Application of a Metamodel-Based Optimization Approach for Toll Optimization and its comparison with Metaheuristics-based Model Optimization via a Case Study.

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SHORT SUMMARY

First, a concept of metamodel-based optimization, in which a transport economics inspired model acts as a metamodel over an underlying set-up of directly interfaced transport models, is discussed. Then, a toll optimization scenario including a city and its neighboring rural municipalities is developed and a case study concerning its cooperative version is presented. The metamodel for this case study involves the player(s) optimizing their objective based on a schematic network, and simplified cost and demand functions, whereas the underlying set-up is a Static Traffic Assignment over the physical network with physical origin-destination elastic demand. This new metamodel-based optimization is then compared with traditional metaheuristics-based optimization. Results show that the new approach not only leads to lower computational expense but even outperforms metaheuristics-based optimization in terms of optimality.

Keywords: Game theoretical interactions in mobility, Pricing and capacity optimization, Transport economics and policy, Metamodel-based optimization, Transportation network modelling

1. INTRODUCTION

Road pricing is an important topic for many transportation stakeholders. Comprehensive analysis of tolling schemes should consider interactions between transportation subsystems/stakeholders (travelers, mobility service providers, and local network operators) and other systems (neighboring governments' network operators, housing market, urban design/land-use).

Thus, for a particular tolling scheme, the challenge for modelers is to 1) identify relevant (sub)systems/stakeholders and their interactions. 2) develop models that take into account these interactions in a more elaborate way than fixed inputs or unidirectional influences. 3) develop mechanisms for computation of consistent impacts on all interrelated (sub)systems/stakeholders. There have been attempts to approach solutions based on three major approaches:

1. **All-encompassing micro-models:** These are highly detailed and disaggregate models in which every relevant player is modelled at a micro level e.g., the Multi-Agent Transport Simulation (MATSim) framework (Horni, Nagel, & Axhausen, 2016). Such frameworks can provide information at a very disaggregate level to the stakeholders allowing analysis from equity as well as efficiency perspective. However, developing, extending,

calibrating, maintaining, and interpreting such models require substantial effort, and game-theoretical analyses involving multiple stakeholders with different objectives can be prohibitively expensive.

2. **Tailor-made simplified conceptual models**: On the other end of the spectrum lie simplified conceptual models which are often used by transport economists (B. De Borger, Dunkerley, & Proost, 2007; Bruno De Borger & Proost, 2021). These are comparatively easier to develop, calibrate, maintain, and interpret. It is also easy to tailor them and focus only on the relevant stakeholders/(sub)systems and their interactions for a particular problem. However, they involve extensive simplifications of the underlying (sub)systems; thus, they only provide highly aggregate and schematic results which is, usually, not sufficient for aiding actual decisions.
3. **Tailor-made directly interfaced traditional mono-disciplinary models:** Traditionally, a toll optimization problem is solved by using a bi-level optimization framework in which toll is altered in an outer loop around the Static Traffic Assignment (STA) (Ekström, Rydergren, & Sumalee, 2014). Nowadays, to account for interactions with other (sub)systems, dedicated interfaces between relevant mono-disciplinary transportation models are being built within the inner loop e.g., the connection between activity-based demand model and STA in Strategisch Personen Model Vlaanderen (Vanderhoydonc & Borremans, 2020). This approach may provide flexibility and produce disaggregate results. However, solving complex optimization problems involving several stakeholders can still be extremely sluggish because: a) As the number of models increase, attaining consistency via bi-directional interfaces becomes computationally quite expensive. b) Due to the possibility of only marginal steps in the optimization variables, the risk of getting stuck in local stationary points is quite high.

It can be appreciated that none of these three approaches offers the combination of scalability, detail, and flexibility required by contemporary transportations problems.

Inspired from the complementary characteristics of the 2nd and 3rd approaches mentioned above, **we aim to use a transport economics inspired conceptual model as a metamodel to find an optimal toll for an underlying set-up of directly interfaced traditional transport models**. In this way, the underlying set-up only has the computational load related to achieving consistency between the directly interfaced models, whereas the computational load for optimization lies completely at the metamodel level. At every iteration, the underlying set-up is used to (re-)calibrate the metamodel which includes a simpler and more aggregate version of all the relevant stakeholders/(sub)systems. Toll optimization is performed for the meta-model and optimal tolls are transmitted to the underlying set-up. At the new tolls, the underlying set-up is evaluated again, and the meta-model is recalibrated at the new point. This sequence is repeated until a certain level of convergence is achieved in the optimal toll values.
The objective of this paper is to present the development and results of a **proof of** this **concept** and thereby, determine 1) the optimality and 2) the computation speed of this framework.

2. METHODOLOGY

The fictional problem considered for the proof of concept is as follows: the city municipality is looking to impose two non-discriminatory cordon tolls i.e., an entry toll each for radial and ring roads with the intention of curbing the use of city infrastructure by transit traffic. To avoid the rerouting of transit traffic to their infrastructure, rural municipalities come together to charge an entry toll for the neighboring rural territory.

