## A fast algorithm to optimize meeting-point-based electric first-mile feeder services with capacitated charging stations

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## SHORT SUMMARY

This paper addresses the meeting-point-based electric demand-responsive-transport routing and charging scheduling problem under charging synchronization constraints. The problem considered exhibits the structure of the location-routing problem, which is more difficult to solve than conventional electric vehicle routing problems. We propose to model the problem using a mixed integer linear programming approach based on a layered graph structure. A two-stage simulated annealing-based algorithm is proposed to solve the problem efficiently. A mixture of randomness and greedy partial recharge scheduling strategy is proposed to find feasible charging schedules under the synchronization constraints. The algorithm is tested on 20 instances with up to 100 customers and 49 bus stops. The results show that the proposed algorithm outperforms the best solutions found by a commercial mixed-integer linear programming solver (with a 2-hour computational time limit imposed) for 12/20 test instances and with less than 1-minute computational time on average.

**Keywords:** charging synchronization, demand responsive transport, electric vehicle, feeder service, meeting point, metaheuristics (topics: electrification and decarbonization of transport, operations research application)

## 1. INTRODUCTION

Electric vehicle routing problems consist of deciding vehicle routes and charging schedules to serve a set of customers while satisfying constraints regarding vehicle capacity, time windows and vehicle energy (Kucukoglu et al., 2021). For passenger transportation, the problem is related to the electric dial-a-ride problem (Bongiovanni et al., 2019). Most of the literature assumes that vehicles can be recharged anytime with unlimited capacity of charging stations (Schneider et al., 2014). This assumption is often violated in practice as the number of rapid chargers is very limited due to their high installation costs. The electric vehicle routing problem with capacitated charging stations (EVRP-CS) is yet more difficult, as it needs to synchronize the charging operations of vehicles to save waiting time at charging stations. Recent research efforts have mainly focused on developing exact methods based on the mixed linear integer programming (MILP), by assuming that the vehicles recharge to full when arriving at charging stations and considering a linear charging speed (Bruglieri et al., 2019). To allow multiple visits of vehicles to the chargers, each charger node has several dummy copies. Charging capacity is ensured by deferring the current visit of a charger to at least the full charging time of the previous visit of another vehicle in ascending order of visits. The authors propose a path-based MILP formulation and use the cutting plane method to solve exactly the test instance with less than 20 customers within 1-hour computational time. Froger et al. (2021) propose a path-based MILP formulation by considering piecewise linear charging functions and partial recharge, and propose a matheuristic method to solve the EVRP-CS exactly. Their solution method first generates a pool of initial routes without considering the capacity constraints of the charging stations, and then in the second step, they try to recombine these routes to find a solution satisfying the capacity constraints. Their problem assumes that the vehicles are homogeneous (battery size) and fully recharged before starting the service. They were able to solve exactly most of the test instances with 10 customers. Lam et al. (2022) propose a branch-and-cut-and-price algorithm to solve the EVRP problem with time windows and capacitated charging station constraints (EVRPTW-CS) by considering both vehicle partial recharge and piecewise linear charging functions. The charging scheduling synchronization subproblem is handled by applying the constraint programming technique. Their exact method can solve the problem with the larger test instances of up to 100 customers. To the best of our knowledge, the existing literature mainly focuses on exact methods that can be adopted to small-scale instances. There are still no efficient algorithms to address the related EVRP or electric dial-a-ride problem (e-DARP) at large scale.

In this paper, we aim to address the above issue and focus on a variant of EVRPTW-CS, an electric DRT (feeder) system which provides passenger transport service to connect to transit stations. This type of service is mainly applied in rural areas where public transport service is poor (Ma et al., 2021). To enhance efficiency of the electric DRT system, the meeting point concept (customers may board/alight at pre-defined stops near to their origins/destinations) is considered (Czioska et al., 2019). The problem needs to decide jointly where to pick up customers and how to route vehicles under various constraints. The problem is more complicated due to interactions between customer-to-bus-stop assignment and the subsequent vehicle routing and charging synchronization.

