On the Computation of Accessibility Provided by Shared Mobility

Severin Diepolder*[1], Andrea Araldo[2], Tarek Chouaki[3], Santa Maiti[4], Sebastian Hörl[5], and Costantinos Antoniou[6]

[1]Student, Technical University of Munich, Germany
[2]Associate Professor, SAMOVAR, Institut Polytechnique de Paris, France
[3]PhD Candidate, SystemX, France
[4]Postdoc, Technical University of Munich, Germany
[5]PhD, SystemX, France
[6]Professor, Technical University of Munich, Germany

Short summary

Shared Mobility Services (SMS), e.g., Demand-Responsive Transit (DRT) or ride-sharing, can improve mobility in low-density areas, often poorly served by conventional Public Transport (PT). Such improvement is mostly quantified via basic performance indicators, like wait or travel time. However, accessibility indicators, measuring the ease of reaching surrounding opportunities (e.g., jobs, schools, shops, ...), would be a more comprehensive indicator. To date, no method exists to quantify the accessibility of SMS based on empirical measurements. Indeed, accessibility is generally computed on graph representations of PT networks, but SMS are dynamic and do not follow a predefined network. We propose a spatial-temporal statistical method that summarizes observed SMS trips in a graph on which accessibility can be computed. We apply our method to a MATSim simulation study concerning DRT in Paris-Saclay.

Keywords: Accessibility; Public Transport; Shared Mobility;

1 Introduction

Location-based accessibility measures the ease of reaching surrounding opportunities via transport (Miller (2020)). Accessibility provided by conventional PT is generally poor in low-demand areas, e.g., suburbs (Badeanlou et al. (2022)), because a high frequency and high coverage service in such areas would imply an unaffordable cost per passenger. Poor PT accessibility in the suburbs makes them car-dependent, which prevents urban regions from being sustainable ((Saeidizand et al., 2022, Section 2.2)). SMS, e.g., Demand-Responsive Transit (DRT), ride-sharing, carpooling, car-sharing, are potentially more efficient than conventional PT in the suburbs (Calabrò (2023)). However, their current deployment is commonly led by private companies targeting profit maximization. This may turn SMS into additional source of congestion and pollution (Henao & Marshall (2019); Erhardt et al. (2019)).

We believe that SMS deployment should be overseen by transport authorities under the logic of accessibility improvement. To this aim, a method is needed, able to compute impact of SMS on accessibility, based on empirically observed trips. To the best of our knowledge, this paper is the first to propose such a method. Chandra et al. (2013) study how DRT improves connection to conventioal PT stops, without considering the impact on accessing opportunities. Nahmias-Biran et al. (2021) and Zhou et al. (2021) calculate based accessibility from Autonomous Mobility on Demand, based on utilities perceived by agents within simulation. By contrast, our method computes accessibility solely based on observed SMS trip times, either from the real world or simulation. A first attempt of integrating SMS into the graph-based description of PT is done by Le Hasif et al. (2022). However, they use analytic models to model SMS performance and thus fail to give real insights adapted to the areas under study. Our effort consists instead of estimating accessibility from empirical observations via spatial-temporal statistics. General Transit Feed System (GTFS) is the standard data format for PT schedules. Recently, the GTFS-Flex extension allows also describing SMS (Craig & Shippy (2020)). Although uur estimates could thus be fed into GTFS-Flex data, for the sake of simplicity, we use plain GTFS instead.

Our contribution consists in developing a spatial-temporal statistical pipeline to transform SMS trip data observations in a graph representation, on top of which well-established accessibility

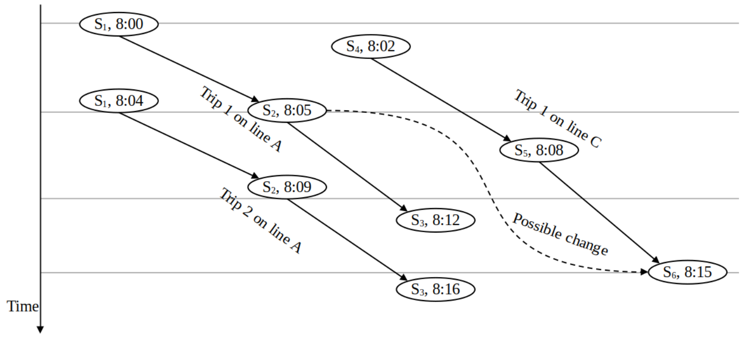

Figure 1: Time-expanded graph, representing two trips on line A and one trip on line B, as well as a potential change.

computation can be performed. The observations that can be taken as input might come from real measurements or from simulation. This paper's observations come from a MATSim simulation study of DRT deployment in Paris Saclay, from Chouaki et al. (2023).
By providing a first method to compute the accessibility of SMS on empirical observations, this work can contribute to a better understanding of the potential of SMS and guide their future deployment.

2 Methodology

Accessibility

As in (Biazzo et al. (2019)), the study area is tessellated in hexagons with a grid step of 1km, whose centers $\mathbf{u} \in \mathbb{R}^2$ are called centroids and denoted with set $\mathcal{C} \subseteq \mathbb{R}^2$. Each hexagon contains a certain quantity of *opportunities*, e.g., jobs, places at school, people. With $O_\mathbf{u}$ we denote the opportunities in the hexagon around $\mathbf{u}$ and with $T(\mathbf{u}, \mathbf{u}', t)$ the time it takes to arrive in $\mathbf{u}'$, when departing from $\mathbf{u}$ at time t. As in Miller (2020), accessibility is the amount of opportunities that one can reach departing from $\mathbf{u}$ at time of day t within time τ:

$$acc(\mathbf{u}) \equiv \sum_{\mathbf{u}' \in \mathcal{C}(\mathbf{u},t)} O_{\mathbf{u}'}. \tag{1}$$

$\mathcal{C}(\mathbf{u}, t) = \{\mathbf{u} \in \mathcal{C} | T(\mathbf{u}, \mathbf{u}', t) \leq \tau\}$ is the set of centroids reachable within τ. By improving PT, such set can be enlarged such as to consent to reach more opportunities. In this work, the opportunities are the number of people (residents) that can be reached. $T(\mathbf{u}, \mathbf{u}', t)$ is always computed on a graph representation of the transport network. However, SMS are not based on any network. Our effort is thus to build a graph representation of SMS, despite the absence of a network model.

Time-Expanded Graph Model of conventional PT

Inspired by Fortin et al. (2016) and Le Hasif et al. (2022), we model PT as a time-expanded graph $\mathcal{G}$, compatible with the GTFS format. The nodes of $\mathcal{G}$ are *stoptimes*. Stoptime $(\mathbf{s}, t)$ indicates the arrival of a PT vehicle at stop $\mathbf{s} \in \mathbb{R}^2$ (modeled as a point in the plane) at time $t \in \mathbb{R}$. Different trips on a certain line are represented as sequences of different stoptimes, as in Figure 1, as well as potential line change, within 15 minutes walk, assuming 5 Km/h walk speed, if it is possible to arrive at the new line on time. When a user departs at time t_0 from location $\mathbf{x}$ for location $\mathbf{x}'$, they can simply walk (but no more than the maximum walk time). Or they can walk to $\mathbf{s}$, board a PT vehicle at t (corresponding to a stoptime $(\mathbf{s}, t)$, use PT up to a stoptime $(\mathbf{s}', t')$ and from there walk to $\mathbf{x}'$. The arrival time at $\mathbf{x}'$ will be t' plus the time for walking. Users are assumed to always choose the path with the earliest arrival time. Path computation is performed within CityChrone (Biazzo et al. (2019)). No capacity constraints are considered.