The framework has three main parts: 1) The underlying Set-up 2) the metamodel 3) the calibration interface between underlying set-up and the metamodel. **Figure 1** shows a block diagram representing a basic instance of this framework.

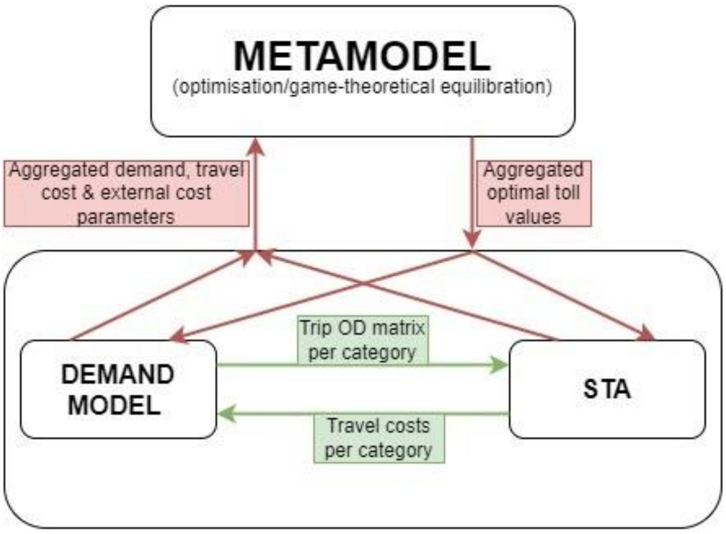

Figure 1: Basic instance of framework

Underlying Set-up

The underlying set-up, for this exercise, consists of an STA for a fictional city and a simple demand model with linear elasticities per OD.

<u>Static Traffic Assignment (STA):</u>
Details of STA are mentioned in **Table 1**.
Figure 2 and **Figure 3** show the network and the zoning respectively.