## 2. METHODOLOGY

### *2.1. Problem description*

We consider a DRT feeder service in a rural area provided by an operator using a heterogeneous (in terms of capacity, battery size, and energy consumption rate) fleet of electric buses (also called vehicles hereafter) to complement the public transport system. To enhance system efficiency and reduce operational costs, the DRT system adopts the concept of **meeting points** i.e. customers are offered a limited number of pick-up/drop-off meeting points, rather than a door-to-door service (Czioska et al., 2019; Ma et al., 2021) and the service is **punctuated** (e.g. the vehicle arrives at a transit station every 10-20 minutes to drop off the transit passengers). The system is operated as follows. For a given planning period, customers submit their ride requests in advance indicating their origin, the transit station to be dropped off, and their desired arrival time (corresponding to the pre-defined arrival timetable of the DRT buses). The operator collects these ride requests and communicates whether customers' ride requests are accepted, their pickup time, and suggested bus stop (meeting point). The operator's objective is to optimize vehicle routes so as to arrive at transit stations within a fixed buffer time (e.g. $\leq 10$ minutes before the announced arrival timetable at transit stations). We assume that customers are willing to walk from their origins to the suggested meeting points, up to some maximum acceptable walking distance. The state of charge of the vehicles cannot fall below the reserve battery level throughout the route. Vehicles can be recharged only at operator-owned charging stations; each station has a limited number of chargers. Charging operations cannot overlap at any charger, i.e., a vehicle is not allowed to wait at a charger/charging station.

The meeting-point-based electric feeder service problem with charging synchronization constraints (**MP-EFCS**) problem is formulated as a mixed-integer-linear programming (MILP) problem as an extension of the electric dial-a-ride problem (e-DARP)(Bongiovanni et al., 2019), but adopting the concept of meeting points, allowing customers to be rejected (with a high penalty costs) under vehicle charging synchronization constraints. Given a set of customer requests, the objective is to optimize vehicle routes to meet these requests while considering the trade-off between system costs and customer inconvenience. The objective function minimizes the weighted sum of total vehicle travel time and total vehicle charging time, customer's total walking time, total vehicle waiting time at transit stations before the acceptable fixed buffer time, and the total penalty of unserved requests. The computational time for solving the MP-EFCS exactly needs to enumerate all possible customer-bus-stop assignments and then solve each corresponding e-DARP with charging synchronization constraints problem (**e-DARP-CS**) to find the global minimum. This is possible only for very small problem size. To solve it efficiently, we propose a layered (directed) graph model (Fang and Ma, 2022) according to the sorted arrival timetable at transit stations and prune infeasible arcs or layers to reduce the problem size (see an illustrative example in Figure 1).

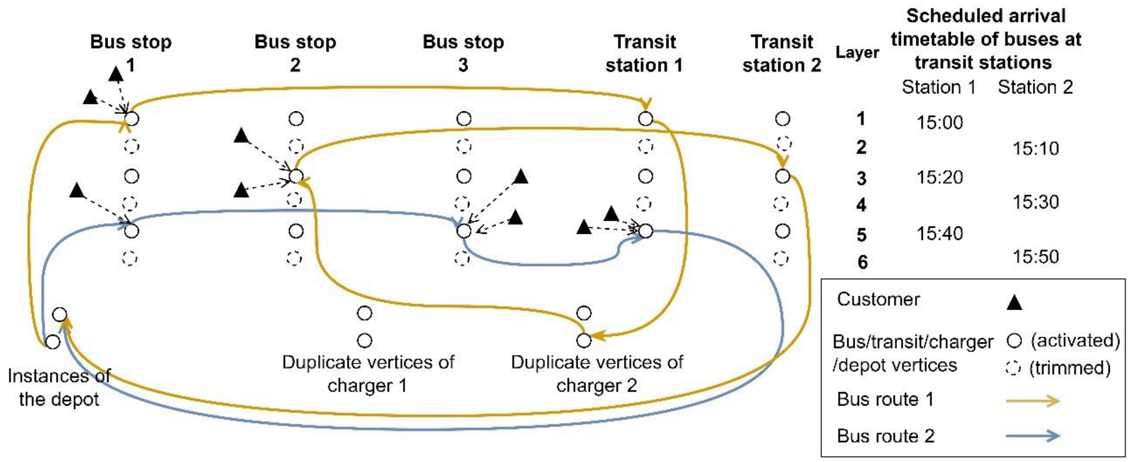


Figure 1. An illustrative example of the layered directed graph (arcs are omitted) for modeling the meeting-points-based electric feeder service with the charging synchronization constraints.