Integration of shared mobility into the time-expanded graph

SMS is assumed to provide a feeder service to traditional PT. In a feeder area $\mathcal{F}(\mathbf{s}) \subseteq \mathbb{R}^2$ around some selected stops $\mathbf{s}$ (which we also call *hubs*), SMS provide connection to and from $\mathbf{s}$. The set

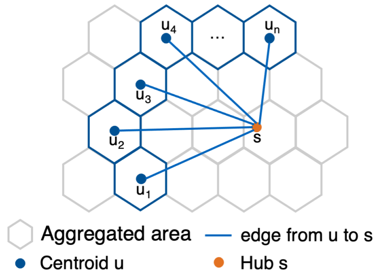

Figure 2: Hub and virtual trips provided by SMS

of centroids in such an area is $\mathcal{C}(\mathbf{s}) = \mathcal{C} \cap \mathcal{F}(\mathbf{s})$. In this section, we will focus on access trips (from a location to a PT stop) performed via SMS. The same reasoning applies to egress trips, *mutatis mutandis*. We assume to have a set $\mathcal{O}$ of observations. Each observation $i \in \mathcal{O}$ corresponds to an access trip and contains:

- Time of day $t_i \in \mathbb{R}$ when the user requested a trip to the flexible service
- Location $\mathbf{x}_i \in \mathbb{R}^2$ where the user is at time t_i
- Station s_i where the user wants to arrive via the SMS feeder service
- Duration w_i indicating the wait time before the user is served: it can be the time passed between the time of request and the time of pickup from a vehicle, in case of ride-sharing, DRT or carpooling; it can be the time to wait until a vehicle is available at the docks in a car-sharing or bike-sharing system.
- Travel time y_i: time spent in the SMS vehicle to arrive at $\mathbf{s}$.

We interpret y_i and w_i as realizations of spatial-temporal random fields (Handcock & Wallis (1994)): for any time of day $t \in \mathbb{R}$ and physical location $\mathbf{x} \in \mathcal{F}(\mathbf{s})$, random variables $W^{\mathbf{s}}(\mathbf{x}, t), Y^{\mathbf{s}}(\mathbf{x}, t)$ represent the times experienced by a user appearing in t and $\mathbf{x}$, for any stop $\mathbf{s}$. In the following subsection we will compute estimations $\hat{w}^{\mathbf{s}}(\mathbf{u}, t), \hat{y}^{\mathbf{s}}(\mathbf{u}, t)$ of expected values $\mathbb{E}[W^{\mathbf{s}}(\mathbf{u}, t)], \mathbb{E}[Y^{\mathbf{s}}(\mathbf{u}, t)]$ at centroids $\mathbf{u} \in \mathcal{C}(\mathbf{s})$.
To integrate SMS into PT graph $\mathcal{G}$, SMS are represented as a set of "virtual" trips, running between centroid $\mathbf{u} \in \mathcal{C}(\mathbf{s})$ and hub $\mathbf{s}$ (Figure 2). Each trip has travel time $\hat{y}^{\mathbf{s}}(\mathbf{u}, t)$. The access connection between centroid $\mathbf{u}$ and hub $\mathbf{s}$ is modeled as a sequence of trips, corresponding to stop times $(\mathbf{u}, t_j)$, for different values of departure time. We thus have to compute the list of such departure times. To do so, we interpret the inter-departure time between sich trips as a random field $H^{\mathbf{s}}(\mathbf{x}, t)$, which represent a "virtual" headway. The value of such an interval in $\mathbf{x}$ and t is also a spatial-temporal random field. We use the common approximation $H^{\mathbf{s}}(\mathbf{x}, t) = 2 \cdot W^{\mathbf{s}}(\mathbf{x}, t)$, idealizing the SMS headway as regular so as to apply (2.4.28) from Cascetta (2009). Therefore, we separate stoptimes by $2 \cdot w^{\mathbf{s}}(\mathbf{u}, t)$. More precisely, the stoptimes corresponding to access trips departing from centroid $\mathbf{u}$ to hub $\mathbf{s}$ are:

$$\begin{array}{lll} (\mathbf{u}, t_0), & & \\ (\mathbf{u}, t_j) & \text{where } t_j = t_{j-1} + 2 \cdot \hat{w}^{\mathbf{s}}(\mathbf{u}, t_{j-1}) \text{for } j = 1, 2, & \text{until 11:59 pm}, \\ (\mathbf{u}, t_j) & \text{where } t_j = t_{j+1} - 2 \cdot \hat{w}^{\mathbf{s}}(\mathbf{u}, t_{j+1}) \text{for } j = -1, -2, & \text{until 00:00 am}. \end{array} \tag{2}$$

Correspondent stoptimes are added to represent the arrival of access trips $(\mathbf{s}, t_j + \hat{y}^{\mathbf{s}}(\mathbf{u}, t_j))$ and an edge between each departure stoptime and the respective arrival stoptime is added. A similar process is applied for egress trips. At the end of the described process, time-expanded graph $\mathcal{G}$ is enriched with stoptimes and edges representing SMS trips. Having done so, it is possible to reuse accessibility calculation methods for time-expanded graphs, such as CityChrone Biazzo et al. (2019), with no modifications required.

Estimation of Waiting and Travel Times

We now explain how we construct estimation $\hat{w}^{\mathbf{s}}(\mathbf{u}, t)$ used in the previous subsection, for access SMS trips only. Similar reasoning can be applied to $\hat{y}^{\mathbf{s}}(\mathbf{u}, t)$ and egress trips. We assume random field $W^{\mathbf{s}}(\mathbf{x}, t)$ is approximately temporally stationary within each timeslot:

$$W^{\mathbf{s}}(\mathbf{x}, t) = W^{\mathbf{s}}(\mathbf{x}, t_k), \qquad \forall \mathbf{x} \in \mathbb{R}^2, \forall t \in [t_k, t_{k+1}[, \forall \text{ station } \mathbf{s} \tag{3}$$

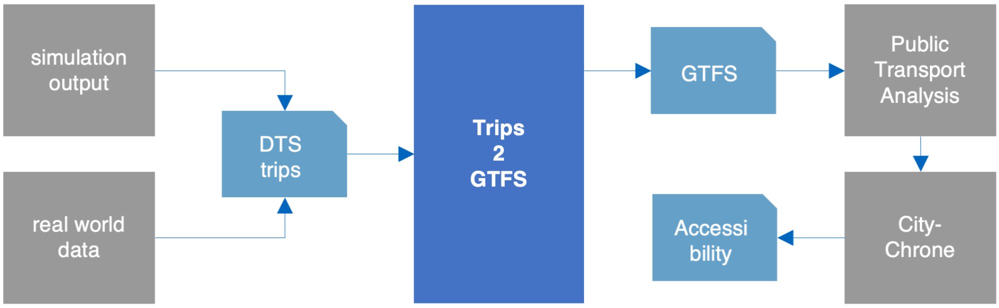

Figure 3: Implementation pipeline. CityChrone is from Biazzo et al. (2019).