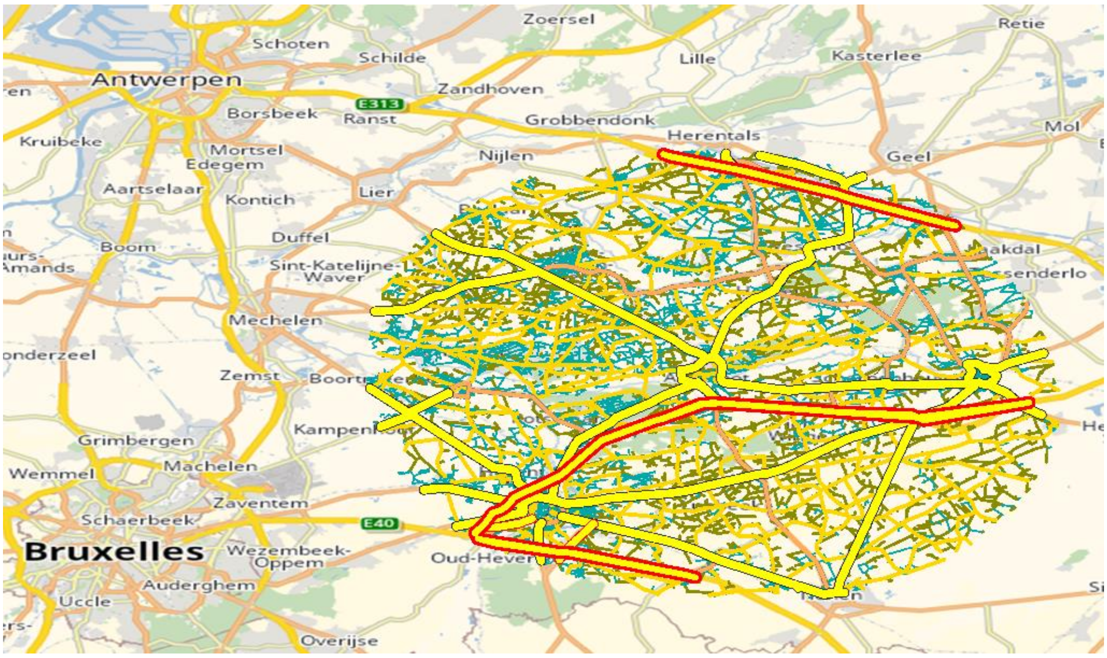

Figure 2: **Network shown against the background of OpenStreetMap**

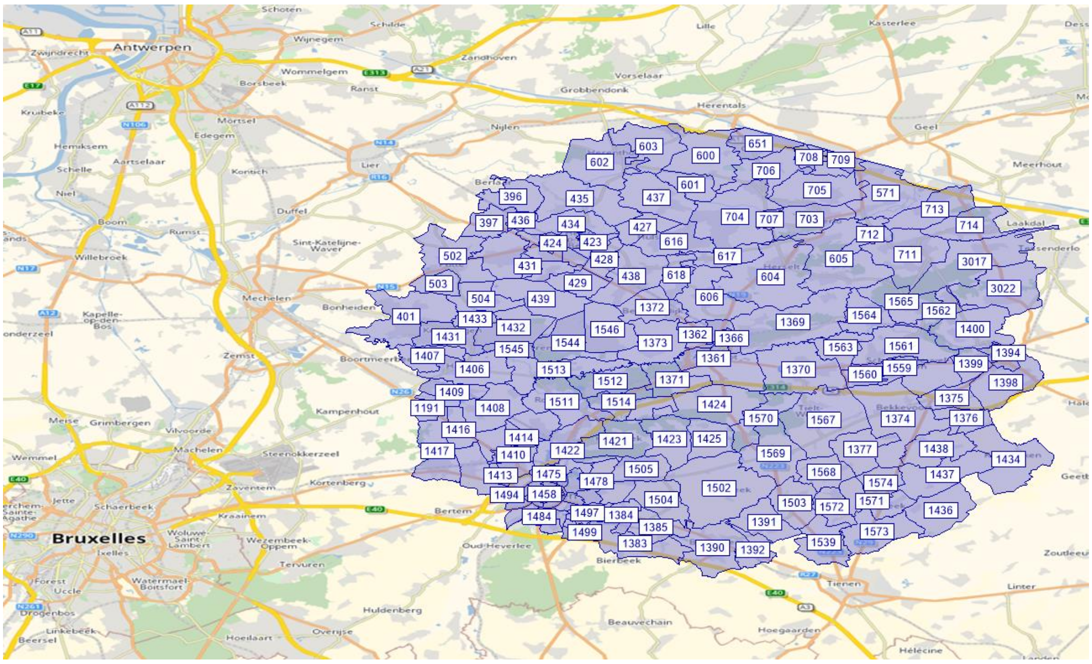

Figure 3: 173 zones used for the study

Table 1: Static Traffic Assignment details [1: (Boeing, 2017), 2: (Vanderhoydonc & Borremans, 2020), 3: (Gentile, 2014)]

Network		
	Centre	Aarschot
	Buffer	17 kms
	No. of nodes	43440
	No. of links	143368
	Source	OpenStreetMap
	Comment	using OSMnx (1)
Demand Data		
	Centre	Aarschot
	Buffer	17 kms
	No. of zones	173
	OD Matrix size	173 X 173
	Source	Belgium-wide data by Flemish Road Authority (2)
	Comment	Additional in-house mining on top of source
Assignment		
	Software	PTV Visum
	Method	LUCE (3)

To include transit traffic through Aarschot, a proportion of the external demand of interest is projected onto the periphery zones. The city and the neighboring municipalities are assumed to have jurisdiction over Territory 6 and Territory 5 respectively (**Figure 4**). The three tolls are added in units of time to travel costs (BPR) of the appropriate **entry** links.

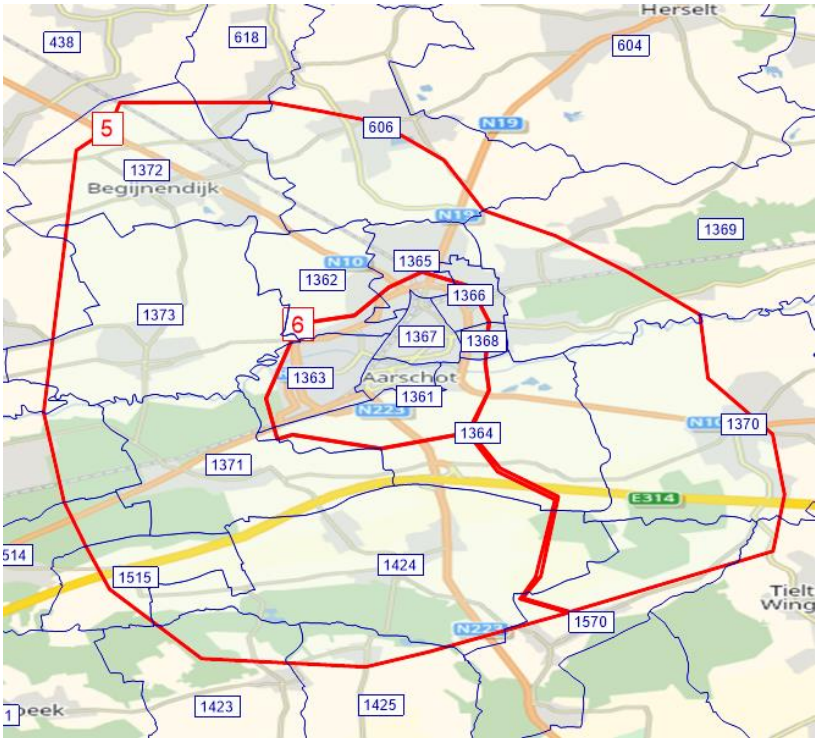

Figure 4: Territory 6 is under city jurisdiction and Territory 5 is under jurisdiction of rural municipalities.

