Charging demand for the unserved — an agent-based model approach

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Short summary

Deploying charging infrastructure requires identifying the most effective placement and size of charging facilities. This is particularly challenging when electric vehicles (EVs) are gradually introduced, as it creates a dynamic target that must be met to ensure successful adoption of EVs. This paper introduces an agent-based simulation model that tracks movements of EVs in space and time. Our model is based on a choice model of charging behaviour, which is integrated with non-parametric queues and information-sharing of waiting time. Our simulation captures demand resulting from choice of charging and the unserved demand represented as charging intentions that were not met. It is demonstrated that unserved demand varies over time and across locations, and that it can be greatly reduced by our information-sharing strategy. The model is applied to Copenhagen where we examine changes in charging infrastructure requirements between 2021-2030 when going from EV shares of 2% to 30%.
Keywords: agent-based simulation, censored demand, electric vehicles, information-sharing

1 Introduction

The recent worldwide uptake of EVs has led to an increasing interest for the EV charging situation and the design of the corresponding charging infrastructure.
Many papers have considered the placement and sizing problem of chargers from an operational research perspective (Hengsong Wang et al., 2010; Kuby & Lim, 125). Included in these types of models are the maximum set covering models as considered in Wang & Lin (2009) and Wang & Wang (2010). A common challenge, however, when applying such models, is that it requires simplifications when stating the demand-side. As demonstrated in Luo & Qiu (2020) and Kavianipour et al. (2021), it often involves use of simple queuing systems, limited heterogeneity, and few or no interactions between users.
An alternative and increasingly popular approach, however, is the use of agent-based simulation models to explore heterogeneous inputs and the consequences it has for different types of users. Specifically, the impact of flexible arrivals, vehicle heterogeneity (e.g. battery range and charging speed) and different driving patterns. This is the approach taken here, as will be discussed in more detail below.

The challenge

When planning a charging infrastructure for future years, several non-trivial challenges exist. Before considering the solution approach, it is timely to dive into some of these challenges.
A general problem is the prediction problem of charging demand in future years. This is complicated because it depends on many heterogenous inputs, which in itself are difficult to project. Some inputs, such as battery range and charging speed, are dependent on technological innovations and buying preferences of users. Other input's, such as car ownership and population growth, depend on the income development as well as the urban development. Because of the accumulated uncertainty from all of these inputs, the idea that a single unified solution exists renders itself useless. Rather, the solution to the problem should be seen as a distribution of solutions over a parameter

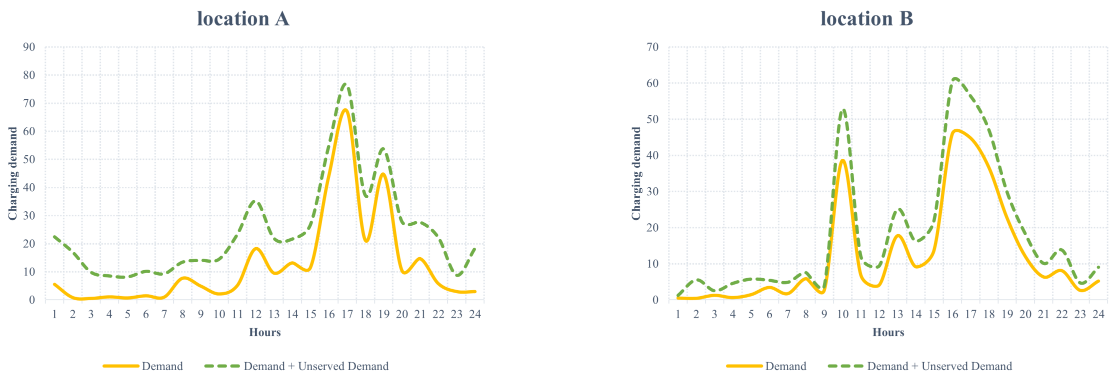


Figure 1: Unserved and served charging demand at different locations.

space, where some solutions are more likely than others. This observation, in our view, suggests that the agent-based simulation approach is the more suitable solution approach.

Another challenge, which exists for all capacity-constrained demand problems, is that supply deficiencies will cause users to suppress demand. This is particularly relevant in the context of charging because unserved charging demand will fluctuate in space and in time. An attempt is made to illustrate this in Figure 1 below.