### *2.2. Solution algorithm*

We propose an efficient **two-stage solution scheme** by finding a good customer-bus-stop assignment in the first stage. In the second stage, a **simulated annealing (SA) based metaheuristic** (Braekers et al., 2014) **with a post-optimization** procedure is proposed to solve the routing problem with charging synchronization constraints. The new challenge is how to optimize vehicles' charging schedules with synchronization constraints. The **customer-bus-stop assignment** problem on the first stage is formulated as an MILP formulated, as a variant of the capacitated facility location problem, to minimize the weighted sum of the total customer walking time and bus travel time between the activated (with positive assigned customers) bus stops. Given the solution obtained from the customer-bus-stop assignment problem, we construct an e-DARP-CS instance by trimming off unused bus stop nodes and arcs connected to them, based on the layered graph model. An initial feasible solution is generated as the best feasible solution found for *n* random solutions using a greedy insertion approach. The SA-based algorithm applies a randomly selected local search operator on the current solution and obtains a temporary solution. If the cost of the temporary solution is smaller than that of the current solution plus the threshold value *T* (temperature), and there are no charging operation conflicts, a vehicle exchange operator is applied on the temporary solution to further reduce the charging time of the vehicles of the temporary solution. If the resulting vehicle exchange and rescheduled charging operations (if any) has an improved cost without charging conflicts for all vehicles, then update the current solution. We track the number of times that the current best solution has stagnated. If this exceeds a pre-defined limit, the algorithm returns the current best solution. Otherwise, randomly selected local search operators are applied until a maximum number of iterations is achieved. This early stop criterion helps to reduce the computation time. As we allow customer requests to be rejected, unserved customers are managed in a pool, which is regarded as a virtual route, allowing customers to be removed from the vehicles. Note that charging schedule updating is applied at the end of each local search operation. Given the layered graph structure, we can efficiently screen-out infeasible insertion positions by checking whether the layer of a customer to be inserted is (in)compatible with the layer of the current inserted position of the route. This conflict check can be done in O(1) and reduces the computational time significantly. We propose seven local search operators, including relocate ensemble, two-opt*, two-opt, exchange-segment, exchange-customer, four-opt, and create-route.

The e-DARP-CS instance is optimized based on the first stage customer-bus-stop assignment. It might be possible to accommodate unserved customers by changing their assigned bus stops, then re-inserting them into the current bus routes. In doing so, bus routes and charging schedules need to be updated accordingly. In the case that there are unserved customers, we propose an efficient post-optimization procedure to re-optimize the best solution obtained from the SA algorithm. Our numerical results show that this post-optimization procedure can improve the final solution and reduce the number of unserved customers with little additional computational effort.

## 3. RESULTS AND DISCUSSION

To test the algorithm, we consider two 4-hour scenarios, corresponding to peak (P) and off-peak (OP) demand profiles. Scenario P simulates a peak-hour situation where customers' desired arrival times at transit stations are concentrated around a peak hour, while OP reflects the opposite situation when customers' desired arrival times are uniformly spread over a longer operating period. In each scenario, we generate 10 instances spanning the range of 10-100 customers. These test instances have a single vehicle depot, two train stations, and four chargers. Meeting points (potential bus stops) are generated as a grid with a separation distance of 1 km and customers maximum walking distance is 1.5 km. Punctuated services are provided for the two train stations with three services per hour throughout the analysis period. In total, there are 26 layers with 25 to 49 activated bus stops per layer (bus stops within the maximum walking distance of the customers). A customer may have up to 7 potential bus stops within walking distance. Consequently, the possible customer-bus-stop assignment combinations are very large, providing non-trivial tests of algorithm performance. We consider two types of vehicles with different passenger capacity, battery capacity, and energy consumption rate.

The performance of the algorithm is compared with the solution obtained by a state-of-the-art MILP solver (Gurobi, version 9.1.2) with a 2-hour computational time limit. Our algorithm and the MILP model are both implemented using the Julia programming language. We run the experiments on a laptop with Intel(R) Core(TM) i7-11800H processor and 64 GB memory using a single thread.

