The Bus Rapid Transit Investment Problem

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Short summary

Bus Rapid Transit (BRT) systems can be of great value to attract passengers towards public transport, as they offer an attractive service at relatively low investment costs. Often, BRT lines are created by giving the bus a dedicated right of way along segments of an existing bus line. This paper focuses on quantifying the trade-off between the number of attracted passengers and the available investment budget when upgrading a line. Motivated by the construction of a new BRT line around Copenhagen, we consider multiple municipalities that invest in the line. We additionally allow restrictions on the number of connected components to be upgraded to enforce connectedness. We suggest two passenger responses to determine the number of attracted passengers and propose an ϵ-constraint based algorithm to enumerate all non-dominated points. Moreover, we perform an extensive experimental evaluation on artificial instances and a case study for the BRT line around Copenhagen.
Keywords: Bus Rapid Transit, Network Design, Operations Research Applications, Public Transport

1 Introduction

Increasing the modal share of public transport is seen as one of the paths to reducing greenhouse gas emissions, even when considering the electrification of private cars (Messerli et al., 2019). Bus Rapid Transit (BRT) systems can contribute to this goal, as BRT lines provide a fast and reliable service to passengers due to having a dedicated right of way for buses along a large share of their route. However, while BRT investment costs are lower than for rail-based alternatives, these investments are still substantial for local authorities both in terms of cost and usage of city space that cannot be used for other purposes. Hence, careful planning is needed to decide which segments of the existing line should be upgraded to a BRT standard.
In this paper, we focus on quantifying the trade-off between the number of attracted passengers and the investment budget by determining the optimal sets of segment upgrades. Motivated by a new BRT line being built in the Greater Copenhagen area, we consider a problem setting in which multiple municipalities are responsible for different segments of the line and each municipality has a budget limit. To prevent frequent switching between upgraded and non-upgraded segments due to fragmented investments, which could reduce reliability and thus deter passengers, we additionally allow restricting the number of upgraded connected components on the line. We refer to this problem as the BRT investment problem.
The BRT investment problem relates to the well-studied network design problem for public transport (Laporte et al., 2000; Laporte & Mesa, 2019). While the network design problem generally focuses on constructing a network from scratch, numerous papers also look at the upgrading of existing public transport networks. Particularly relevant for us are those papers looking at the allocation of dedicated bus lines within existing transport networks, many of which focus on the trade-off between the benefits for public transport passengers and the congestion on the road network (Khoo et al., 2014; Bayrak & Guler, 2018; Tsitsokas et al., 2021). Another addition to the standard network design problem that is relevant for our application in the Greater Copenhagen region is the inclusion of multiple investing parties, which was studied by Wang & Zhang (2017) within a game-theoretical setting. Moreover, the underlying mathematical structure of the BRT investment problem is similar to the more general network improvement problem, which consists

of choosing edges in a network to be upgraded while minimizing costs or satisfying a budget constraint (Krumke et al., 1998; Zhang et al., 2004; Murawski & Church, 2009). Compared to existing literature, our work distinguishes itself by studying the combination of a trade-off between the number of attracted passengers and the investment budget, the inclusion of multiple investing parties, and the inclusion of constraints ensuring connectedness of the upgraded segments.
This paper builds on the work of our earlier conference paper (Hoogervorst et al., 2022), in which we looked at the single-objective problem of upgrading bus line segments under a budget constraint and without a constraint on the number of upgraded components. In this paper, we instead propose a new bi-objective mixed-integer programming model to solve the BRT investment problem that allows us to construct the Pareto curve between the total investment budget and the number of attracted passengers. We do so under two possible passenger responses, one corresponding to a linear relation and the other to a threshold relation between segment upgrades and the attracted share of passengers. We show how the set of all non-dominated points can be found under these passenger responses and test our proposed algorithm in a numerical study for both artificial instances and a case study for the BRT line in Greater Copenhagen.

2 Methodology

In this section, we first formally define the problem and afterwards describe the used solution methods.