The figure illustrates several things. Firstly, that the share of unserved demand varies in space and time. This suggests that from a charger location perspective, it is not only relevant to look at charging locations from the sole perspective of observed demand. Unserved demand should also be taken into account as it may influence the business case for operators and waiting time observed on the user side. Hence, a charging infrastructure that is designed with the unserved demand in mind, might look differently from a charging infrastructure solely based on what can be observed. It follows from this that the share of unserved demand can be considered as an important performance metric. Essentially, if measurable, the gap represents important information regarding the spatial and temporal match between demand and supply. The relevance of this is particularly useful when considering the gradual integration of EVs into the vehicle fleet over an extended period. This is because the balance between demand and supply presents itself as a moving target (in space and time) where insufficient supply may hinder EV adaptation in certain areas. As a result, it is highly important to monitor this balance over time and make sure that the inconvenience resulting from charging is minimal.

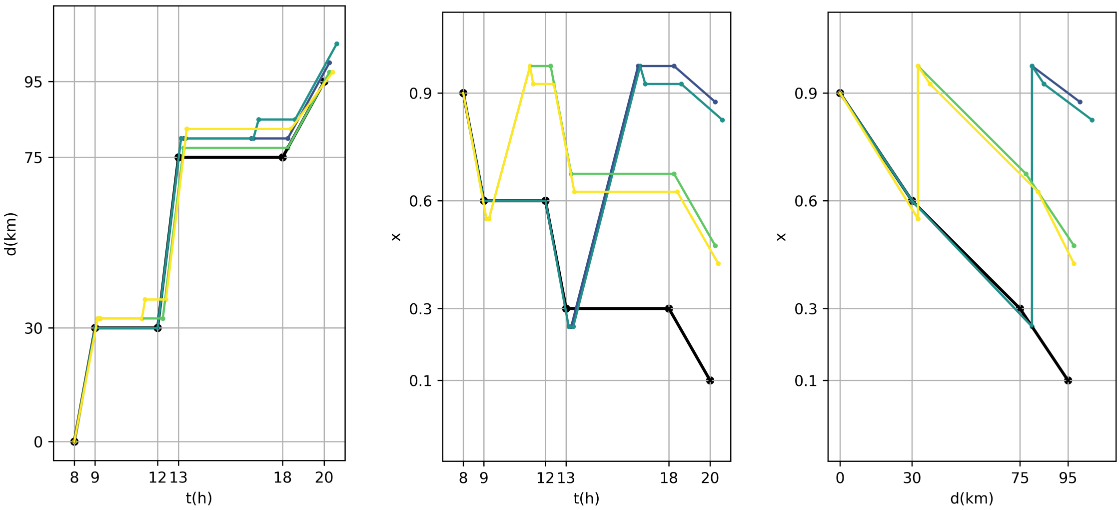
Sketch of a solution

Following the challenges as described in Section 1, it is relevant to delve into possible methodologies that allow consistent monitoring of unserved demand.

A requirement is that the modelling framework is agent-based, where utility functions and choice probabilities are monitored in space and time as a function of the different event triggers, allowing the systematic monitoring of unserved demand. Figure 2 attempts to illustrate the trajectories of a hypothetical agent's diary over time, driven distance and SoC level. The solid black line represents the unperturbed diary path, i.e. no charging, whereas the coloured lines indicate some options to charge during the morning and afternoon activities. The colour coding indicates the ordering of the probabilities for each option, from yellow being the most likely to dark purple the least. The details regarding the evaluation of the utility of each option and decision process are addressed in §2, while the agent generation of the process is discussed in §2.

Realistic monitoring of the state-of-charge in the space-time domain as well as the waiting time, requires, on the one hand realistic arrival patterns among users, but also flexibility resulting from queuing spill-back at the level of chargers. Specifically, for users to realistically react to queuing dynamics and trade different charging options to circumvent waiting time, it must be assumed that users possess certain forward-looking properties. This, in turn, necessitates an information-sharing system where waiting time is shared and where people react to this information. Our approach is based on the most recent state-of-art as presented in Vandet & Rich (2021).

Moreover, the tracking of behaviour over a range of heterogeneous inputs requires these to be properly modelled and forecasted. One example is that we use flexible battery state-of-charge distributions, which is based on recent research in Hipolito et al. (2022). Another example is that



(a) Time *vs.* distance (b) Time *vs.* SoC (c) Distance *vs.* SoC

Figure 2: Phase space trajectories over time, driven distance and SoC level, for a hypothetical agent with 4 charging options. Black lines trace non-charging path.