MILP solutions obtained by Gurobi are reported in Table 1. The instance name cxx means that there are xx customers in that instance. To ensure scenarios where vehicles need to recharge, initial battery levels of vehicles are set as low as 20%, 30%,…, and 80% of the battery capacity.

For each instance, the MILP results give the best feasible solution found within 2 hours, along with the lower bound. The third column shows the number of unserved customers. The solver can obtain (near) optimal solutions for small instance of 10 customers. The number of unserved customers increases dramatically for instances with more than 50 customers. To account for the random elements within the two-stage SA-based algorithm, results are based on the average over 5 runs with random seeds. For each instance, we report the average objective function value and its gap to the best-known solution (BKS) found by the solver. The last two columns report the number of unserved customers and the average computational time (per run). The results show that the proposed algorithm outperforms the BKS for 12/20 test instances with less than 1-minute computational time on average.

**Table 1: Computational results obtained using Gurobi solver and the two-stage SA-based algorithm on the test instances.**

| | **MILP** | | | **Two-stage SA based algorithm** | | | |
|---|---|---|---|---|---|---|---|
| Instances | Best known solution | Gap to the lower bound | Num. of unserved customers | Avg. obj. value | Gap to BKS | Num. of unserved customers | cpu time (sec.) |
| Scenario Off-Peak (OP) | | | | | | | |
| c10 | 107.91 | 1.10% | 0 | 107.91 | 0.00% | 0 | 4 |
| c20 | 233.02 | 14.40% | 0 | 234.96 | 0.83% | 0 | 11 |
| c30 | 327.46 | 16.30% | 0 | 340.08 | 3.85% | 0 | 36 |
| c40 | 450.17 | 31.30% | 1 | 424.03 | -5.81% | 0 | 39 |
| c50 | 696.13 | 44.70% | 2 | 611.32 | -12.18% | 0.4 | 36 |
| c60 | 816.88 | 43.04% | 4 | 659.31 | -19.29% | 0 | 120 |
| c70 | 755.46 | 26.35% | 0 | 782.41 | 3.57% | 0 | 84 |
| c80 | 1107.52 | 43.35% | 7 | 906.83 | -18.12% | 0 | 123 |
| c90 | 1525.81 | 58.33% | 15 | 969.28 | -36.47% | 0 | 155 |
| c100 | 1689.13 | 55.57% | 20 | 1074.65 | -36.38% | 0 | 154 |
| Scenario Peak (P) | | | | | | | |
| c10 | 112.31 | 18.00% | 0 | 112.56 | 0.22% | 0 | 5 |
| c20 | 284.79 | 41.49% | 1 | 311.36 | 9.33% | 0 | 9 |
| c30 | 340.85 | 33.16% | 1 | 365.03 | 7.09% | 1 | 22 |
| c40 | 455.81 | 44.10% | 2 | 472.6 | 3.68% | 1 | 27 |
| c50 | 705.36 | 55.10% | 5 | 698.18 | -1.02% | 2.2 | 36 |
| c60 | 996.41 | 60.11% | 11 | 692.2 | -30.53% | 0 | 37 |
| c70 | 1023.21 | 55.22% | 15 | 769.55 | -24.79% | 0 | 60 |
| c80 | 1514.25 | 66.05% | 24 | 892.82 | -41.04% | 0 | 42 |
| c90 | 1600.89 | 68.04% | 24 | 1009.5 | -36.94% | 0 | 3 |
| c100 | 1369.96 | 53.34% | 13 | 1101.09 | -19.63% | 0 | 15 |

## 4. CONCLUSIONS

Electric vehicle routing with charging synchronization (under capacitated charging stations) are more difficult to solve and efficient solution algorithms are still underdeveloped for solving medium/large problem instances. In this study, we consider the problem of an electric dial-a-ride feeder system with charging synchronization based on the concept of meeting points and propose a layered graph model and a mixture of randomization and greedy strategy within a two-stage SA-based algorithm framework to solve this problem efficiently. We test the algorithm on 20 test instances with up to 100 customers and 49 bus stops. Results show that the proposed algorithm can find solutions efficiently with good solution quality. Several research directions are ongoing, including algorithmic parameter calibration, sensitivity analysis, charging infrastructure and fleet size planning, and integrated DRT system operational policy optimization.


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