For any timeslot, we thus just need to find estimation $\hat{w}^{\mathbf{s}}_{t_k}(\mathbf{u})$ of the expected values of random field $W^{\mathbf{s}}_{t_k}(\mathbf{x}) \equiv W^{\mathbf{s}}(\mathbf{x}, t_k)$. First the observations $\mathcal{O}$ are projected onto time-slot $[t_k, t_{k+1}]$:

$$\mathcal{O}^{\mathbf{s}}_{t_k} \equiv \{\text{observation } i = (\mathbf{x}_i, w_i, y_i) | i \in \mathcal{O}, t \in [t_k, t_{k+1}[, i \text{ is related to an access trip to } \mathbf{s}\} \quad (4)$$

Estimation $\hat{w}^{\mathbf{s}}_{t_k}(\mathbf{u})$ is computed by Ordinary Kriging (AA.VV. (2018)) on the observations $\mathcal{O}^{\mathbf{s}}_{t_k}$ as a convex combination of observations w_i:

$$\hat{w}^{\mathbf{s}}_{t_k}(\mathbf{x}) = \sum_{i \in \mathcal{O}^{\mathbf{s}}_{t_k}} \lambda_i \cdot w_i \quad (5)$$

In short (details can be found in Section 19.4 of Chilès & Desassis (2018)), coefficients λ_i are computed based on a *semivariogram* function $\gamma^{\mathbf{s}}_{t_k}(d)$, which obtained as a linear regression model, with predictors $d_{i,j}$ (distances between all pair of observations) and labels $\gamma_{i,j}$, which are called *experimental seminariances*:

$$\gamma_{i,j} \equiv \frac{1}{2} \cdot (w_i - d_j)^2 \quad (6)$$

The underlying assumption here is that correlation between wait times in different locations vanishes with the distance between such locations. The semivariogram gives the "shape" of this vanishing slope. In estimation (5), closer observations will have a higher weight. Under hypothesis on spatial stationarity and uniformity in all directions (*Kriging Interpolation* (2023)), Theorem 2.3 of Yakowitz & Szidarovsky (1985) proves that Kriging gives an aymptotically biased estimator: as the number of observations tends to infinite, $\hat{w}^{\mathbf{s}}_{t_k}(\mathbf{x})$ tends to the "true" $\mathbb{E}[W^{\mathbf{s}}_{t_k}(\mathbf{x})]$.

3 Implementation

The methodology of Section 2 is implemented in a Python pipeline, which we release as open source (Diepolder (2023)) and is depicted in Figure 3.

1. We first get centroids and cells performing the tessellation via CityChrone.
2. We read the file containing the observations (SMS trips). Such a file can be a simulation output or measurements of real SMS. Each observation includes the same information as in page 3. Observations are stored in a dataframe.
3. We assume SMS is deployed as feeder (as it is the case for the MATSim simulation on which we perform our analysis). Therefore, we can classify every SMS trip as either access or egress, depending on whether the origin or the destination is a PT stop.
4. To establish the feeder area $\mathcal{F}(\mathbf{s})$, we find among the observations $\mathcal{O}$ the furthest cell from s in which a trip to/from $\mathbf{s}$ has occurred. All cells within such a distance, are assumed to be in $\mathcal{F}(\mathbf{s})$. Observe that feeder areas of different hubs may overlap.
5. We group observations in timeslots (Figure 9).
6. In each time slot $[t_k, t_{k+1}[$ and each centroid $\mathbf{u}$ around each stop $\mathbf{s}$, we perform Kriging via library `pyInterpolate` (Moliński (2022)) to obtain estimations $\hat{w}^{\mathbf{s}}(\mathbf{u})$ and $\hat{y}^{\mathbf{s}}_{t_k}(\mathbf{u})$.

Table 1: Parameters used for the numerical results.

Parameter	Value	Reference
Side of a hexagon (tessellation)	1km	Badeanlou et al. (2022)
τ (Equation (1))	1 hour	Badeanlou et al. (2022)
Total number of DRT trips	14700	
- access trips	5289	
- egress trips	9412	
Total number of hubs	16	
Walk times		Computed via OpenStreetMap
Population Distribution		from the simulation scenario from Chouaki et al. (2023)

7. We obtain stoptimes and edges using the estimations above, as specified in (2). We add stoptimes and edges to the GTFS data of conventional PT, following the specifications in *GTFS Reference Document* (2023).
8. We give the obtained graph to CityChrone, which will give us accessibility scores in all the centroids.

4 Results and discussion

Data Source of the observations

The observation dataset in this study comes from a MATSim simulation, from Chouaki & Puchinger (2021); Chouaki et al. (2023), of door-to-door Demand-Responsive Transit (DRT), deployed as a feeder to and from conventional PT, in Paris-Saclay. The area in which DRT is deployed is depicted in Figure 4, but the entire Paris Region is simulated. Scenario parameters are in Table 1.

Analysis of Temporal and Spatial Patterns of DRT trips

Figure 5 clearly shoes morning peak $[7:00, 10:00[$, evening peak $[16:00, 19:00[$ and off-peak (all the other intervals).
In the following figures, the measures DRT trips toward/from all hubs, without distinguishing between hubs. Figures 6 and 7 are negative results: wait and travel times (figures on the right) do not appear to be spatially stationary (the distribution of values measured close to the related PT stops is different than further). Therefore, our estimations are not guaranteed to be asymptotically unbiased (page 4). In our future work, we will explore indirect estimation of wait and travel times through other indicators, e.g., the detour factor of DRT, which respect the requirements for the unbiasedness of Kriging.
Figure 8 shows that wait time follows expected peak/off-peak patterns. Values are generally slow since the simulation is configured so that a DRT trip is accepted only if it the dispatcher predicts it is possible to serve it within 10 minutes. All wait times exceeding this limits might be due to the dispatcher not taking traffic correctly into account.

Estimation of Waiting and Travel Times

Figure 9 shows that timeslots of 1h preserve the temporal pattern of trips, so 1h should be preferred to smaller timeslots, so as to perform Kriging with as many observations as possible.
Within each timeslot, estimation of wait and travel times is based on Kriging, which exploits spatial correlation. First, we note in Figure 10 that travel times close to hubs are shorter than further away. Then, we note that the experimental semivariance in Figure 11, i.e., the $\gamma_{i,j}$ between pair of observation i, j (Equation (6)), increases with the spatial distance between the observations: the closer the observations, the more similar are the respective travel times measured therein.
Such trends are not as evident for wait times (Figure 12) although similarity between observations still decay with distance (Figure 13).

