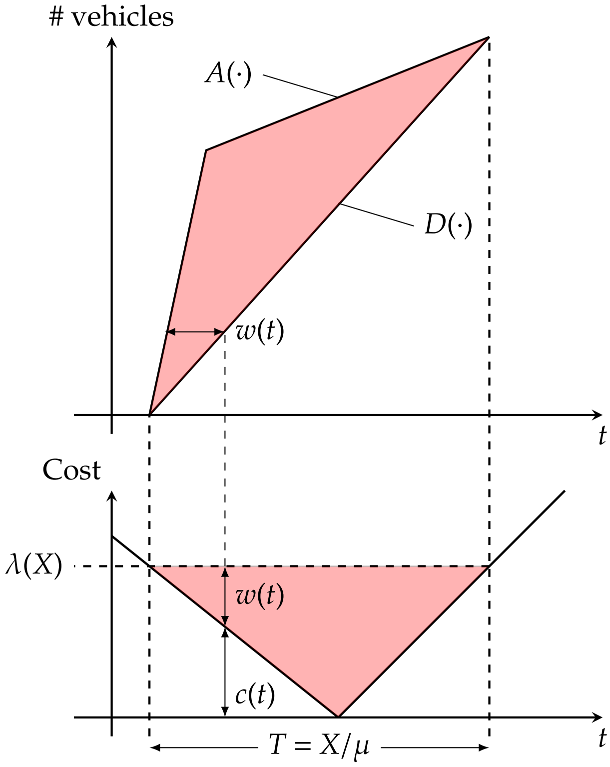

Figure 3: Commuting equilibrium with $K = 1$.

where $w(t)$ is the queuing delay experienced at the bottleneck by commuters with CBD arrival time t. Parameters f and α represent the free-flow travel time and value of time, respectively. This study assumes that $f = 0$ and $\alpha = 1$ for all commuters.

Workers choose their working styles to maximize their own utility. Utility of an office worker whose OWS time is t_k and actual CBD arrival time is t determine $\theta_O - C_k(t)$, where θ_O represents the wage parameter for office workers. In contrast, the utility of a remote worker is θ_R, where θ_R represents the wage parameter for remote workers. We assume that office workers have higher productivity and wages, i.e., $\theta_O > \theta_R$. In the equilibrium resulting from these choices of the workers, the following properties hold: no worker can reduce their utility by unilaterally changing their working style, and no commuter can reduce their commuting costs by unilaterally changing their destination arrival time.

The equilibrium problem can be formulated as a linear complementary problem comprising five equilibrium conditions. First, the worker conservation condition is expressed as follows:

$$\text{(Worker conservation)} \qquad Q_O + Q_R = Q, \tag{3}$$

where Q_O and Q_R represent the numbers of office and remote workers, respectively. Second, the equilibrium condition for workers in terms of working style is expressed as

$$\text{(equilibrium condition for office workers)} \qquad \begin{cases} \rho = \theta_O - \lambda & \text{if} \quad Q_O > 0 \\ \rho \geq \theta_O - \lambda & \text{if} \quad Q_O = 0 \end{cases} \tag{4}$$

$$\text{(equilibrium condition for remote workers)} \qquad \begin{cases} \rho = \theta_R & \text{if} \quad Q_R > 0 \\ \rho \geq \theta_R & \text{if} \quad Q_R = 0 \end{cases} \tag{5}$$

where ρ represents the equilibrium utility, and λ represents the equilibrium commuting cost. Third, the office worker (commuter flow) conservation conditions for the commuting demands must satisfy

$$\text{(commuter conservation)} \qquad \sum_{k \in \mathcal{K}} \int_{t \in \mathcal{T}} q_k(t) \mathrm{d}t = Q_O \qquad \forall k \in \mathcal{K}, \tag{6}$$

where $q_k(t) \geq 0$ is the arrival flow rate of commuters whose CBD arrival time is t and OWS time is