Problem Definition

We consider an existing bus line given by a linear graph (V, E), where $V = \{1, \dots, n\}$ for $n \in \mathbb{N}_{\geq 1}$ denotes the set of stations and $E = \big\{e_i = \{i, i+1\} : i \in \{1, \dots, n-1\}\big\}$ the set of segments between the stations. For each edge $e \in E$, we know the cost $c_e \in \mathbb{R}_{>0}$ for upgrading the edge as well as the improvement, i.e., improvement in travel time, $u_e \in \mathbb{R}_{>0}$ that is realized when the edge is upgraded. Moreover, let $D \subseteq \{(i,j) : i, j \in V, i < j\}$ be the set of origin-destination (OD) pairs for the line, where OD-pair $d = (i,j) \in D$ has the unique path $W_d = \{e_k : k \in \{i, i+1, \dots, j-1\}\}$ along the line. For each OD-pair $d \in D$, we additionally know the number of potential passengers a_d that are attracted when all edges in the path W_d are upgraded.
The set of municipalities that are investing in the BRT line is given by M. For each municipality, we know the set of consecutive edges $E_m \subseteq E$ that lie within the municipality. We will assume that these sets of edges of the different municipalities are pairwise disjoint, i.e., $W_k \cap W_l = \emptyset$ for $k, l \in M, k \neq l$. Moreover, we know the budget share b_m that is allocated to each municipality, i.e., each municipality gets budget $b_m B$ when considering some total budget B. Lastly, to prevent buses from switching too often between upgraded and non-upgraded ones, we enforce a maximum number of BRT components of Z.
While the number of potential passengers attracted is given for each OD-pair $d \in D$ when all edges in W_d are upgraded, it is beforehand unclear how passengers react to partial upgrading of the edges in W_d. We consider two different passenger responses $p_d(F)$ to a set of upgrades $F \subseteq E$:

- The Linear response to upgrades

$$p_d(F) := \frac{\sum_{e \in F \cap W_d} u_e}{\sum_{e' \in W_d} u_{e'}} \cdot a_d,$$

  in which the number of passengers scales linearly with the amount of improvement realized.

- The MinImprov response to upgrades

$$p_d(F) := \begin{cases} a_d & \text{if } L_d \leq \sum_{e \in F \cap W_d} u_e, \\ 0 & \text{otherwise,} \end{cases}$$

  in which all the potential passengers are only attracted when a minimum improvement of L_d is achieved.

Note that the MinImprov response resembles a shortest path based route and mode choice, where passengers only switch to the BRT line in case the upgrade is large enough to make it their option with the shortest travel time.

The BRT investment problem then becomes to find all the non-dominated solutions (F, B), with F the set of upgraded edges and B the total investment budget, that solve:

$$\max \sum_{d \in D} p_d(F) \tag{1}$$

$$\min B \tag{2}$$

$$s.t. \sum_{e \in E_m \cap F} c_e \leq b_m B \qquad m \in M, \tag{3}$$

$$G[F] \textit{ has at most } Z \textit{ connected components}, \tag{4}$$

$$F \subseteq E, \tag{5}$$

$$B \in \mathbb{R}. \tag{6}$$

The first objective (1) maximizes the number of attracted passengers, while the second objective (2) minimizes the investment budget. Constraints (3) enforce the budget limit for each municipality. Moreover, constraints (4) enforce the maximum number of BRT components. Here, $G[F]$ is the graph induced by the set of edges F, i.e., the graph obtained after deleting all edges from G that are not contained in F.