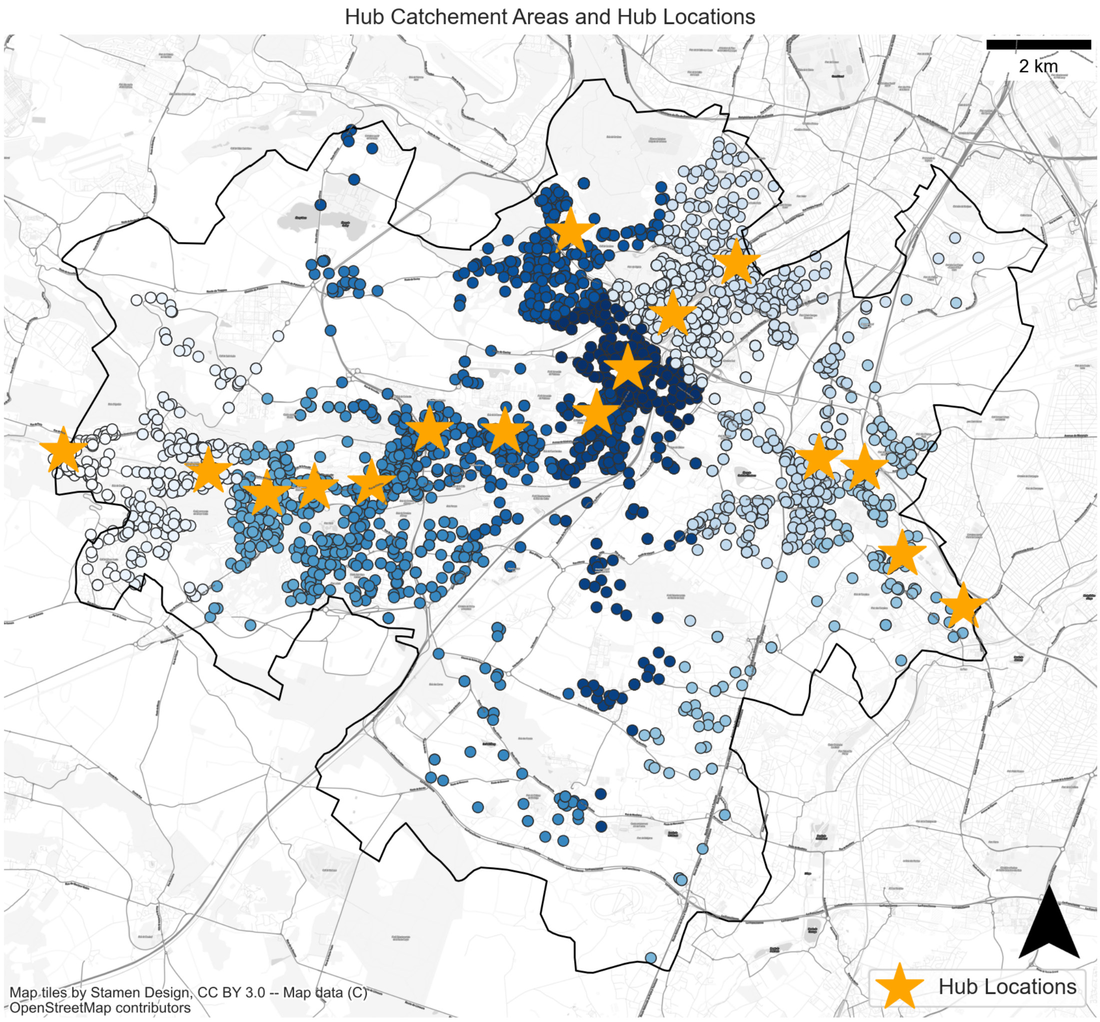

Figure 4: Hub Catchment Areas and Hub Locations. Each dot corresponds to the origin of one trip observed during the simulation. The differentiation in color of the observed trip origins indicates the catchment by different hubs

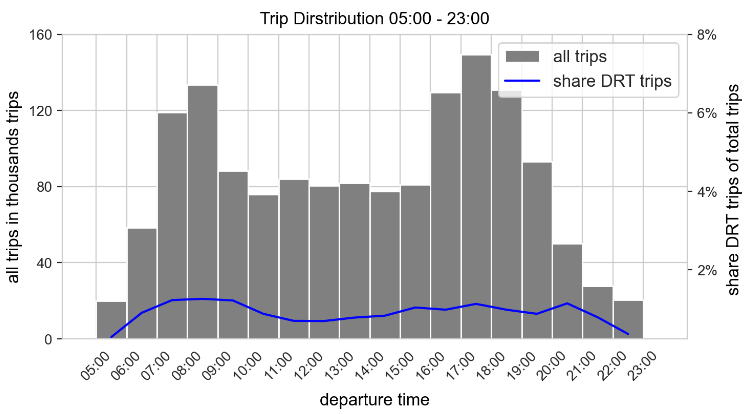

Figure 5: Trips over time. One trip is defined by the departure within the study area of Paris Saclay. A trip consisting out of multiple legs (e.g. walk + drt + PT is considered as one trip)

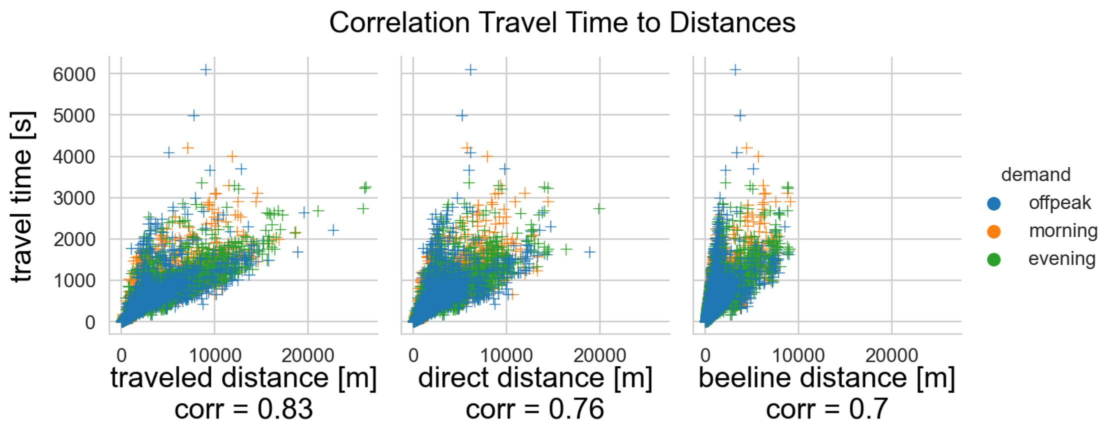

Figure 6: Relation between travel time and distance measures. Traveled distance is the actual Km traveled by the user inside the DRT vehicle. Direct distance is the one from the shortest road network road from the origin centroid to the hub. Beeline is the Euclidean distance.

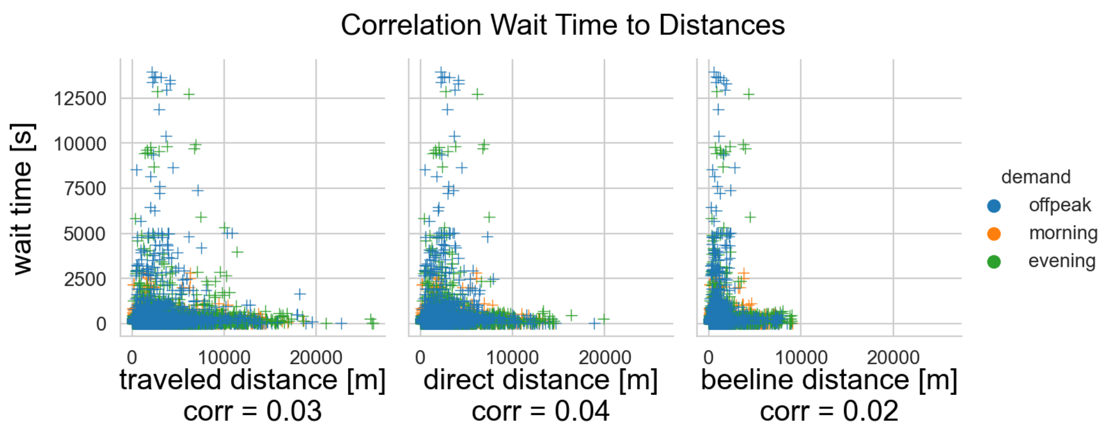

Figure 7: Relation between wait time and distance measures.