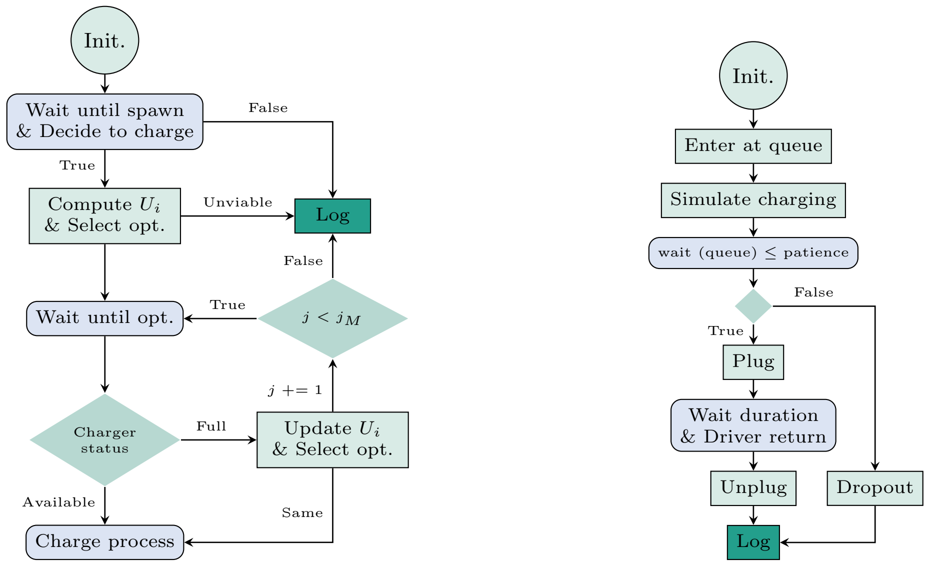
we use probabilistic range projections as applied in Rich et al. (2022).

Literature and contribution

Recent progress on analysis of censored demand Gammelli et al. (2020); Huttel et al. (2022) has highlighted the importance of a proper quantification of the real demand for efficient use of resources in transportation systems. Given the potential large impact of censored data, several authors elect to discard results when censoring is observed Rudloff & Lackner (2014); O'Mahony & Shmoys (2015), or alternatively calibrate their models based on historical data when no censoring was observed Albiński et al. (2018); Jian et al. (2016); Freund et al. (2019). However, previous approaches have been agnostic to queueing dynamics and to the potentially long duration of charging sessions in slow charging infrastructure. Given the large duration of charging events, and the expected queueing at fast charging clusters, it becomes crucial to quantify and track the impact of censored demand. Moreover, by monitoring not only the queueing and charging processes, but also each agent's decision path, we can track and record all instances in which demand is not served, and thus quantify what would otherwise be censored demand, whilst decoupling it from choice substitution. Hence, quantifying unserved demand is a better indicator of the limitations of the charging infrastructure, as performance can suffer significantly from high demand occurring in short periods of time, which are less likely to be identified when analysing performance solely on utilization.

Methodologically, this paper is based on an even-based micro simulation framework and thereby goes in the direction of Pruckner et al. (2017); Q. Yang et al. (2019), who use discrete event-simulators to represent demand and to analyse capacity utilization in charging systems. Moreover, inspired by Rich et al. (2022) and Vandet & Rich (2021) we use up-sampled trip diaries as input to the simulator. The benefits of doing so, in terms of attaining realistic trip and arrival time patterns, clearly outweigh the potential shortcomings, as discussed in more detail in Rich et al. (2022).

The contribution of the paper lies in the focus on unserved demand when dealing with the charging infrastructure problem. Add to this that, by monitoring this KPI over many different heterogeneous inputs, we are able to generate corresponding heterogeneous outputs and identify emergent behaviour in the system. The relaxation of simple Markovian queuing dynamics (van der Kam et al., 2019; Viswanathan et al., 2016; J. Yang et al., 2017) for non-parametric queues of the G/G/s type creates patterns that are closer to reality. Hence, we refrain from the single-solution approach and present a distribution of solutions which is a function of several probabilistic inputs.



(a) Main process

(b) Charge process

Figure 3: Diagrammatic representation of the iterative simulation process. Light blue background and rounded corners indicate the steps that include a waiting period.