Solution Methodology

Formulation (1) – (6) can be transformed into a bi-objective mixed-integer linear programming (MILP) model through introducing variables for all $e \in E$ that depict if a segment is upgraded and, in the case of the MinImprov response, variables y_d for all $d \in D$ that depict if the minimum improvement is realized for an OD-pair. For example, this leads to the following formulation for the MinImprov objective:

$$\max \sum_{d \in D} a_d y_d \tag{7}$$

$$\min B \tag{8}$$

$$s.t. \; L_d y_d \leq \sum_{e \in W_d} u_e x_e \qquad d \in D, \tag{9}$$

$$\sum_{e \in E_m} c_e x_e \leq b_m B \qquad m \in M, \tag{10}$$

$$x_{e_i} - x_{e_{i+1}} \leq z_i, \qquad i \in \{1, \ldots, n-2\}, \tag{11}$$

$$x_{e_{i+1}} - x_{e_i} \leq z_i, \qquad i \in \{1, \ldots, n-2\}, \tag{12}$$

$$x_{e_1} + \sum_{i=1}^{n-2} z_i + x_{e_{n-1}} \leq 2Z, \tag{13}$$

$$x_e \in \{0, 1\} \qquad e \in E, \tag{14}$$

$$z_i \in \{0, 1\} \qquad i \in \{1, \ldots, n-2\}, \tag{15}$$

$$y_d \in \{0, 1\} \qquad d \in D, \tag{16}$$

$$B \in \mathbb{R}. \tag{17}$$

The objectives (7) and (8) maximize the number of attracted passengers and minimize the investment budget, respectively. Constraints (9) determine if the minimum improvement for an OD-pair is realized. The budget limit is enforced for each municipality by constraints (10). Moreover, constraints (11) – (13) enforce the maximum number of BRT components through counting the number of switches on the line between upgraded and non-upgraded segments. A bi-objective MILP model can be obtained for the Linear passenger response in a similar way.
We use the ϵ-constraint method to find the set of non-dominated solutions, i.e., solutions on the Pareto curve, for the proposed bi-objective programming problems. The used algorithm is given in Algorithm 1, which is an adaptation of the algorithm proposed by Bérubé et al. (2009). The idea of the algorithm is to iteratively compute all non-dominated points by solving the single-objective version of the BRT investment problem for a fixed total budget B and to decrease B in each step by a value that is small enough not to cut-off any non-dominated solution. In particular, we can prove that this algorithm generates the set of all non-dominated points on the Pareto curve.

Algorithm 1 Computing the non-dominated points for the BRT investment problem

Input: instance I of the BRT investment problem.
Output: set Γ of all non-dominated points.
As start values set
$\Gamma \leftarrow \emptyset$,
$B \leftarrow \max\limits_{m \in M} \{ \frac{1}{b_m} \cdot \sum\limits_{e \in E_m} c_e \}$,
$v^* \leftarrow \max\limits_{m \in M} \{ \frac{1}{b_m} \cdot \sum\limits_{e \in E_m} c_e \}$,
$p^* \leftarrow \sum_{d \in D} a_d$.
while $B \geq 0$ **do**
    Solve instance I with budget B. Let F be an optimal solution, $\bar{p}$ be the optimal objective value.
    Compute the minimum budget $\bar{v}$ such that F remains feasible.
    Compute step width δ.
    if $\bar{p} < p^*$ **then**
        Set $\Gamma \leftarrow \Gamma \cup \{(p^*, v^*)\}$.
        Set $p^* \leftarrow \bar{p}$.
    end if
    Set $v^* \leftarrow \bar{v}$.
    Set $B \leftarrow \bar{v} - \delta$.
end while
Set $\Gamma \leftarrow \Gamma \cup \{(p^*, v^*)\}$.
return Γ

3 Results and discussion

We perform computational experiments for both a set of artificial instances, based on those introduced in Hoogervorst et al. (2022), and on instances from the proposed BRT line in Greater Copenhagen that motivated our study. The artificial instances differ with respect to the passenger demand over the OD-pairs and the upgrade costs of the segments. The instances for the Greater Copenhagen BRT line are instead based on five proposed line alternatives for the BRT, depicted in Figure 1, and consider different ways of distributing the total budget over the municipalities.