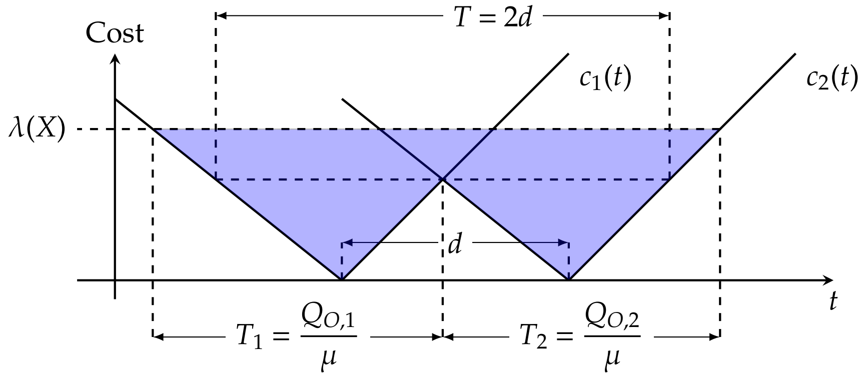

Figure 4: Commuting equilibrium with $K = 2$.

t^k. Fourth, the equilibrium condition for commuters is expressed as

$$\text{(departure time choice condition)} \quad \begin{cases} \lambda = c_k(t) + \alpha(w(t) + f) & \text{if} \quad q_k(t) > 0 \\ \lambda \leq c_k(t) + \alpha(w(t) + f) & \text{if} \quad q_k(t) = 0 \end{cases} \quad \forall k \in \mathcal{K}, \quad \forall t \in \mathcal{T}. \tag{7}$$

Fifth, the queueing condition at the bottleneck is expressed as follows (Akamatsu et al., 2021):

$$\text{(bottleneck queueing condition)} \quad \begin{cases} \sum_{k \in \mathcal{K}} q_k(t) = \mu & \text{if} \quad w(t) > 0 \\ \sum_{k \in \mathcal{K}} q_k(t) \leq \mu & \text{if} \quad w(t) = 0 \end{cases} \quad \forall t \in \mathcal{T}. \tag{8}$$

The equilibrium state represents the collection of variables $\{Q_O, Q_R, \lambda, \rho, w(t), q_k(t)\}$ that satisfy Eqs. (3) to (8).

Based on the methods reported by Iryo & Yoshii (2007); Akamatsu et al. (2021), an optimization problem can obtain the aforementioned equilibrium state. Specifically, we can derive the equilibrium number of office/remote workers and commuter arrival flow patterns as the optimal solution to the following linear programming. In addition, the equilibrium cost pattern, including the equilibrium utility, equilibrium commuting cost, and queueing delay pattern, can also be obtained as the optimal Lagrange variable of the problem.

$$\min_{Q_O, Q_R, \{q_k(t)\} \geq 0} . \quad \sum_{k \in \mathcal{K}} \int_{t \in \mathcal{T}} c_k(t) q_k(t) \mathrm{d}t - \theta_O Q_O - \theta_R Q_R \tag{9}$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}} q_k(t) \leq \mu \qquad \forall t \in \mathcal{T} \quad [w(t)], \tag{10}$$

$$\sum_{k \in \mathcal{K}} \int_{t \in \mathcal{T}} q_k(t) \mathrm{d}t = Q_O \qquad [\lambda], \tag{11}$$

$$Q_O + Q_R = Q \qquad [\rho], \tag{12}$$

where the variables inside the square brackets represent the Lagrangian multipliers for each constraint. We refer to this problem as the equivalent optimization problem. Equivalency can be proven by comparing the first-order conditions with the equilibrium conditions.

