Varying critical time to collision: a perspective of driver space

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Short summary

Collision warnings play a crucial role in preventing crashes based on estimating the critical time to potential accidents. Existing research and applications mainly focus on longitudinal vehicle interaction and headway keeping on highways. However, urban driving with frequent lateral interaction may have different critical time to collision. This study considers both longitudinal and lateral interaction through a two-dimensional spacing measure, driver space, to estimate the average critical time for drivers to respond to a potential collision. With the average spacing between vehicles at different levels of discomfort and in different relative speeds, we estimate the critical time via linear regression. Our experiments on two trajectory datasets find that drivers are more alert to collision dangers on highways compared to urban intersections, and drivers respond to potential collisions more quickly during lateral interaction than longitudinal. These findings emphasise the need of tailored collision warning systems for further improving road safety.

Keywords: Critical time to collision, driver space, driving safety, trajectory data.

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1 Introduction

Autonomous driving and advanced driving assistance systems (ADAS) have been a rapidly evolving research area, aiming to enhance road safety and efficiency. As part of that, forward collision warning and collision avoidance systems play a critical role in preventing crashes. These systems distinguish emergencies based on the critical time to collision, which refers to the amount of time a vehicle has to respond to a potential collision.

Critical time to collision has been typically considered in car following scenarios for longitudinal interaction between vehicles (Bella & Russo, 2011; Tawfeek & El-Basyouny, 2018). As an example, Time to Collision (TTC) is one of the most effective and broadly used indicators for warning rear-end collisions (Lu et al., 2021). It is calculated based on the relative position and relative velocity between two approaching vehicles, assuming that their movement continue without change. Then a critical threshold (denoted as TTC*) is set to distinguish un(safe) situations. ADAS usually set a fixed threshold, but some studies found that TTC* can vary among drivers (Kusano et al., 2015) and in different traffic environments (Arun et al., 2021) due to different human perception.

As the number of ADAS-equipped vehicles increases in urban areas, there is a growing need to consider critical time to collision in a wider range of driving scenarios. In urban traffic, the interaction between vehicles involves more angled movement such as lane changing and turning (Zhao et al., 2020). This entails potential collisions beyond rear-end crashes, such as head-on and side-swipe accidents (Theofilatos et al., 2012).

Does human perception of collision danger differ on highways and in urban traffic, with or without lateral interaction? This study addresses the question from the perspective of driver space. Driver space measures two-dimensional spacing between vehicles. Our previous study (Jiao et al., 2022) presented a method to quantify the 2d spacing in a probabilistic manner, which is based on the accumulative presence of vehicles. The method can locate a set of boundaries of spacing in various driving scenarios, with which we can estimate the average critical time to collision given the relative speed between vehicles.

In the following sections, we will firstly introduce our methods in Section 2. Then we apply the methods to two trajectory datasets collected by drones in the U.S. One is over an expressway and the other is over an intersection. The results and discussion will be presented in Section 3. Section 4 will conclude the study. Our findings are expected to aid the development of collision warning systems.

2 Methodology

This section will at first briefly explain driver space and its quantification. Then we will introduce how to obtain comfortable and uncomfortable spacing from the quantification. Finally, we will present how this study estimates critical time to collision.

Driver space and its quantification

The driver space of a vehicle refers to a set of boundaries where the driver experiences different levels of discomfort. When the driver space is intruded (i.e., when two vehicles are close enough), discomfort is raised. This discomfort motivates the drivers to maintain a proper distance from each other.