Results Artificial Instances

The obtained Pareto plots for the artificial instances are given in Figure 2 for the setting of a single municipality that can invest in all edges. The different columns in the figure indicate the different passenger demand distributions, where each OD-pair has equal demand (*EVEN*), passengers mostly travel to the closest large station (*CENTER*), and passengers mainly travel between the two end-stations (*END*), respectively. The rows instead indicate the different cost patterns, where all segments have equal upgrade cost (*UNIT*), edges towards the middle are most expensive to upgrade (*MIDDLE*), and edges towards the ends are most expensive to upgrade (*ENDS*), respectively.

The Pareto plots in Figure 2 show that there is a noticeably different trade-off between attracted passengers and investment budget for the two passenger responses. For passenger response Linear, we obtain a mostly concave shape for all demand and cost patterns, where the first investments generate the largest number of new passengers. Instead, the shape of the Pareto curves is more variable over the demand and cost patterns for the the MinImprov response. In particular, we can see a clear jump in the Pareto plots for the MinImprov response for the *END* demand pattern, which can be explained by the minimum improvement threshold that needs to be reached for attracting the large number of passengers traveling over the whole line in this demand pattern.

The Pareto plots also allow us to obtain insight into the effect of restricting the number of connected components. Restricting the BRT line to consist of a single component leads to a clear reduction in the number of passengers attracted, especially for the *CENTER* and *END* demand patterns. The reduction is significantly smaller when allowing at least two components, where especially the Pareto plots for allowing three components lie close to the ones where no restriction on the number

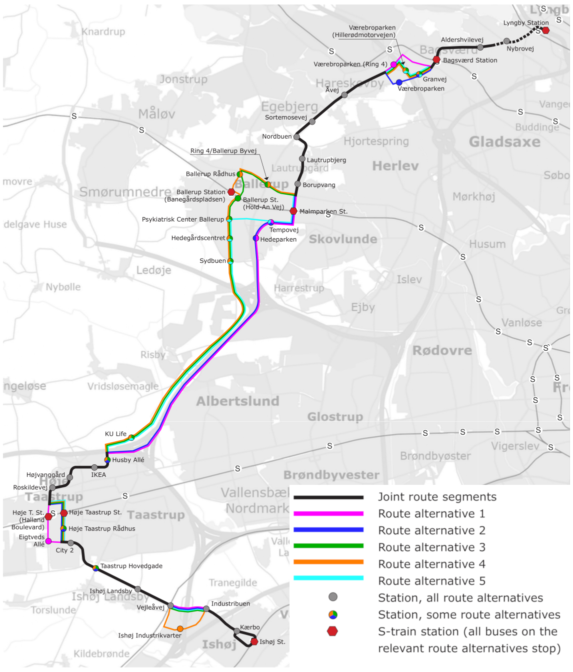


Figure 1: Route alternatives for a new BRT line in Greater Copenhagen. Adapted from Vejdirektoratet et al. (2022).

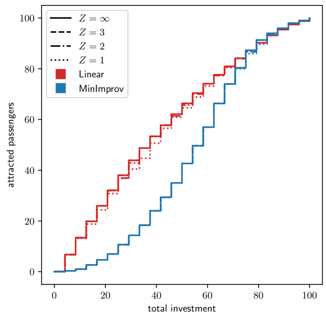
of components is enforced.