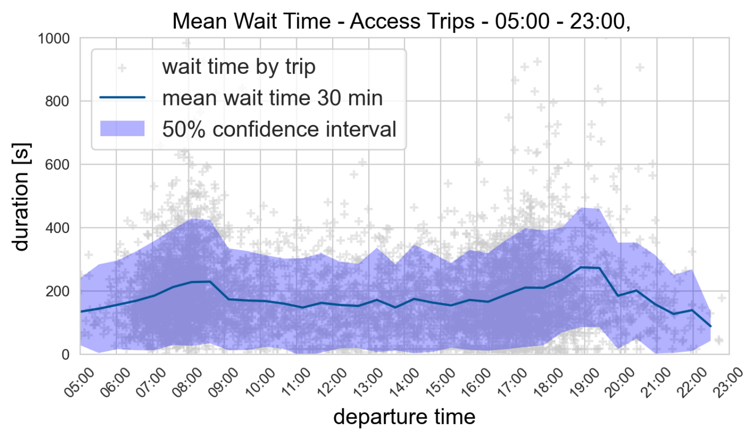

Figure 8: Mean Wait Time - A moving average of wait time during one day

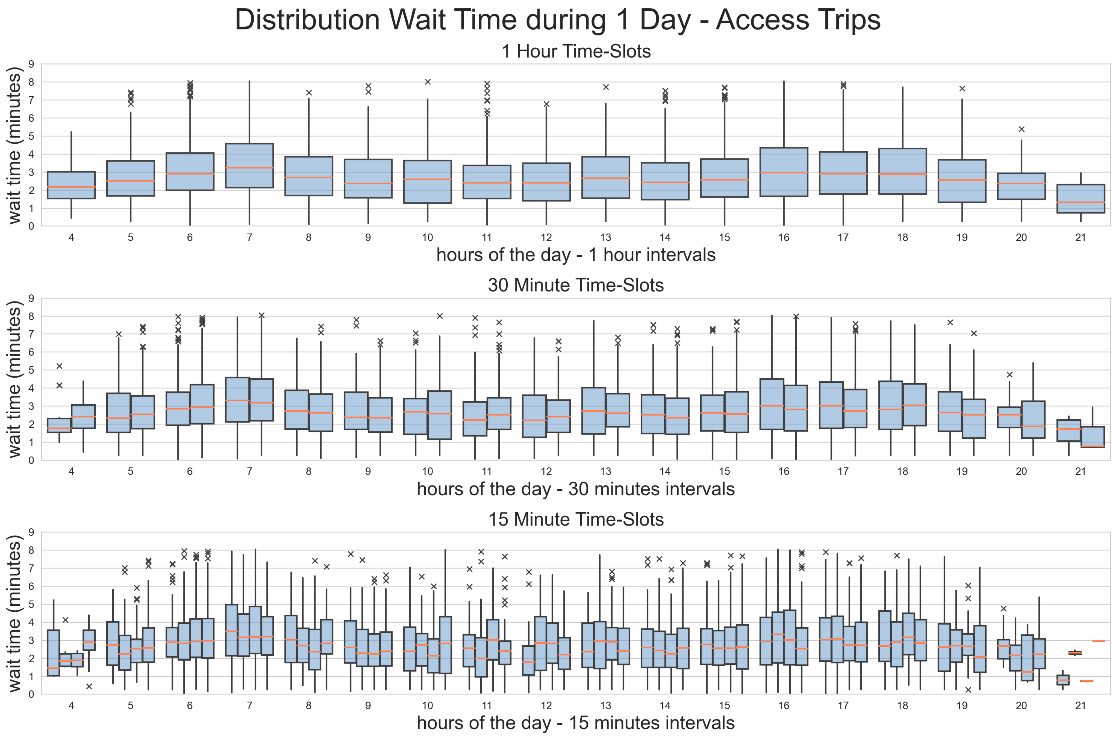

Figure 9: Comparison of different timeslot sizes. Values exceeding 10 minutes are not depicted, as they are due to simulation events unpredictable for the SMS dispatcher

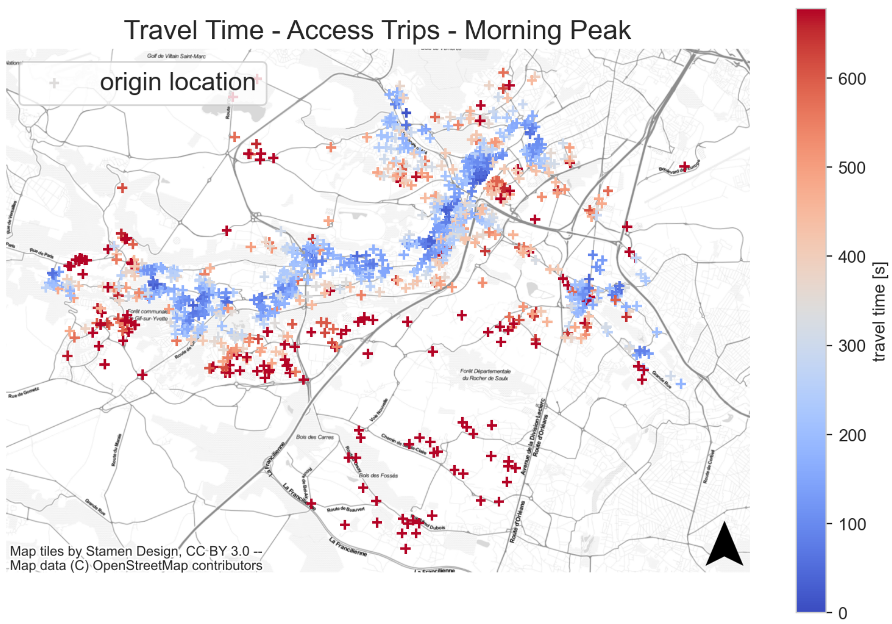

Figure 10: Spatial Trend Travel Time for access time, morning peak (evening and off-peak show similar trends).