We use formula (1) to quantify the varying levels of discomfort caused by driver space intrusion into a value between 0 and 1. The higher the $p(x, y)$, the more discomfort is caused. For two vehicles i and j, x and y are transformed coordinates of j in a system where i is at the origin and the y-axis points to the direction of their relative velocity.

$$p(x, y|\boldsymbol{\theta}) = \exp\left(-\left|\frac{x}{r_x}\right|^{\beta_x} - \left|\frac{y}{r_y}\right|^{\beta_y}\right), \tag{1}$$

where each of $\boldsymbol{\theta} = (r_x, r_y, \beta_x, \beta_y)^\top$ has two components:

$$\begin{cases} \theta = \dfrac{1+\mathrm{sgn}(x)}{2}\theta^+ + \dfrac{1-\mathrm{sgn}(x)}{2}\theta^- \text{ for } \theta = r_x, \beta_x, \\ \theta = \dfrac{1+\mathrm{sgn}(y)}{2}\theta^+ + \dfrac{1-\mathrm{sgn}(y)}{2}\theta^- \text{ for } \theta = r_y, \beta_y. \end{cases} \tag{2}$$

Formula (1) is adapted from the density function of the generalised Gaussian distribution. We use it to parameterise the comfort-discomfort transition during approaching. Among the parameters in formulae (2), $\boldsymbol{r} = \{r_x^+, r_x^-, r_y^+, r_y^-\}$ determine driver space boundaries in different directions, where $p = e^{-1}$; and $\boldsymbol{\beta} = \{\beta_x^+, \beta_x^-, \beta_y^+, \beta_y^-\}$ determine how fast comfort changes to discomfort across the boundaries in different directions.

Driver space, i.e., two-dimensional vehicle spacing, can vary in different scenarios. Correspondingly, given samples of vehicles in various scenarios, $\boldsymbol{\theta}$ can also be different. The inference of $\boldsymbol{\theta}$ is achieved by estimating the density of the accumulative presence of vehicle pairs. In each pair, one of them is considered as an ego vehicle and the other is a surrounding vehicle. By aggregating all pairs in the same scenario, an average ego vehicle is abstracted, and driver space is shaped by the surrounding vehicles. In our previous research, we developed a method that makes consistent inference of driver space. The readers are referred to Jiao et al. (2022) for technical details.

Spacing between approaching vehicles

Driver space reflects the average preference of drivers' spacing behaviour. This includes spacing when vehicles are approaching each other and leaving away. As collision is mainly a result of getting overly close, we only take approaching spacing into account.

As stated above, our quantification of driver space transforms vehicle coordinates. The transformation aligns the y-axis in the target coordinate system to the direction of the relative velocity between two interacting vehicles. This ensures the independence of x and y. For two vehicles approaching each other, the transformed y will be larger than 0. In formula (2), $\boldsymbol{r}$ and $\boldsymbol{\beta}$ define where the intrusion discomfort increases most quickly and how quickly the increase is. Therefore, among the parameters, r_y^+ and β_y^+ characterise the spacing between vehicles that are approaching each other.

With the inferred r_y^+ and β_y^+ in various scenarios, we can correspondingly estimate the spacing between approaching vehicles under different extent of discomfort. Given a certain extent of discomfort indicated by p, the approaching spacing $s(p)$ can be computed by solving the inverse of equation (1). As x and y are independent from each other, $s(p)$ is solved as

$$s(p) = \hat{r}_y^+(-\ln(p))^{1/\hat{\beta}_y^+}. \tag{3}$$

For vehicle samples in different situations and at different relative speeds, approaching spacing can be computed correspondingly, and then we can estimate the critical time to collision.

Critical time to collision

Our quantification of driver space is based on the assumed negative correlation between the presence of vehicles and the extent of discomfort. During the approaching between vehicles, drivers can feel increasing discomfort, and this increase is assumed to be fastest when $p = e^{-1}$. When p is close to 0, drivers are comfortable with other vehicles' presence. When p is close to 1, the vehicles are so close that a collision is imminent.

In this study, we consider two kinds of critical time. One is the time from comfort to collision, the other is the time from discomfort to collision. The former represents the time from when a driver starts to feel discomfort due to approaching to a potential collision, and the latter represents the time from when a driver experiences clear discomfort and seeks for change to a potential collision.