Results BRT Line Greater Copenhagen

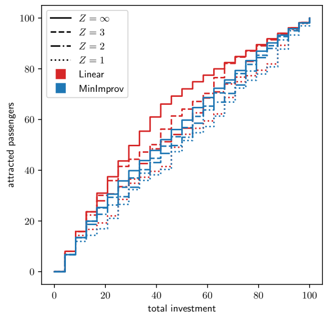
Pareto plots for the instances based on the Greater Copenhagen BRT line are given in Figure 3. To evaluate the impact of multiple investing parties, these Pareto plots have been split into the case with multiple investing municipalities (MIM) and a case with a single investing party (SOC) that can spend the whole investment budget B. For the MIM case, the total budget is split both according to the number of passengers in a municipality (*pass*) and the costs of the edges in a municipality (*cost*). Note that no restriction is enforced on the number of BRT components.
Figure 3 shows that there is not a universal ordering of the line alternatives but that the best alternative depends on the investment level. For the SOC case, line alternatives 4 and 5, e.g., lead to the highest number of passengers for higher investment levels under both passenger responses. On the other hand, line alternatives 1 and 2 perform well for low investment levels, in particular for the MinImprov response. When comparing the SOC and MIM cases, it can additionally be seen that the introduction of a budget per municipality leads to a clear reduction in the number of attracted passengers. This reduction seems to be strongest when passengers behave according to the MinImprov response, for which we again see a more convex shape of the Pareto curve when moving to the MIM case. Lastly, a comparison between the two budget assignments shows that the *cost* budget distribution often seems to lead to the highest number of passengers for high investment levels, while *pass* often performs well for lower investment level.

4 Conclusions

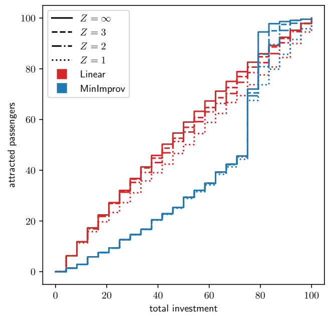
In this paper, we studied the BRT investment problem, which is focused on finding the trade-off between attracted passengers and investment budget when upgrading an existing bus line to a BRT line. We formulated the problem formally and suggested an ϵ-constraint based algorithm to enumerate the full set of non-dominated points. The algorithm was tested on both artificial instances and instances coming from a BRT line case study in Greater Copenhagen. Our artificial results give insight into the trade-off between the number of passengers and investment budget for different instance settings and show that the trade-off clearly depends on the assumed passenger response to upgrades. Moreover, they show that especially the limitation to a single BRT component leads to fewer passengers, while the impact is significantly lower if more components are allowed. Our results for the Greater Copenhagen case study show how the best line alternative can