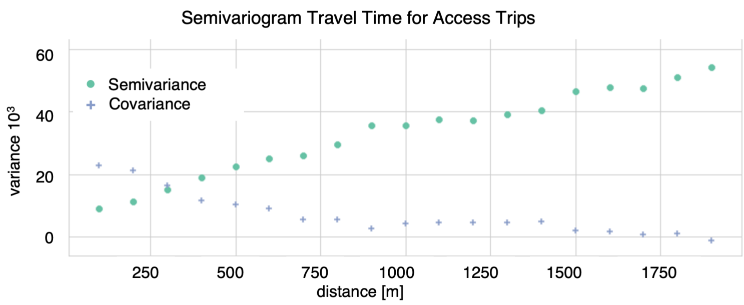

Figure 11: Spatial correlation of travel time observations

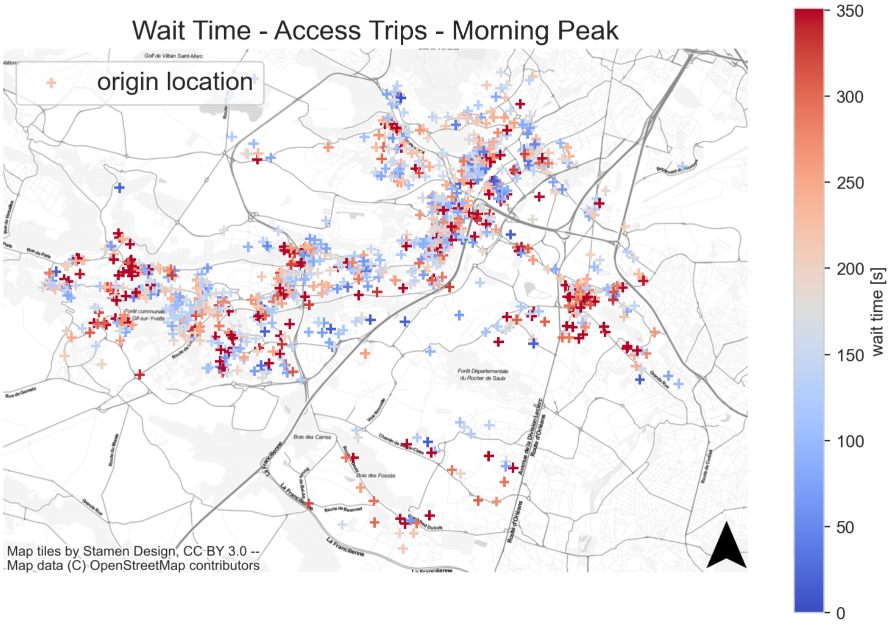

Figure 12: Spatial Trend Wait Time - No clear pattern can be identified, indicating low spatial autocorrelation

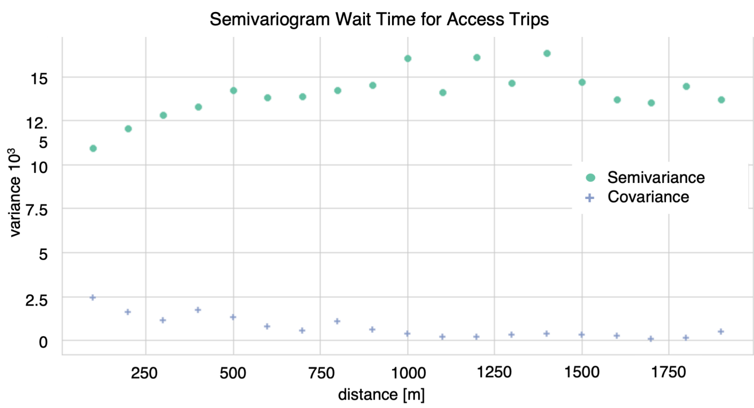

Figure 13: Spatial correlation of wait time observations

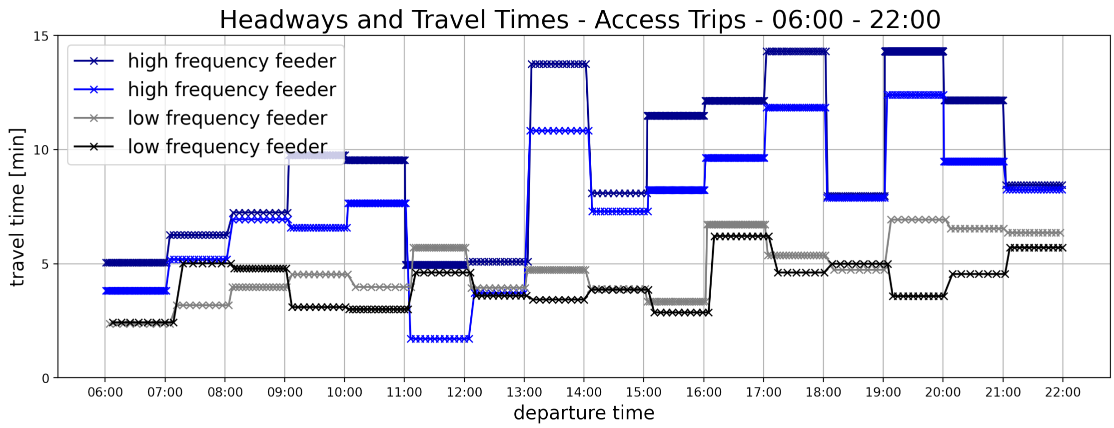

Figure 14: Headway and Travel Times of some examples of virtual DRT trips. Each departure time of a virtual DRT trip is indicated by a cross. The respective travel time is indicated by the y-axis value.

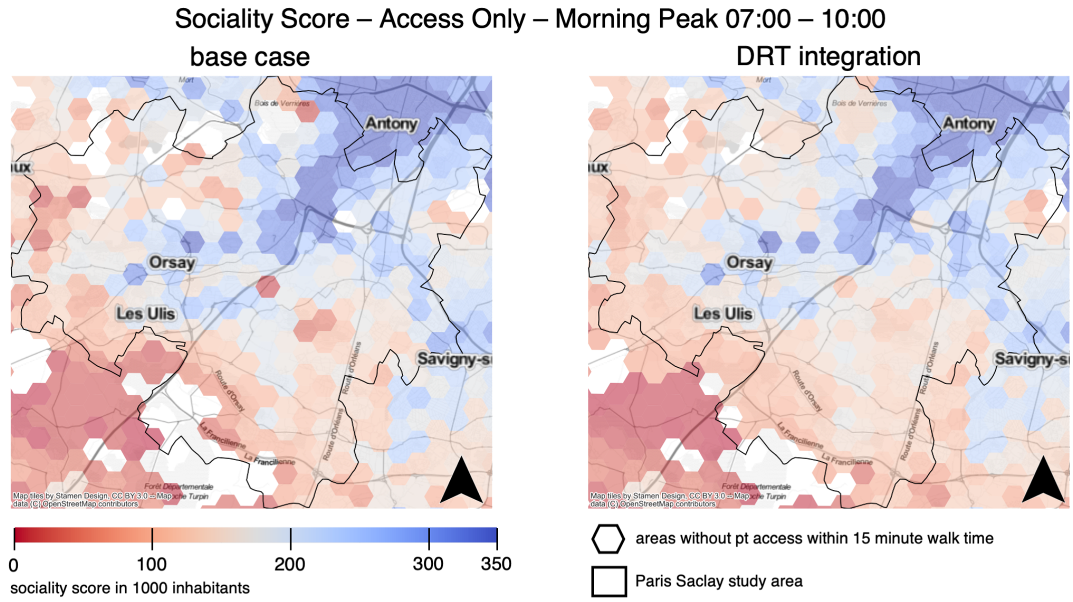

Figure 15: Sociality Score - Access Only - Morning Peak 07:00 - 10:00

Improvement of Accessibility Brought by DRT

Figure 14 shows headway and travel times of the virtual DRT trips added to the PT graph. We can then compute accessibility on this graph. Note that accessibility varies with the time of day (1). However, in the following figures we show averages over the time periods mentioned.

First, we study a system with DRT access services only (no egress). Figure 15 shows that the catchment area is expanded, especially in the south: hexagons with no access to PT within 15 minutes walk, can now use PT. Figure 16 shows more clearly the improvement in accessibility brought by improved access to PT thanks to DRT. As only access SMS feeder is added in Paris Saclay, the areas outside Saclay do not show any changes, except sligth improvement in some locations, for instant south of Versaille, possibly due to the possibility for travelers starting from there to make changes in Saclay, which are enhanced by DRT.

Accessibility improvements are even greater in peak hours (Figures 17 and 18, as DRT compensates for the low frequency of conventional PT.