<u>Demand Model:</u>
The demand model is a standard linear inverse demand function of the type mentioned in **Equation 1** and it is used to get the demand (D_{od}) for each of the 173*173 = 29929 OD pairs as a function of their cost skims (C_{od}).

$$A_{od} - B_{od} * D_{od} = C_{od} \qquad (1)$$

For this study, A_{od} and B_{od} have been derived by using the reference OD matrix and reference Cost Skim matrix (obtained after assigning reference OD matrix) and a realistic assumption about the maximum possible demand (at zero cost) for each OD pair.

The horizontal interface between the STA and the demand model shown in **Figure 1** is solved as a fixed-point problem with Method of Successive Averages (MSA) smoothening. In future, more advanced demand models, such as activity-based demand models, may replace this basic demand model.

Metamodel

The metamodel is inspired from transport economics models (Bruno De Borger & Proost, 2021). The schematic network chosen for this case study is shown in **Figure 5** and other details are mentioned in **Table 2.**

Table 2: Details of metamodel network

No. of nodes	9
No. of links	13
Centroid nodes	1, 4, 7, 8 and 9
Left to Right links	All except 9 and 12
Right to left links	9 and 12
Radial links	5 and 6
Ring links	7, 4 and 8
Rural links	3, 9, 10, 11 and 12
Radial toll	link 5
Ring toll	link 7
Rural toll	links 10 and 12

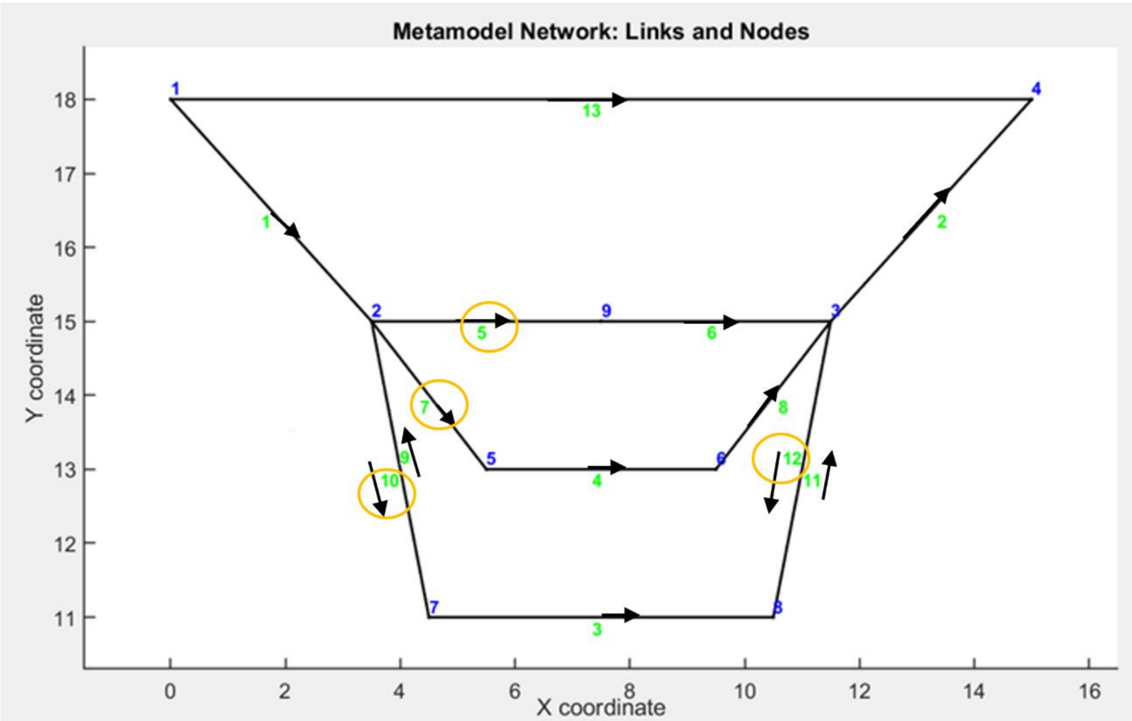

Figure 5: **Network for metamodel: Links (green labels), Nodes (blue labels) and Tolled Links (yellow circles)**

Links are assumed to have a linear congestion cost function (lc_i) of the type:

$$lc_i = a_i + b_i * f_i \qquad (2)$$

Demand is assumed to be comprised of ten schematic OD pairs between the five centroids (see **Table 3**). There are nineteen paths for the ten OD pairs. The paths using radial links, ring links and rural links are mentioned as ‘r’, ‘R’, and ‘M’ respectively. For OD pair 1_4, the path that escapes all three tolls is mentioned as ‘e’ in **Table 3**. The demand in metamodel is also assumed to be elastic with linear inverse demand functions of the type mentioned in **Equation 1** with A_{od} and B_{od} (for each of the 10 OD pairs) being calibration parameters for the metamodel.