3 Welfare impact of remote and flexible working policies

The model formulated in the previous section describes the equilibrium under various situations by fixing certain parameters. In this section, we first develop a general approach to obtaining equilibrium by solving the equivalent optimization problem. We then derive the equilibrium in the following four situations by fixing the appropriate parameters (This paper assumes the two OWS times differ by d in flexible working situation, i.e., $K = 2$ and $t_2 - t_1 = d$.):

- Scenario (1): No policies ($\theta_R = -\infty$, $K = 1$)

- Scenario (2): Remote working ($K = 1$)
- Scenario (3): Flexible working ($\theta_R = -\infty$, $K = 2$)
- Scenario (4): Remote working and flexible working ($K = 2$)

Finally, by comparing equilibrium in scenarios (1)-(4), we analyze the effect of the interaction between remote and flexible working on traffic congestion.
To derive the equilibrium solutions systematically, let us decompose the equivalent optimization problem into a hierarchical optimization problem consisting of the master problem and sub-problem. The master problem determines the mass of office workers Q_O and remote workers Q_R. The sub-problem determines the commuting departure flow patterns $q_k(t)$, in which Q_O is the given parameter. We here introduce the hierarchical optimization problem below.

$$\text{[Master]} \quad \min_{Q_O, Q_R \geq 0}. \quad TC(Q_O) - \theta_O Q_O - \theta_R Q_R \tag{13}$$

$$\text{s.t.} \quad Q_O + Q_R = Q \qquad [\rho]. \tag{14}$$

$$\text{[Sub]} \quad TC(Q_O) \equiv \min_{\{q_k(t)\} \geq 0}. \quad \sum_{k \in \mathcal{K}} \int_{t \in \mathcal{T}} c_k(t) q_k(t) \mathrm{d}t \tag{15}$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}} q_k(t) \leq \mu \qquad \forall t \in \mathcal{T} \quad [w(t)], \tag{16}$$

$$\sum_{k \in \mathcal{K}} \int_{t \in \mathcal{T}} q_k(t) \mathrm{d}t = Q_O \qquad [\lambda]. \tag{17}$$

By solving the sub-problem, we first derive the equilibrium commuting cost as a function of the total commuting demand (the number of office workers) determined by the master problem. Subsequently, we solve the master problem using the equilibrium commuting cost.
The sub-problem has the same structure as the single bottleneck model, and we can analytically solve it by using the condition that all commuters incur the same commuting costs ($w(t) + c_k(t)$) in the equilibrium state, as shown in Figures 3 and 4. Figure 3 illustrates the equilibrium arrival/departure flow pattern at the bottleneck and the equilibrium cost pattern with $K = 1$ when the total commuting demand is X. Similarly, Figure 4 illustrates the equilibrium commuting cost pattern with $K = 2$ when the total commuting demand is X. In both figures, $\lambda(X)$ represents the equilibrium commuting cost derived as follows:

$$\lambda(X) = \begin{cases} \dfrac{X}{\mu}\delta & \text{if} \quad K = 1 \\ \min\left\{ \dfrac{X}{2\mu}\delta, \quad \dfrac{X}{\mu}\delta - d\delta \right\} & \text{if} \quad K = 2 \end{cases}, \tag{18}$$

where $\delta = \beta\gamma/(\beta + \gamma)$.
Based on the equilibrium commuting cost $\lambda(X)$, we obtain the solutions to the master problem, i.e., Q_O and Q_R. Specifically, from the optimality condition of the master problem, we find that the following relationship holds in equilibrium:

$$Q_O = \min\{\lambda^{-1}(\theta_O - \theta_R), \quad Q\}, \tag{19}$$

where $\lambda^{-1}(\cdot)$ is the inverse function of $\lambda(X)$. Thus, "the solution of equation $\theta_O - \theta_R = \lambda(X)$" or "$Q$", whichever is smaller, is the total commuting demand Q_O. By combining Q_O and the conservation condition of the workers, we find the number of remoter workers Q_R. This mathematical approach corresponds to a graphical approach determining the intersection between $\theta_O - \lambda(X)$ and θ_R. Figure 5 illustrates the intersection between θ_R and $\theta_O - \lambda(X \mid K)$. In Figure 5, intersections $\mathrm{I}^{(1)}$, $\mathrm{I}^{(2)}$, $\mathrm{I}^{(3)}$, and $\mathrm{I}^{(4)}$ represent equilibrium states for each scenario. By finding the coordinates of these