Figure 19 shows the improvement in accessibility when both access and egress trips are added, averaged over the entire day. Improvement is much greater than the access-DRT only case. Moreover, improvement is also visible also outiside Saclay: users from everywhere can now reach opportunities in Saclay faster, thanks to DRT egress connections.

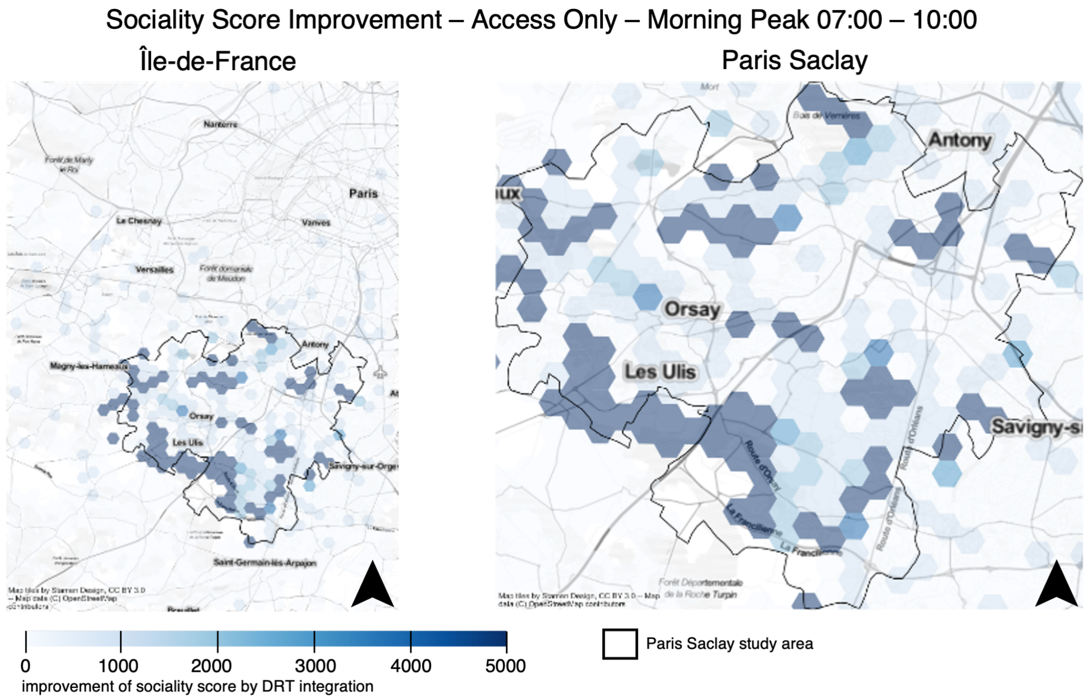

Figure 16: Sociality Score Improvement - Access Only - Morning Peak 07:00 - 10:00