We firstly consider spacing from comfort to collision as $s_c = s(p = 0.1)$ and spacing from discomfort to collision as $s_d = s(p = e^{-1})$. Generally denote a series of spacing under different relative speeds $\boldsymbol{v}$ as $\boldsymbol{s}$. For each relative speed condition v, there is a corresponding s. Given the linear physical relationship between speed and distance, we assume that $\boldsymbol{s}$ and $\boldsymbol{v}$ are linearly correlated as equation

(4) and the coefficient t^* is the critical time.

$$s = s_0 + vt^* + \epsilon, \tag{4}$$

where s_0 is the spacing when vehicles are relative static, and ϵ is the random error that satisfies $E(\boldsymbol{\epsilon}) = 0$.

We then use least squares fitting to estimate $\hat{s}_0$ and $\hat{t}^*$. The unbiased estimation of them are solved as equations (5).

$$\begin{cases} \hat{t}^* = \dfrac{\sum_{i=1}^{n}(v_i - \overline{v})(s_i - \overline{s})}{\sum_{i=1}^{n}(v_i - \overline{v})^2}, \\ \hat{s}_0 = \overline{s} - \overline{v}\hat{t}^*. \end{cases} \tag{5}$$

In addition, the variance of $\hat{t}^*$ can also be estimated as shown in equation (6)

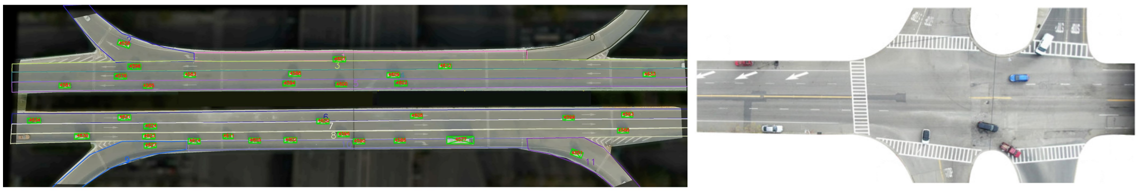
$$V(\hat{t}^*) = \frac{V(\boldsymbol{\epsilon})}{\sum_{i=1}^{n}(v_i - \overline{v})^2} = \frac{V(\boldsymbol{s})}{\sum_{i=1}^{n}(v_i - \overline{v})^2}. \tag{6}$$

In this way, based on s_c and s_d, respectively, we can estimate the critical time from comfort to collision, denoted as $\hat{t}_c^*$, and the critical time from discomfort to collision, denoted as $\hat{t}_d^*$.

3 Results and discussion

Datasets

We apply the proposed approach to two trajectory datasets. Both of them are collected in the U.S., of which the videos of vehicle movement were recorded by drones, and then computer vision algorithms were used to process the videos into numerical coordinates. As shown in Figure 1, one of them is a weaving segment of an expressway marked with *A* (Zheng et al., 2022), where 9,956 vehicles in 1.44 hours were recorded; and the other is an intersection denoted as *GL* (Zhan et al., 2019), where 10,510 vehicles in 4.34 hours were recorded. This study considers vehicle-vehicle interaction only, so pedestrians and cyclists in the intersection GL are excluded.



(a) Expressway A

(b) Intersection GL

Figure 1. Two road segments that are analysed. (a) and (b) are respectively reused from Figure 8 in Zheng et al. (2022) and Fig.2 in Zhan et al. (2019).

We consider lateral interaction at the intersection *GL*. If the angle between the moving directions of the two vehicles in a pair is smaller than 15 degrees or larger than 165 degrees, they are considered to interact only in the longitudinal direction (i.e., car following); otherwise, their interaction is considered to also involve the lateral direction (e.g., lane-changing and turning). In this way, our analysis considers three situations: interaction in the expressway, longitudinal interaction at the intersection, and lateral interaction at the intersection. They are referred to as *Expressway A*, *Non-lateral GL*, and *With-lateral GL* in the following.