2 Methodology

We present an agent-based simulation framework to predict the demand for EV charging over many heterogeneous inputs and for agents that interact through an information-sharing system (Vandet & Rich, 2021). The framework is designed with the intent of determining potential configurations of future charging infrastructure that satisfy the growing adoption of EVs, while minimizing disruption to present day travelling patterns and infrastructure deployment costs. To achieve such goal, we develop a synthetic population of EV users by combining trip dairies, and additional household information, based on the Danish National travel survey Baescu & Christiansen (2020). The origin-destination of all agents and their timing of trips (e.g. arrivals and departures) are calibrated to origin-destination (OD) traffic flow data and assigned to the OpenStreetMap road network using the shortest path algorithm. Similar applications of synthesized trip diary data have been presented in Rich et al. (2022); Kavianipour et al. (2021).

Clearly, the fact that we estimate demand for EV charging in the future based on calibrated trip diaries of today can be criticised. The underlying premise, then, is that the trip demand patterns are invariant with respect to the replacement of conventional cars with EVs. As argued in Rich et al. (2022), if this were not true, it would imply that people would have to change their otherwise optimal behaviour (where battery restrictions are absent) to a different behaviour, which is currently not known. In our view, designing a system where people can maintain a similar behaviour as today, reduces the burden associated with transitioning and thus facilitates adoption of EVs. Therefore, it represents a reasonable compromise when designing a societal optimal infrastructure for the future.

The simulation algorithm is represented diagrammatically in Figs. 3a and 3b, with the latter depicting exclusively the charging process. The intricacies of the simulation algorithm, as well as the agent synthesis process, will be discussed in a forthcoming manuscript, where we explore all branching options that lead to unserved demand. By monitoring the entire system over a period greater than 24 h, it is possible to estimate a range of probabilistic system KPIs, e.g. such as charging station utilization and waiting time at chargers. As we are monitoring, not only charging choices but also intentions to charge in cases where this is not possible, we can also track unserved charging demand in the system. This, in turn, is an extremely powerful performance indicator for the system as it quantifies an otherwise censored part of the demand, thus providing an important metric to plan the development of an efficient of public charging infrastructure.

At the programming level, this model is framed as a discrete event-based simulator, based on the SimPy package for Python Scherfke et al. (2022), allow us to track and coordinate the agent's movement in space and time.

Table 1: The price of electricity v_e for charging EV in DKK/kWh per charger type.

	normal	fast	private
P_{ch}	< 50	≥ 50	11
v_e	3.50	4.90	1.12

Decision model

The decision framework adopted here makes use of two models deployed in sequence. The first assesses the decision to charge Hipolito et al. (2022), while the second selects the desired opportunity. Both models are stochastic in nature, offering the agents the opportunity to make suboptimal choices at each step, introducing variance to the overall decision process, thus allowing the simulation path to branch away from the local minima. To select the an option, the utilities U_i are assessed, and the respective probabilities are determined by a classical multinomial logit model

$$p_i = \frac{e^{-\beta U_i}}{\sum_i e^{-\beta U_i}} , \tag{1}$$

where $\beta = 1$ is the global scaling factor. The choice set i, as well as the full form of all terms contributing to the utilities, will be described in detail in a forthcoming paper, below follows a brief summary. The selection process relies on a weighted random draw, where the weights are defined by p_i. The utilities are defined in generalized time form, where monetary costs are expressed as $U_i = c_i/\nu_\tau$, considering the general value-of-time (VoT) $\nu_\tau = 91$ DKK/h DTU (2022); Rich & Vandet (2019). The utilities include several contributions

$$U_i = \left[c_{i\epsilon} + \nu_i\left(\Delta t_i + w_i\right) + c_{ip} + c_{ih} + c_{ia}\right]/\nu_\tau \, . \tag{2}$$