Table 3: Metamodel OD pairs

S.No.	OD pair (from_to)	Available paths	Explanation (from_to)	Code name
1	1_4	M,R,r,e	external_external	OD
2	1_7	M	external_closerural	ON
3	1_8	M,R,r	external_opprural	ONopp
4	1_9	r	external_city	OA
5	7_4	M,R,r	opprural_external	ND
6	8_4	M	closerural_external	NoppD
7	9_4	r	city_external	AD
8	7_8	M,R,r	rural_rural (opp.)	MoMd
9	7_9	r	rural_city	MoA
10	9_8	r	city_rural	AMd

Since the scope of the paper is only to provide a proof of concept, the metamodel is used only for solving joint optimization problem of the city and rural municipalities as opposed to solving Nash-Cournot or Stackelberg competitions. The objective function for this joint optimization (minimization) is a quadratic function of the optimization variables i.e., three tolls (T_{Tli}) and nineteen path flows (X_{pi}). It is a sum of (negative) user welfare, total costs including tolls, external costs and (negative) total revenue from tolls. Objective function and constraints are shown in **Equations 3-11**.

$$Obj = -Wlf + TC + EC - TR \qquad (3)$$

where:

$$Wlf = \sum_{od=1}^{10}(A_{od} * D_{od} - 0.5 * B_{od} * D_{od}^2) \qquad (4)$$
$$TC = \sum_{pi=1}^{19}(C_{pi} * X_{pi}) \qquad (5)$$
$$EC = \sum_{li=1}^{13}(\lambda_{li}c_{li} * f_{li}) \qquad (6)$$
$$TR = \sum_{Tli=1}^{4}(T_{Tli} * f_{Tli}) \qquad (7)$$

$$subject\ to:$$
$$all\ X_{pi} \geq 0, all\ T_{Tli} \geq 0 \qquad (8)$$
$$Wardrop's\ equilibrium\ (route - choice) \qquad (9)$$
$$Elastic\ demand\ equilibrium \qquad (10)$$
$$all\ T_{Tli} \leq T_{Max} \qquad (11)$$

Wardrop's equilibrium condition makes the problem highly non-convex because of which special checks are required for ensuring global optimality. Details on the solution methods for the metamodel are the subject of a forthcoming paper (Malik & Tampère, n.d.).

Interface

Calibration Interface:

At the beginning of each new metamodel optimization routine, the metamodel is (re-)calibrated by using the underlying set-up. This is done by the calibration interface (represented by red lines going up in **Figure 1**). Specifically, it calibrates: 1) elastic demand parameters for the metamodel i.e., the ten A_{od} and B_{od} mentioned in the previous section, 2) cost parameters of the thirteen metamodel links i.e., a_i and b_i (**Equation 2)** and 3) thirteen external cost parameters i.e., λ_{li} (**Equation 6)**.

The zeroth step in this calibration is classifying each of the 29929 physical OD pairs as belonging to either one of the ten metamodel OD pairs. This is performed as a preprocessing step using geometrical logic. **Figure 6** provides an example of the classification for the categories of 1_4, 1_7, 1_8 and 1_9. Red area represents the city territory and pink + violet areas represent rural territory. For an external origin **1**, an OD pair is classified based on the area in which the destination lies i.e., 7, 8, 9 or 4. A similar process is followed for the six remaining OD pair categories.

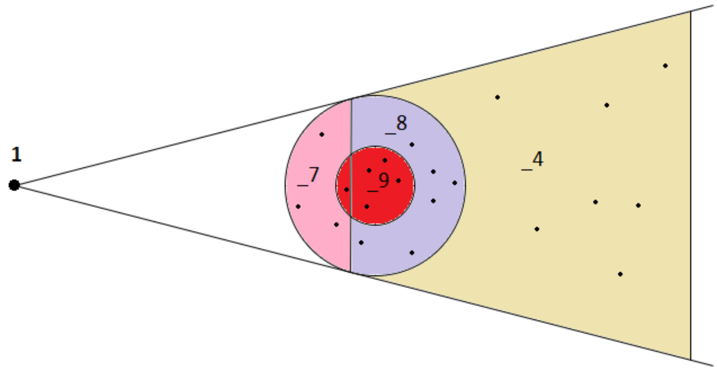

Figure 6: **Geometric logic for classification of physical OD pairs as 1_4, 1_7, 1_8 and 1_9 in metamodel.**

Then, for each meta-OD category, demands of all belonging physical OD pairs are summed together to give category demand (=metamodel demand for that OD pair) and category cost is given by the average of belonging physical OD cost skims weighted by their corresponding maximum possible demand levels. We do this at two closest elastic demand equilibrium points of the underlying set-up and use them to find locally linearized aggregated inverse demand parameters i.e., the ten A_{od} and B_{od} for the metamodel.