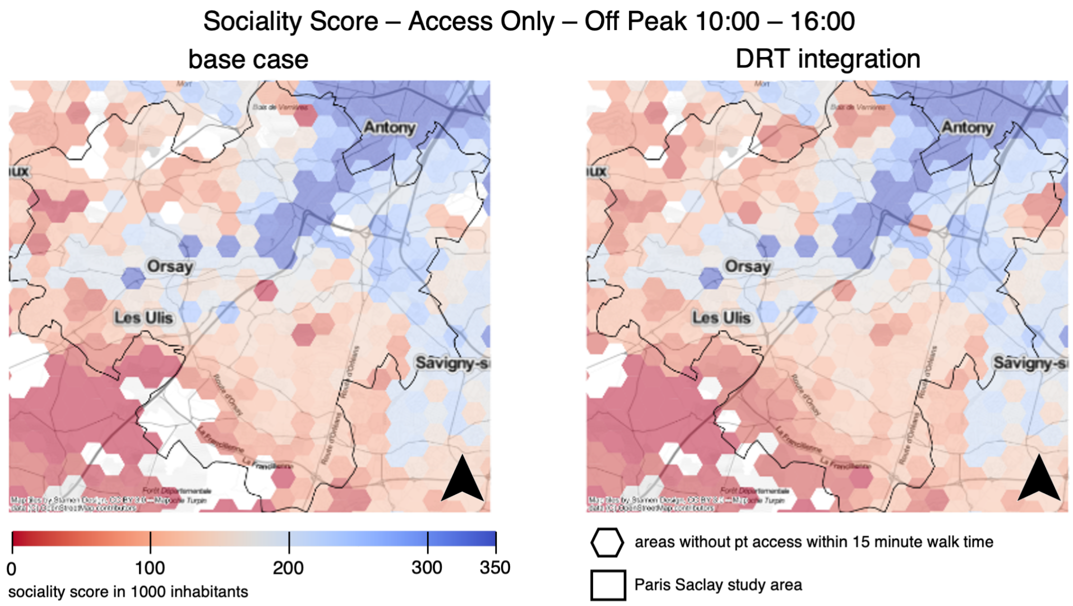

Figure 17: Sociality Score - Access Only - Off Peak 10:00 - 16:00

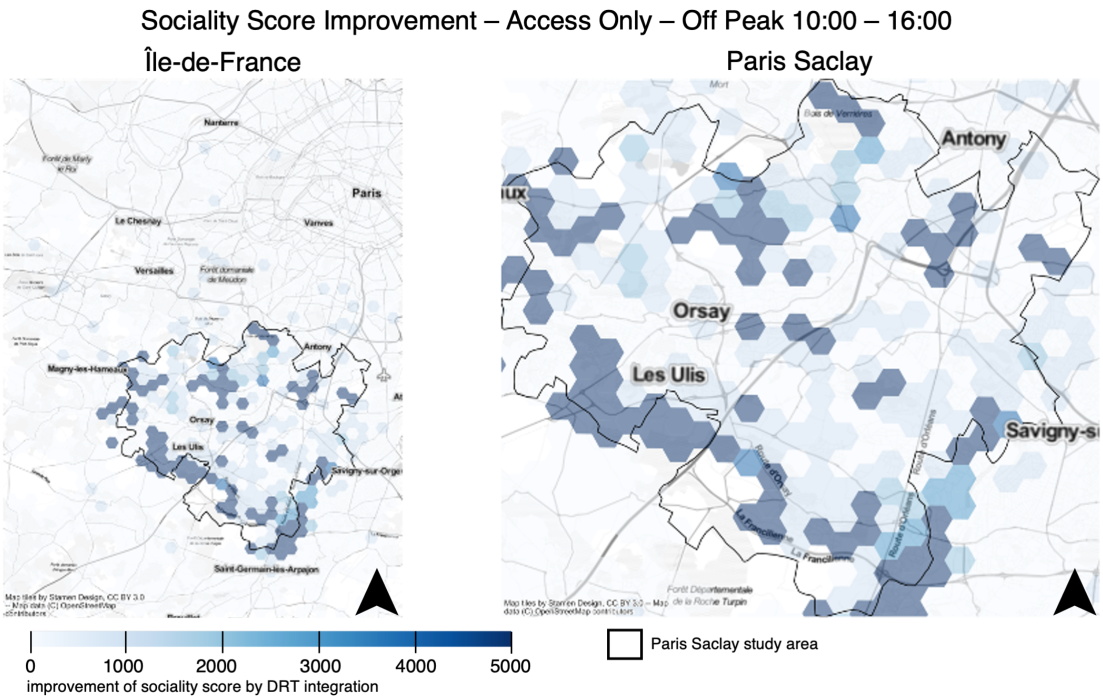

Figure 18: Sociality Score Improvement - Access Only - Off Peak 10:00 - 16:00

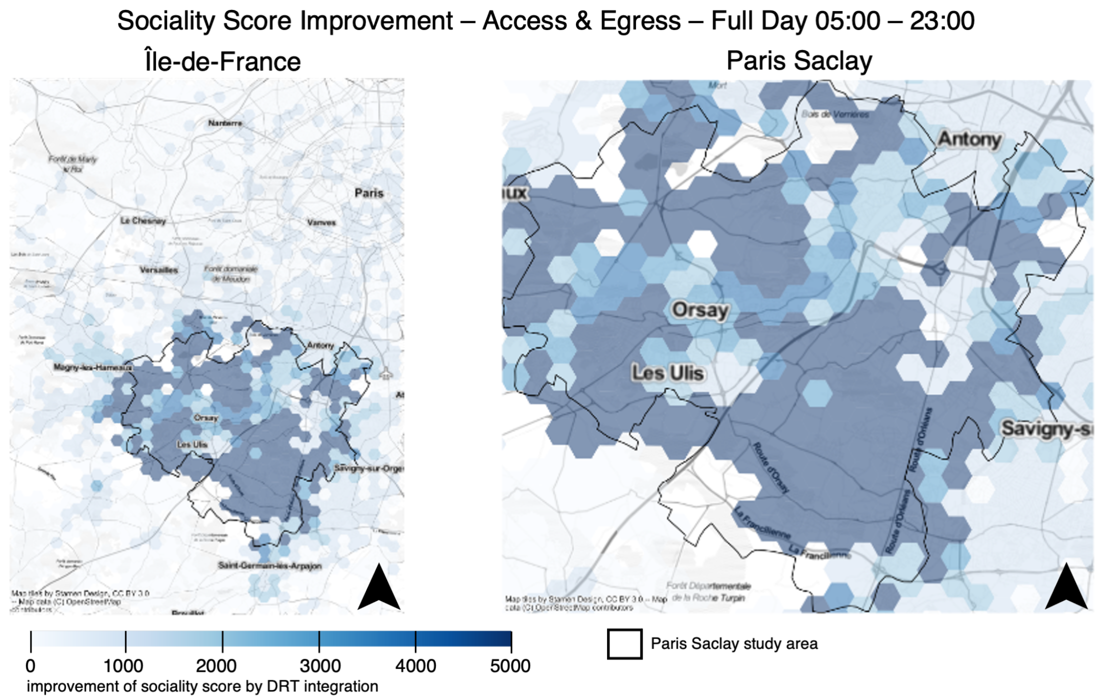

Figure 19: Sociality Score Improvement - Access & Egress - Full Day 05:00 - 23:00

5 Conclusions

We proposed a method to compute the impact of SMS on accessibility, based on empirical observations of SMS trips. Our method can support transport agencies and authorities in future deployment of SMS. In our future work, we will empirically validate the results by running simulations where we replaced simulated SMS with our estimated virtual trips. Finally, we will apply our method to car- or bike-sharing feeder and, possibly, on observations from real deployments.

Acknowledgements

This work has been supported by The French ANR research project MuTAS (ANR-21-CE22-0025-01) and by BayFrance.

References

AA.VV. (2018). Advances in Sensitivity Analysis of Uncertainty to Changes in Sampling Density When Modeling Spatially Correlated Attributes. In B. S. D. Sagar, Q. Cheng, & F. Agterberg (Eds.), *Handbook of mathematical geosciences* (chap. 19). Springer Open.

Badeanlou, A., Araldo, A., & Diana, M. (2022). Assessing transportation accessibility equity via open data. *hEART*.

Biazzo, I., Monechi, B., & Loreto, V. (2019). General scores for accessibility and inequality measures in urban areas. *Royal Society open science*.

Calabrò, G. (2023). Adaptive Transit Design: Optimizing Fixed and Demand Responsive Multi-Modal Transport via Continuous Approximation. In *Transportation Research Part A.*

Cascetta, E. (2009). *Transportation systems analysis: models and applications.* Springer Science & Business Media.

Chandra, S., Bari, M. E., Devarasetty, P. C., & Vadali, S. (2013). Accessibility evaluations of feeder transit services. *Transportation Research Part A*.

Chilès, J.-P., & Desassis, N. (2018). Fifty years of kriging. In B. Daya Sagar, Q. Cheng, & F. Agterberg (Eds.), *Handbook of Mathematical Geosciences.* Springer International Publishing.

Chouaki, T., Hörl, S., & Puchinger, J. (2023). Towards reproducible simulations of the grand paris express and on-demand feeder services. In *Trb.*

Chouaki, T., & Puchinger, J. (2021). Agent based simulation for the design of a mobility service in the paris-saclay area. *Transportation Research Procedia*.

Craig, T., & Shippy, W. (2020). Gtfs flex–what is it and how is it used? *National Center for Applied Transit Technology*.

Diepolder, S. (2023). *Accessibility of dynamic transport.* Retrieved 01.03.2023, from `https://github.com/severindiepolder/AccessibilityOfDynamicTransport`

Erhardt, G. D., Roy, S., Cooper, D., Sana, B., Chen, M., & Castiglione, J. (2019). Do transportation network companies decrease or increase congestion? *Science Advances*(5).

Fortin, P., Morency, C., & Trépanier, M. (2016). Innovative gtfs data application for transit network analysis using a graph-oriented method. *Journal of Public Transportation*.

GTFS Reference Document. (2023). Google. Retrieved 01.03.2023, from `https://developers.google.com/transit/gtfs/reference?hl=en`

Handcock, M. S., & Wallis, J. R. (1994). An approach to statistical spatial-temporal modeling of meteorological fields. *Journal of the American Statistical Association*(426).

Henao, A., & Marshall, W. E. (2019). The impact of ride-hailing on vehicle miles traveled. *Transportation*(6).

Kriging interpolation. (2023). Columbia University. Retrieved from `https://www.publichealth.columbia.edu/research/population-health-methods/kriging-interpolation`

Le Hasif, C., Araldo, A., Dumbrava, S., & Watel, D. (2022). A graph-database approach to assess the impact of demand-responsive services on public transit accessibility. In *15th acm sigspatial international workshop on computational transportation science.*

Miller, E. J. (2020). Measuring accessibility: Methods and issues. International Transport Forum.

Moliński, S. (2022). Pyinterpolate: Spatial interpolation in python for point measurements and aggregated datasets. *Journal of Open Source Software*.

Nahmias-Biran, B.-h., Oke, J. B., Kumar, N., Lima Azevedo, C., & Ben-Akiva, M. (2021). Evaluating the impacts of shared automated mobility on-demand services: An activity-based accessibility approach. *Transportation*.

Saeidizand, P., Fransen, K., & Boussauw, K. (2022). Revisiting car dependency: A worldwide analysis of car travel in global metropolitan areas. *Cities*.

Yakowitz, S. J., & Szidarovsky, F. (1985). A comparison of kriging with nonparametric regression methods. *Journal of Multivariate Analysis*.

Zhou, M., Le, D.-T., Nguyen-Phuoc, D. Q., Zegras, P. C., & Ferreira Jr, J. (2021). Simulating impacts of automated mobility-on-demand on accessibility and residential relocation. *Cities*.