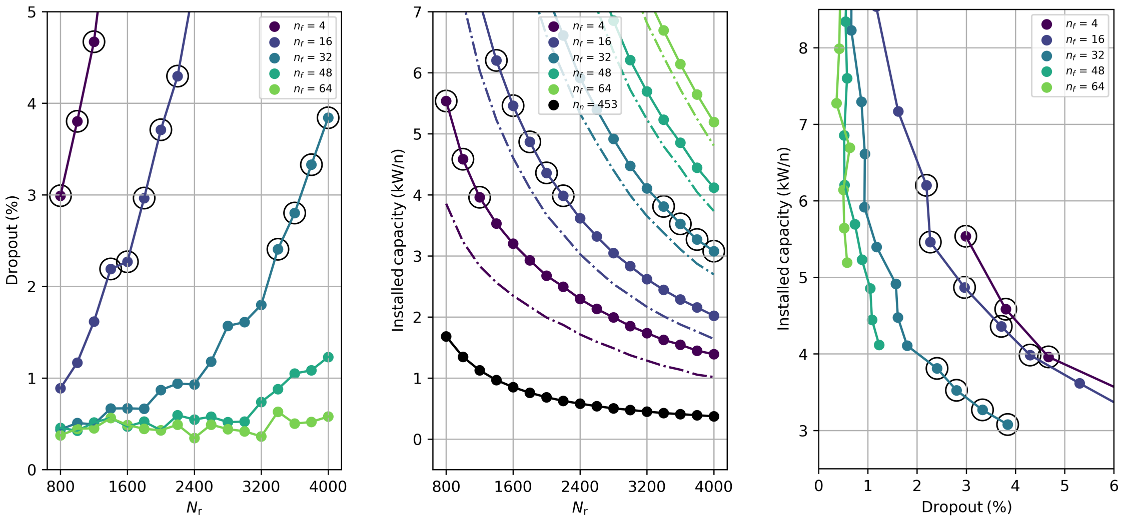
including the cost of electricity c_ϵ, the value of time for waiting w_i, detours and charging, costs of parking c_{ip} and idle charger time c_{ih}, and finally the cost associated with interrupting an activity.

3 Results

Here, we analyse a baseline scenario that offers a reference to validate the simulator against real-world data. We then proceed with an assessment of the viability of meeting growing demand in a dense urban municipality by expanding fast charging infrastructure.

Baseline scenario

To compare the simulator results against real-world utilization data, we make use of data for a charge station in the municipality of Frederiksberg covering a period of 390 days since the fall of 2021, reporting a mean charged energy of $\delta\epsilon = 17.2$ kWh at a mean power of $P = 7.42$ kW. The charging duration and total parking time are $\Delta t_{ch} = 3.0$ and $\Delta t = 6.2$ h, with utilization rate of $u = 0.52$ events per outlet per day.
To compare, we performed simulations covering a period of one week in the fall of 2022, considering a population of $N_\mathrm{r} = 800$ resident and $N_\mathrm{v} = 1200$ visitor EVs Danmarks Statistik (2023); Baescu & Christiansen (2020). In addition, the VoT is set as in DTU (2022), the costs of electricity are Tab. 1, and we follow municipality's parking constraints, whereby parking is free, but limited to 3h between 9h00 and 20h00. Finally, the estimate for the mean charge duration is set at $\tau_c = 0.5, 6$ h. Our simulations indicate that EVs would charge 16.9 and 23.9 kWh on the normal and fast infrastructures, with a utilization of 0.45 and 5.36, respectively. Charging and total duration at the stations are 2.3 and 7.6 h in normal charging infrastructure, while at fast the durations are 42 and 58 min. Agents with home charging (~ 40 %) mostly charge at home, with fewer than 1 % opting for public infrastructure.
The results for the normal charging infrastructure are in line with observed utilization of real infrastructure, but we lack data to validate the results found in the fast charging infrastructure. Discrepancies such as the reduced charge duration in the simulation, 2.3 h, can be traced to two main factors. The first is a higher prevalence of EV that only support charging in single- phase at 3.6kW and load sharing at the charge station.



(a) (b) (c)

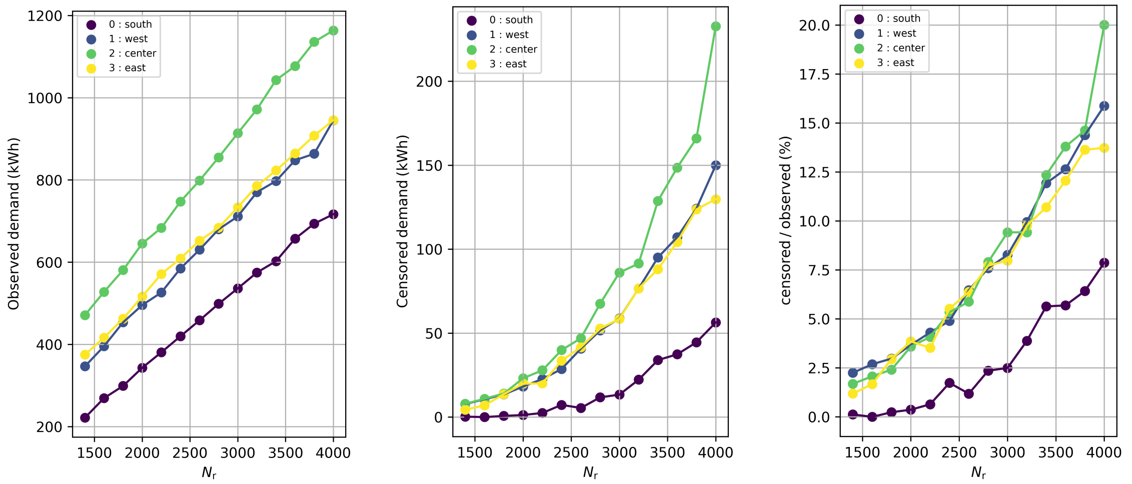
Figure 4: Dropout rate and installed capacity per EV. Lighter colours indicate growing number of fast outlets. Large circles indicate configurations with a dropout rate in $[2, 5]$ %.