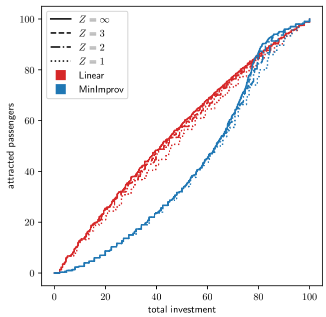
(a) UNIT, EVEN



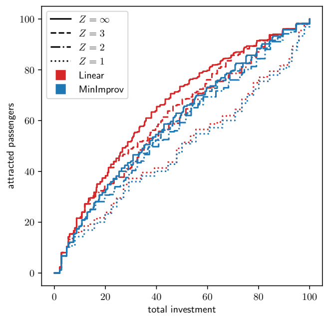
(b) UNIT, CENTER



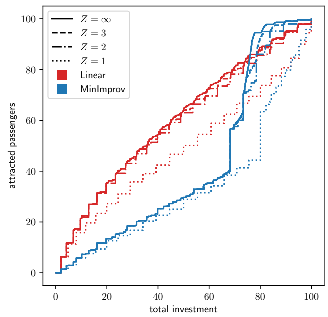
(c) UNIT, END



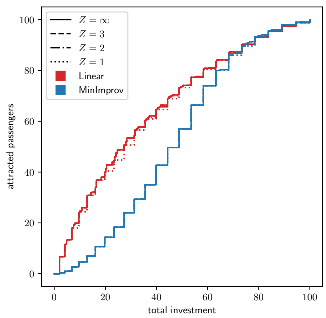
(d) MIDDLE, EVEN



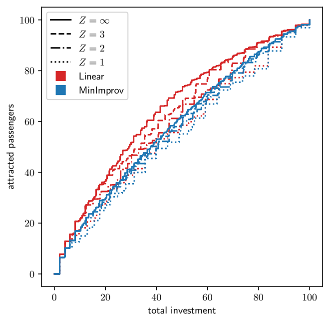
(e) MIDDLE, CENTER



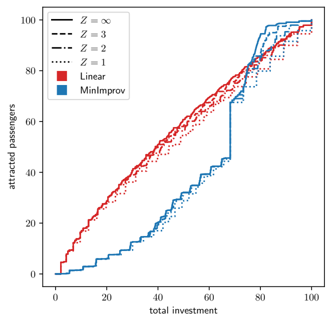
(f) MIDDLE, END



(g) ENDS, EVEN

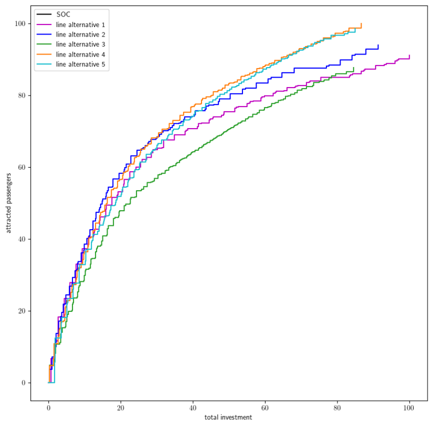


(h) ENDS, CENTER

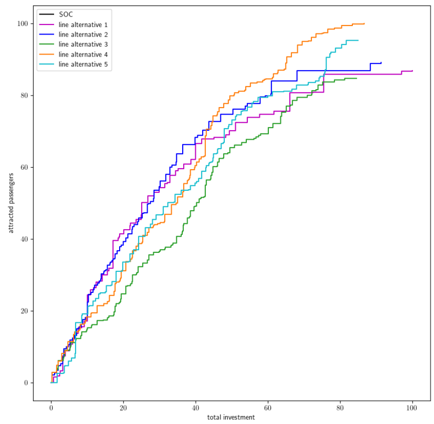


(i) ENDS, END

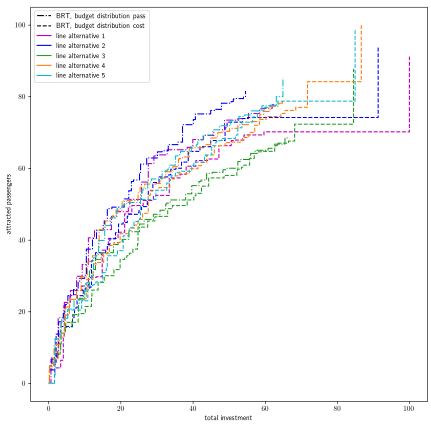
Figure 2: Non-dominated points for the LINEAR (red) and MINIMPROV (blue) passenger response. Solid lines represent the case $Z = \infty$, dashed lines $Z = 3$, dashed-dotted lines $Z = 2$ and dotted lines $Z = 1$. Attracted passengers and total investment are given as a percentage of the total number of potential passengers and costs for upgrading all segments, respectively.



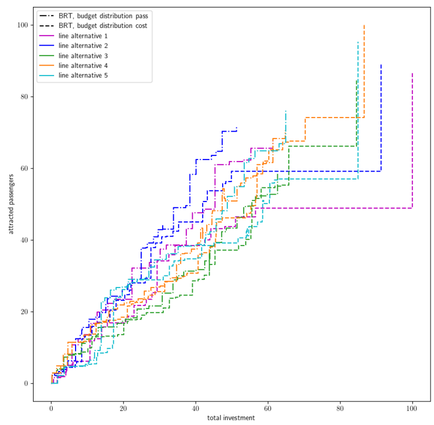
(a) SOC: Linear



(b) SOC: MinImprov



(c) MIM: Linear



(d) MIM: MinImprov

Figure 3: Comparing investment costs and attracted passengers for the different route alternatives for $Z = \infty$.

differ per investment level and show the impact that having multiple investing municipalities has on the number of attracted passengers. The latter is shown to depend on the passenger response, where the impact is strongest in the case of the threshold-based passenger response MinImprov.

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