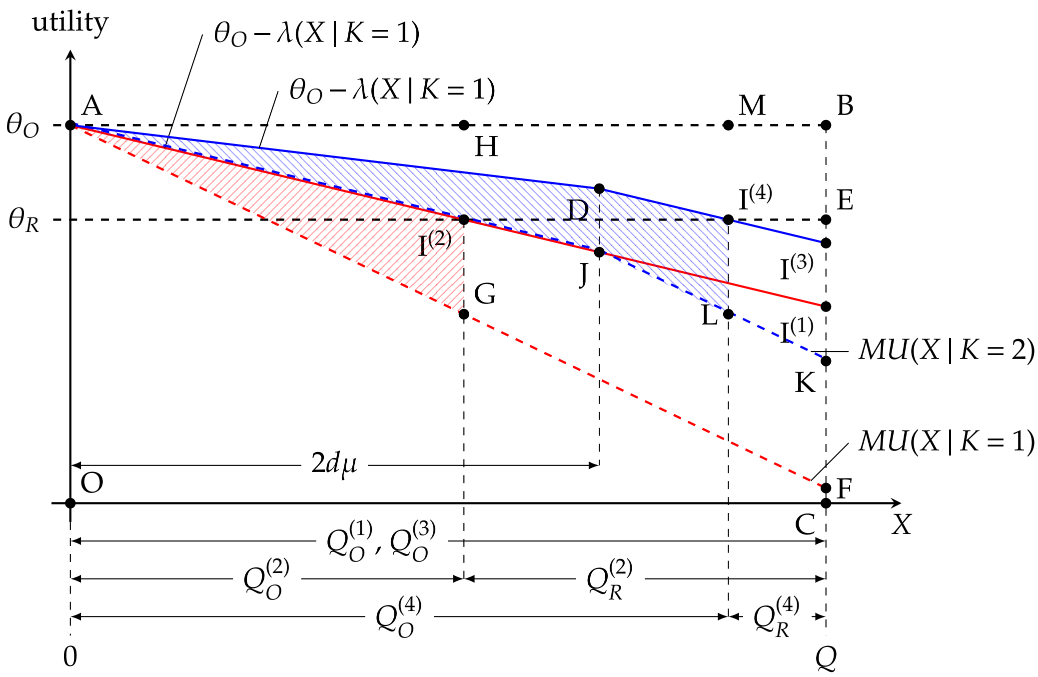

Figure 5: Equilibrium number of office/remote workers in scenarios (1)-(4).

intersections, the analytical solution for each scenario can be derived as follows[1]:

$$\text{Scenario (1):} \quad Q_O^{(1)} = Q, \quad Q_R^{(1)} = 0, \quad \lambda^{(1)} = \frac{Q}{\mu}\delta, \quad \rho^{(1)} = \theta_O - \lambda^{(1)}, \tag{20}$$

$$\text{Scenario (2):} \quad Q_O^{(2)} = \frac{\mu}{\delta}(\theta_O - \theta_R), \quad Q_R^{(2)} = Q - Q_O^{(2)}, \quad \lambda^{(2)} = \theta_O - \theta_R, \quad \rho^{(2)} = \theta_R, \tag{21}$$

$$\text{Scenario (3):} \quad Q_O^{(3)} = Q, \quad Q_R^{(3)} = 0, \quad \lambda^{(3)} = \frac{Q}{\mu}\delta - d\delta, \quad \rho^{(3)} = \theta_O - \lambda^{(3)}, \tag{22}$$

$$\text{Scenario (4):} \quad Q_O^{(4)} = \frac{2\mu}{\delta}(\theta_O - \theta_R), \quad Q_R^{(4)} = Q - Q_O^{(4)}, \quad \lambda^{(4)} = \theta_O - \theta_R, \quad \rho^{(4)} = \theta_R. \tag{23}$$

Using Eqs. (20) to (23), we can calculate the social surplus and queueing loss in each scenario. The social surplus and queueing loss correspond to the areas of the regions in Figure 5, as shown in Table 1. In Figure 5, $MU(X \mid K=1)$ and $MU(X \mid K=2)$ are social marginal utility functions for $K=1$ and $K=2$, respectively.