The next step is to calibrate link cost parameters of **Equation 2.** We do this by estimating two flow-cost points. The flows of each of the metamodel links are estimated by using their physical interpretation e.g., flow on link 1 represents all traffic that's entering the two territories shown in **Figure 4**. Then, the corresponding metamodel link costs can be estimated by using the total vehicle hours spent on the real physical network on links associated with a particular metamodel link. By equating the total vehicle hours on the real and metamodel links, we find the metamodel link costs after already having estimated metamodel link flows. Repeating this for a marginally higher demand level gives us another metamodel link flow-cost point. This allows us to find a_i and b_i i.e., locally linearized meta-model link cost parameters.

For calibrating the external cost parameters λ_{li} for **Equation 6**, we need to assume certain external costs in the underlying set-up first. We used a parameter Λ each for the rural, radial and ring physical links. Then, the obvious choice of λ_{li} for: 1) links 5 and 6 is Λ_{radial}, 2) links 7, 4 and 8 is Λ_{Ring}, and 3) links 9, 10, 11 and 12 is Λ_{rural}. λ_{li} for remaining metamodel links is zero.

<u>Toll Interface:</u>
This interface is represented by red lines coming down in **Figure 1**. After the calibration interface has calibrated the metamodel, the metamodel computes optimal tolls for that version of the metamodel. The Toll interface applies an MSA smoothening step on these optimal tolls using the

optimal tolls of the last iteration. The smoothened tolls are then implemented in the underlying set-up as tolls for the next iteration.

Metaheuristics-based optimization for benchmarking

We aim to evaluate optimality and speed of this metamodel-based optimization. This necessitates the benchmarking of this framework against a state-of-the-art metaheuristics-based optimization framework. Traditionally, metaheuristics like Simulated Annealing (SA) and Genetic Algorithms (GA) are used for optimization of such a large-scale network. However, we use SHERPA ("SHERPA," n.d.) as it has been shown to significantly outperform SA and GA both in terms of efficiency and robustness.

As compared to **Figure 1**, the same underlying set-up is used in this case as well. However, the interface i.e., the calibration, and the toll interface and the metamodel are replaced by a toll optimization outer loop. As mentioned in the introduction, computational loads for both achieving horizontal consistency as well as finding optimal tolls are combined in this case. The objective function for optimization is formed completely analogous to **Equations 3 -7**; however, in this case, it is formulated using 29929 OD pairs and 143,368 links.

3. RESULTS AND DISCUSSION

We obtained results for the case when both the city and the rural municipalities cooperatively optimize the three tolls. Bounds of $0 \leq T \leq 4$ were used for the tolls. Λ_{radial}, Λ_{Ring} and Λ_{rural} were set to 40, 0 and 10 respectively. For SHERPA based optimization, the total number of evaluations was set to 25. The value of objective function for successive designs is shown in Figure 7 and Figure 8. The design with Design Id = 23 proved to be the best design with an objective value of 128.877 vehicle-hours and the corresponding best tolls were T = [3.92, 0.64, 1.4] hours.

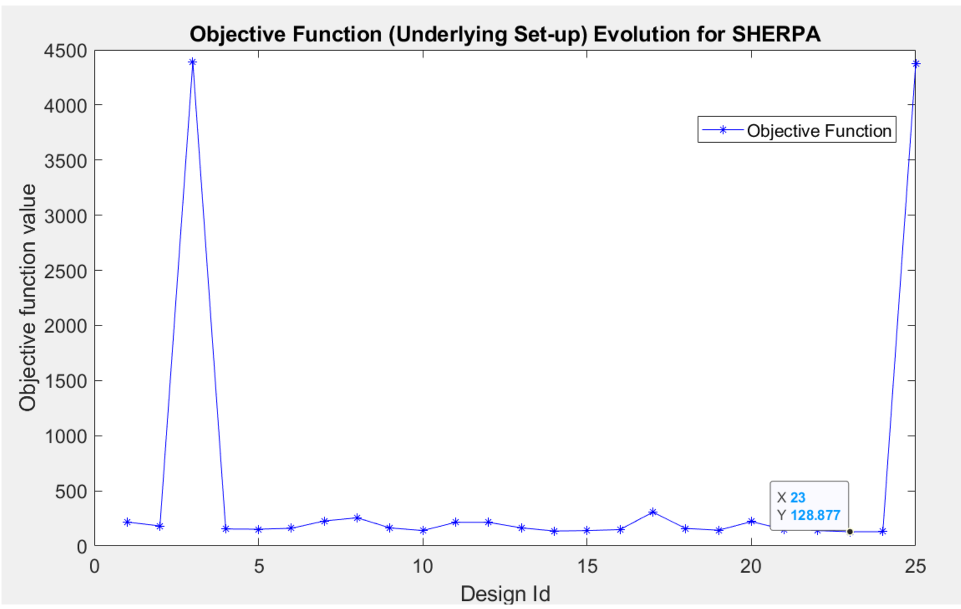

Figure 7: Objective Function (veh.-hr.) for successive designs in SHERPA-based Optimization (Data Tip shows the Best Design)