Inferred driver space

We first sample vehicle pairs present at the same frame, and then infer the driver spaces that they accumulatively shape in different scenarios. Figure 2 shows several inference results. Driver spaces in different situations and under 2, 4, 6, 8, 10 m/s of the relative speed are drawn. In each subplot, the yellow dots indicate surrounding vehicle positions, and the driver spaces that they shape are plotted as contours at different levels of intrusion discomfort.

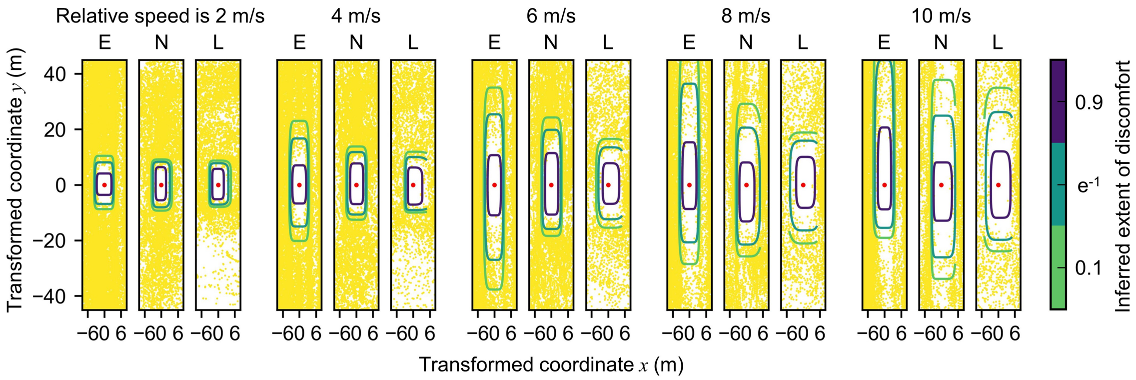


Figure 2. Inferred driver space in different scenarios. E: Expressway A; N: Non-lateral GL; L: With-lateral GL.

Figure 2 provides an intuitive impression of how drivers maintain distance in various scenarios. Driver space expands as the relative speed between the vehicles increases. While at the same relative speed, the average driver space is largest on the expressway, followed by that formed by vehicles at the intersection during non-lateral interaction, and the driver space formed during lateral interaction is the smallest.

The difference in driver space in different scenarios imply that drivers perceive and react differently during interaction. For instance, when two vehicles are moving in the same direction on the expressway at a similar relative speed, they maintain a larger spacing compared to when they are driving at the intersection with non-lateral interaction. In the former case, drivers tend to be more cautious and distant from each other. Similarly, if the two vehicles were to encounter each other during a lateral interaction (i.e., they are in angled directions but the relative speed value is still similar), their spacing would be even smaller. Such differences reflect drivers' different perception about the approaching between each other, which results in different reaction.

Estimated critical time to collision

In the three situations of *Expressway A*, *Non-Lateral GL*, and *With-lateral GL*, we compute the spacing at various relative speeds and then estimate the corresponding critical time to collision. The results are presented in Figure 3. Each column in this figure represents a specific situation. The upper plot in each column displays the computed spacing, and the bottom plot displays the estimated time and its variance.