Table 1: Comparison of the scenarios

	Social Surplus SS	Queueing Loss QL	Schedule Loss SL
Scenario(1)	$SS^{(1)}$ = AFCO	$QL^{(1)}$ = AI$^{(1)}$F	$SL^{(1)}$ = ABI$^{(1)}$
Scenario(2)	$SS^{(2)}$ = AGI$^{(2)}$ECO	$QL^{(2)}$ = AI$^{(2)}$G	$SL^{(2)}$ = AHI$^{(2)}$
Scenario(3)	$SS^{(3)}$ = AJKCO	$QL^{(3)}$ = ADI$^{(3)}$KJ	$SL^{(3)}$ = ABI$^{(3)}$D
Scenario(4)	$SS^{(4)}$ = AJLI$^{(4)}$ECO	$QL^{(4)}$ = ADI$^{(4)}$LJ	$SL^{(4)}$ = AMI$^{(4)}$D

By comparing these areas, we obtain the following theorem:

Theorem 3.1 (Remote work effect)**.** Implementing remote work causes lower queuing loss and higher social surplus than not implementing remote work.

$$QL^{(2)} < QL^{(1)}, \quad SS^{(2)} > SS^{(1)} \tag{24}$$

[1] Strictly speaking, the cases must be divided according to θ_R. Because of space limitations, this study shows only the most standard cases.

Theorem 3.2 (Flexible work effect)**.** Implementing flexible work causes lower queuing loss and higher social surplus than not implementing flexible work.

$$QL^{(3)} < QL^{(1)}, \quad SS^{(3)} > SS^{(1)} \tag{25}$$

Theorem 3.3 (Remote work paradox)**.** Implementing remote and flexible work causes higher queuing loss with equal social surplus than implementing only remote work.

$$QL^{(2)} < QL^{(4)}, \quad SS^{(4)} = SS^{(2)} \tag{26}$$

Theorem 3.1 and Theorem 3.2 state the positive effects of remote and flexible work, respectively. In contrast, Theorem 3.3 describes a paradoxical phenomenon in which the simultaneous implementation of flexible and remote working may cause higher queueing losses. This paradox is due to the induced demand created by decreasing commuting costs for flexible working. That is, even if the flexible working spreads out the OWS time, additional traffic congestion around each OWS time occurs because the utility of office workers is equal to (to balance) the utility of remote workers in the equilibrium state.

This paradox of the relationship between flexible and remote working styles can be prevented by implementing a dynamic pricing scheme. Specifically, the road manager imposes a time-varying congestion toll that mimics the queuing delay pattern in scenario (4) to the commuters. In equilibrium under this dynamic pricing, bottleneck congestion is completely eliminated, and the utility of all workers maintain the same as before the implementation of the pricing. Since this pricing scheme gives the road manager toll revenue equal to queuing loss, the utility of the workers increases if the toll revenue is appropriately returned to workers. These results imply that combining multiple policies can affect efficiency and highlight the importance of analyzing the combined effects of multiple policies.

4 Conclusion

This study investigated the impact of flexible and remote work on traffic congestion using the bottleneck model. We first formulated an integrated equilibrium model that simultaneously considered three worker choices: office/remote work, OWS time, and departure time choices. Furthermore, we elucidated that the equilibrium model had an equivalent optimization problem. We derived the equilibrium solution using a hierarchical decomposition approach and calculated the social surplus and queuing loss under various situations. Comparing these situations showed a paradoxical phenomenon in which queuing losses were higher while social surplus remained constant when flexible and remote work policies were considered simultaneously than when only remote work policies were considered. We conclude that the cause of this paradox is the demand induced by reduced commuting costs for flexible working.

Future studies may investigate a more general model and reveal the impact of the relationships between various working styles on traffic congestion. Specifically, we may extend the network structure to corridor networks with multiple residential areas. This extension analysis allows us to understand the impact of the relationships between flexible and remote working styles on the location choice of workers. We also must model the agglomeration economics of office work and investigate the policy effects more precisely, considering the trade-offs between the office and remote work.

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