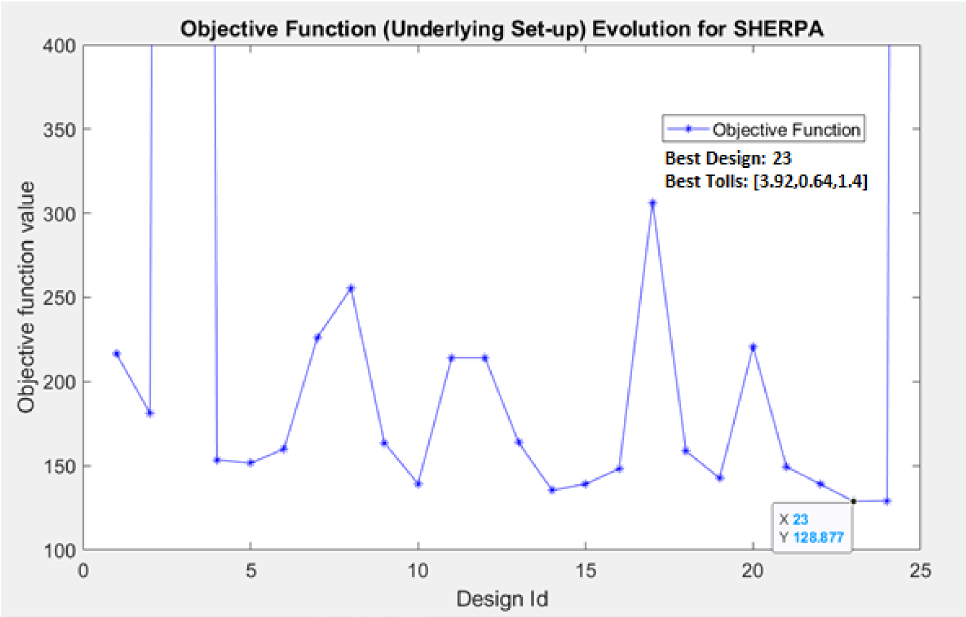

Figure 8: Objective Function (veh.-hr.) for successive designs in SHERPA-based Optimization (ignoring outliers)

For metamodel-based optimization, the number of evaluations was set to 10. The evolution of objective function and tolls values over successive iterations is shown in **Figure 9** and Figure 10 respectively. The iterations start with an educated guess of T0 motivated from the values of $[\Lambda_{radial}, \Lambda_{Ring}, \Lambda_{rural}]$. This initial guess proved to be a highly favorable point as the objective value is extremely low; however, the tolls (and consequently the objective function) move quite aggressively away from this point in next MSA-iteration. Regardless, it is remarkable that in the subsequent iterations, the model, almost monotonously, manages to find its way back to values significantly lower than those suggested by the best design of SHERPA-based optimization. It is even more encouraging that it did that within the first 6 iterations while not directly optimizing the actual objective function on which optimality is evaluated but instead an aggregated and linearized version of it.

Results of the two approaches are summarized in **Table 4**. It should be noted that for additional players e.g., in Nash Cournot/ Stackelberg scenarios, computational time for SHERPA/metaheuristics-based approach will increase exponentially but for metamodel-based approach, it will stay practically the same.