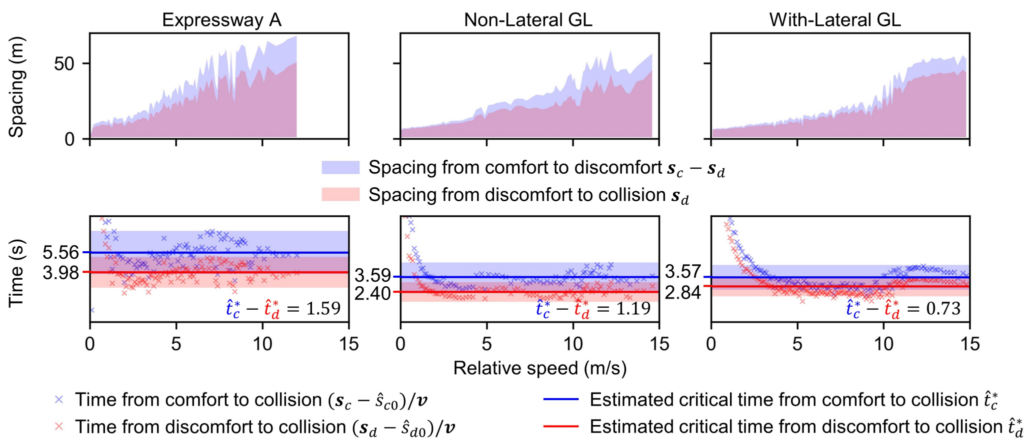


Figure 3. Average spacing and estimated critical time to collision in different situations.

The red lines indicate the estimated critical time from discomfort to collision $\hat{t}_d^*$. Our results show that this value is 3.98s on the expressway A, but reduces to 2.40s at the intersection GL during non-lateral interaction and to 2.84s during lateral interaction. This estimated time represents the duration that a driver realises the danger before a potential collision. A higher value means that the driver perceive the potential danger earlier. Therefore, the results suggest that drivers tend to perceive potential collisions more quickly when they drive on the expressway than when they drive at the intersection. This makes sense as the traffic environment on highways is generally simpler than in urban areas, so that a potential collision is more predictable.

The difference between the blue lines and the red lines (i.e., $\hat{t}_c^* - \hat{t}_d^*$) indicates the estimated time from when a driver begins to feel discomfort to when the driver feels clear discomfort that motivates behaviour change. This time difference reflects how quickly the driver responds to a potential collision. Our results show that drivers respond most quickly when they drive at the intersection during lateral interaction, with a time difference of 0.73s. However, during non-lateral interaction, the time difference is 1.19s at the intersection and 1.59s on the expressway. This could be due to that lateral interaction typically takes place in a closer distance, which requires faster response of drivers. Corroborating support can be seen from the spacing plots, where the spacing from comfort to discomfort is larger for longitudinal interaction than for lateral interaction.

4 Conclusions

This study offers a driver space perspective to analyse the critical time to collision and the issuing of collision warning. Firstly the averaged two-dimensional spacing of drivers in different situations is quantified based on the accumulative presence of vehicles. Then we use least squares to estimate the critical time to collision by fitting the linear relationship between the approaching spacing and the corresponding relative speeds. We analysed three situations in this study, including highway interaction, longitudinal interaction at an intersection, and lateral interaction at the same intersection. Our results show that drivers may take different amount of time to perceive and react to a potential collision in different situations. Specifically, drivers perceive the danger of a potential collision sooner on highways than at urban intersections, which implies that they are more sensitive to potential collision when driving on highways. Meanwhile, drivers take quicker reaction during lateral interaction than longitudinal interaction. These results suggest that collision warning systems need to be tailored to specific driving situations, with a greater warning sensitivity on highways and a proper time design to avoid distracting drivers' proactive reaction. By accounting for the variation of driving situations and interactions, drivers can get more reliable reminders and the road safety can be improved for all road users.

In the next step, we will apply the approach to more intersection situations. Figure 3 shows two thresholds for the situation of *With-Lateral GL*. The lower threshold is seen when the relative speed is smaller than around 10 m/s, and the higher one appears when the relative speed is larger. This implies that, during lateral interaction, drivers may have even slower awareness of potential collisions if their speed difference is smaller. Such implication poses a concern as the delayed awareness may result in inadequate reaction time to avoid potential collisions. Therefore, we still need to examine whether this phenomenon is common across different intersections and to explore the underlying causes.

This study contributes to the growing research field on ADAS and autonomous driving by extending the scope of critical time to collision to a wider range of driving scenarios, beyond car-following that is typically focused. However, it is important to note that the critical time to collision is just one aspect of safe driving. Other factors, such as vehicle-to-vehicle communication, pedestrian and bicycle safety, and the ability to detect and respond to unexpected situations on the road, also need to be considered in developing more advanced collision avoidance systems.

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