The baseline scenario indicates that our simulations are consistent with reality, but indicate a 3 % dropout rate from the charging queue. There is little data on quality of service, but anecdotal reports of occasional long waiting, as well as sporadic cases of full charger utilization in the real-world data, indicate that waiting times could easily exceed the lower bounds of the agent's patience, which manifest in our simulations as dropouts.

Fast charging scenarios

Baseline results indicate that the fast infrastructure can serve up to 10 times more EV than normal charging, indicating that developing this type of infrastructure could reduce the urban impact of EV charging. To explore this hypothesis, we simulate the impact of introducing 3 new fast charging clusters within the municipality towards the West, South and East, in addition to the current cluster at its centre, adding equal number of outlets at each cluster. In Fig. 4a, we show the change in dropout rate for each infrastructure configuration. Results indicate that the baseline dropout rate ~ 3 % can be maintained at least up to $N_\mathrm{r} \sim 4000$ with total of 32 fast charging outlets. Moreover, as shown in Fig. 4b and 4c, the efficiency of the infrastructure increases as we observe that more EV can be served while maintaining the baseline performance. This is corroborated by utilization data which increases to 1.99 an 7.66 events per normal and fast outlet per day. Several insights can be drawn from this analysis. First, in all scenarios, the required installed power per EV is larger than the European guideline of 1 kW per EV {Secrétariat général du Conseil} (2022). This discrepancy can be understood in light of population density and the relatively low share of residents with access to private parking in this municipality. Second, upon combined analysis of Fig. 4a and 4b, it is possible to verify that for configurations with similar dropout rate, the required power per EV decreases with the increase in the number of EV, indicating that the infrastructure becomes more efficient as in grows in size.

The daily demand is shown in Fig. 5. While Fig. 5a indicate that some degree of geographical variation is observed, the unserved/censored demand Figs. 5b and 5c is mostly location-independent. At first glance, one would expect variation as depicted in Fig. 1, yet note that information-sharing and the short distances between fast charging infrastructure, facilitate the redistribution of demand, which in turn further increases the efficiency of the infrastructure. Moreover, the discrepancies found in south charger can be understood in light of the lower density of that part of the municipality, as well as the proximity to to municipality boarder leads to increase slip-over demand being transferred to other stations in the neighbour municipalities. Accounting for an effectively reduced number of EV using this charger, we can verify that the demand is in line with results in the other 3 fast chargers.



(a) (b) (c)

Figure 5: Daily observed (a) and censored (b) demand at four fast chargers with 4 outlets each. Ratio of censored against observed demand (c).

4 Concluding remarks

We present an agent-based approach to simulate the EV demand, tracking it over space and time. By virtue of introducing branching options in the simulation algorithm, we can decouple the demand into the observed and censored parts. The censored part offers a quantification of the unserved demand, that can be used to improve rollout strategies of charging infrastructure, that satisfy demand, while minimizing impact on society.
Moreover, the results strongly suggest that the charging demand, both observed and censored, can be evenly distributed, thanks to the introduction of an information-sharing strategy, that leads agents to avoid overcrowded stations.

Future research perspectives

In addition to an expansion of the present analysis, a forthcoming publication will extend the analysis discussed here beyond the spatial distribution of demand at the fast charging infrastructure. It will include, among other goals, an analysis of the intraday variation of demand, to characterize the distinct rhythms of utilization of each type of infrastructure. In addition, the simulation region will be expanded beyond the current municipality, to include the whole city of Copenhagen, which will offer a better platform to quantify the role of information-sharing and additional strategies to balance demand.

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