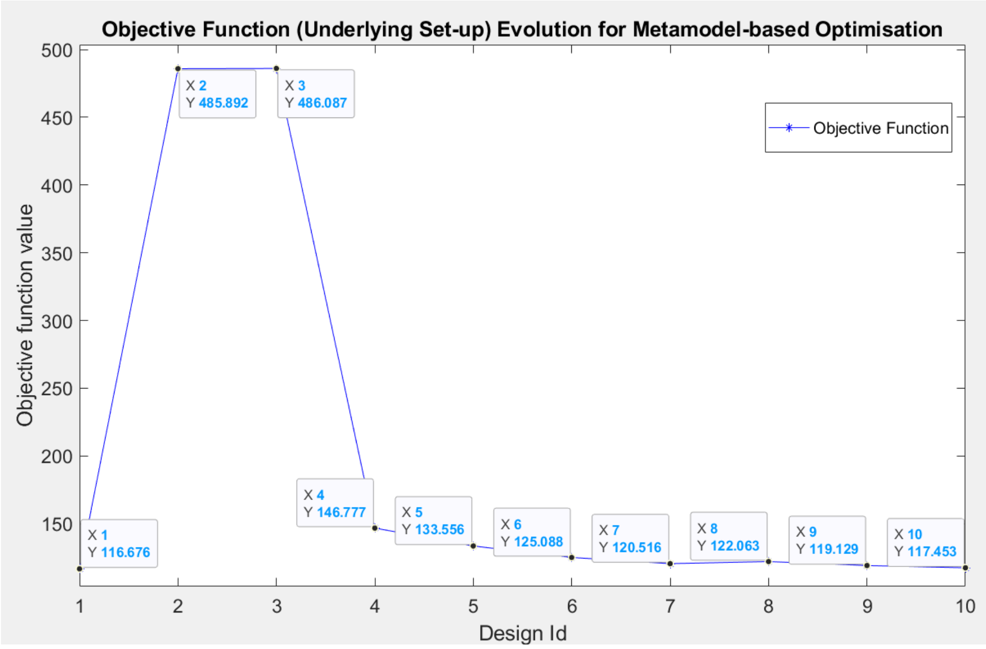

Figure 9: Evolution of Objective Function (veh.-hr.) for Metamodel-based Optimization

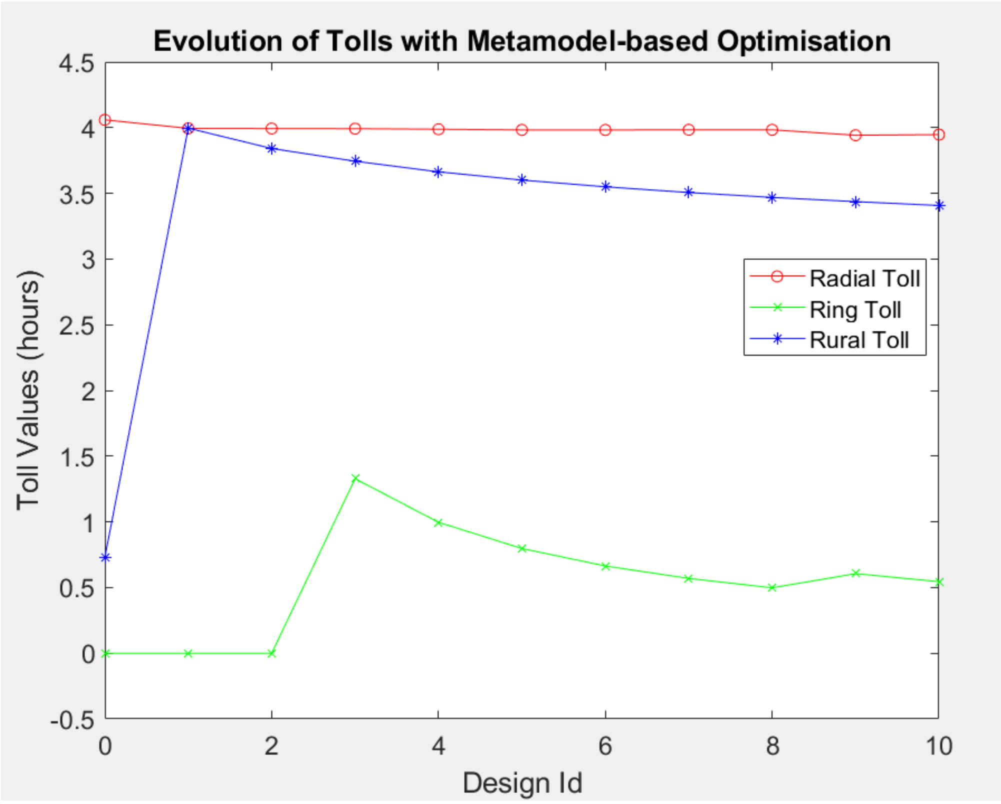

Figure 10: Evolution of Tolls with Metamodel-based Optimization

Table 4: Comparison of two approaches

Optimisation type	SHERPA-based	Metamodel-based
No. of iterations	25	10
Computation time per iteration	13-25 mins	15-26 mins
Best Tolls (hr.)	[3.92, 0.64, 1.4]	[3.94, 0.61, 3.43]
Objective function value of best design	128.77 veh. hr.	117.453 veh. hr
Total computation time (both run parallely)	8h 6m 5s	3h 39m 58s

4. CONCLUSIONS

This paper discussed a new metamodel-based optimization approach in which a transport economics inspired metamodel is used for optimizing the underlying directly interfaced traditional transport models. A case study about a joint toll optimization problem of a fictional city municipality and its neighboring rural municipalities is developed and presented as a proof of concept of this approach. Preliminary results suggests that the problem can be solved at a much lower computational cost and with appreciable accuracy in terms of optimality of tolls. This study serves as a motivation for interfacing additional models in the underlying set-up as well as solving Nash Cournot and Stackelberg competition scenarios in the metamodel where traditional metaheuristics-based optimization can be prohibitively expensive.

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