# Resilience-Oriented Design for Public Transport Networks

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## SHORT SUMMARY

Public transport systems are typically designed based on estimated passenger demand and supply patterns, yet may often be called to operate under vastly different operational settings. To systematically design resilient transit systems, it is necessary to "weave" resilience-oriented thinking into the established public transport network design process, moving from an abstract concept to an implementable methodology. This study aims to effectively and efficiently design resilient public transport networks through the integration of Reinforcement Learning (RL), Local Search operators and Particle Swarm Optimization. We present a redundancy indicator and integrate it within a hybrid RL-enhanced metaheuristic solution framework to design more resilient route structures. We apply the proposed Memetic algorithm to an established benchmark from the literature and validate the proposed approach under a series of random and targeted attacks, simulating link disruptions. Results demonstrate that resilience can be enhanced through redundancy without adversely impacting average travel times.

**Keywords:** Transit Route Network Design, Resilience, Vulnerability, Redundancy, Memetic algorithms, Reinforcement learning.

## 1. INTRODUCTION

Most existing systems are over-optimized to predefined design inputs. However, as a great deal of uncertainty persists, these systems become fragile to ever-changing external conditions. This is especially true when it comes to transportation systems, and particularly public transport networks (Mattsson and Jenelius, 2015). Indeed, these systems were once designed for predicted passenger demand and supply patterns, yet are now called to operate under vastly different operational settings. Still, medium and long-term disruptions (e.g. road closures, maintenance works) can induce significant changes in supply and trigger shifts in passenger demand, rendering public transport systems vulnerable and often unviable. In such cases, associated costs incurred by passengers and operators are unaccounted for in the design process, albeit significant. Due to rigid constraints and limitations of the underlying network structure, strategic interventions are limited in such cases, with disruptions associated with wide and sustained implications (Jenelius and Cats, 2015).

Under this scope, an important consideration refers to the ability of transport networks to withstand perturbations and maintain their serviceability. Towards this goal, several concepts have been introduced in the literature to evaluate the performance of transport networks under disruption, notably robustness, vulnerability, and resilience (Ge et al., 2022). In transportation-related literature, vulnerability and resilience are viewed as core properties of public transport systems, both considering the decrease in network performance under perturbations (Ge et al., 2022; Mattsson and Jenelius, 2015).

Under this scope, a large stream of studies sought to identify critical network links/segments, a posteriori investigating the link between network design and vulnerability (Cats, 2016; Mattsson and Jenelius, 2015; Rodríguez-Núñez and García-Palomares, 2014; von Ferber et al. 2012). Despite these observations, in terms of a priori designing resilient transit networks, there is a large gap in the respective literature. The problem of optimally designing surface public transportation systems, referred to as Transit Route Network Design Problem (TRNDP), has attracted the interest of the research community for over five decades (Iliopoulou et al. 2019). Still, despite the vast literature on the general TRNDP, methods for designing resilient transit networks are lacking.

Motivated by the gap in the respective literature, this study presents a methodological framework for enhancing resilience within the TRNDP, without negatively impacting performance. To achieve this, we incorporate Reinforcement Learning (RL) within a metaheuristic solution framework to reinforce the resilience of transit networks, under the planning paradigm for enhanced connectivity. In particular, the availability of trip alternatives, i.e. the amount of redundancy offered, allows for the impacts of link-based incidents to be absorbed **(**Rodríguez-Núñez and García-Palomares, 2014). To that end, a particle swarm optimization (PSO) algorithm integrated with neighborhood operators which manipulate the degree of connectivity, referred to as Memetic PSO, is developed and enhanced through RL to reinforce path redundancy. We adopt the view that resilience is related to the network's performance decrease, as computed based on demand coverage and transport efficiency, under a shift in operating conditions and develop a set of bi-directional link-based disruption scenarios to be investigated, representing full link closures (Cats, 2016).

## 2. METHODOLOGY

This section presents a general formulation for the TRNDP, the proposed redundancy indicator and outlines the components of the developed algorithmic framework.

### *The TRNDP*

In general, there is no commonly accepted mathematical programming formulation for the TRNDP due to its inherent complexity and discrete nature (Iliopoulou et al., 2019). A high-level mathematical formulation for the TRNDP can be given as follows. Let:

| | |
|---|---|
| $ATT_{RS}$: | Average Travel Time for route set *RS* |
| **C:** | the vector of path costs on the transit network |
| $d_{0RS}$: | Percentage of passenger demand satisfied without transfers for route set *RS* |
| $d_{1RS}$: | Percentage of passenger demand satisfied with one transfer for route set *RS* |
| $d_{2RS}$: | Percentage of passenger demand satisfied with two transfers for route set *RS* |
| $d_{unRS}$: | Percentage of unsatisfied demand for route set *RS* |
| *R:* | Route $\in RS$ |
| $\hat{\mathbf{RS}}$: | Vector of optimal routes |
| *RM:* | Maximum number of routes *R* in *RS* |
| *RS:* | Set of routes *{R}* |
| ***Q***: | the vector of segment flows on the transit network |
| $p_{RS}$: | Path redundancy score for route set *RS* |
| $s_R$: | Number of stops per route *R* |

$s_{min}$: Minimum number of stops
$s_{max}$: Maximum number of stops
$U$: the user route choice model function
$\omega$: Weighting factor

$$(\hat{\mathbf{RS}}) = \arg\min Z \ (\mathbf{RS}, \mathbf{Q}) \tag{1}$$

$$Z = ATT_{RS} - \omega p_{RS} \tag{2}$$

s.t.

$$(\mathbf{Q}, ATT_{RS}, d_{0RS}, d_{1RS}, d_{2RS}, d_{unRS}, p_{RS}) = U(C(\mathbf{RS})) \tag{3}$$

$$s_{\min} \leq s_R \leq s_{\max}, \quad \forall R \in RS \tag{4}$$

$$R \neq K \qquad \forall R \in RS \tag{5}$$

$$|RS| \leq RM \tag{6}$$

$$d_{unRS} = 0 \tag{7}$$

The problem seeks to determine the route set that minimizes the objective function (Eq. 1). The latter represents the user cost associated with the route set, defined in this case as a score resulting from the weighted difference of average travel time (*ATT*) and the redundancy indicator (Eq. 2). In this case, *ATT* is the most important metric, as it also reflects direct ridership due to transfer penalization (Fan and Mumford, 2010). The values of *ATT*, the redundancy indicator and other route evaluation criteria, are derived from the transit assignment process, which is represented by Equation (3). Equation (4) specifies the minimum and the maximum number of stops for routes. Equation (5) states that two individual routes cannot coincide, while Equation (6) specifies the maximum number of lines. Last, Equation (7) states that the percentage of unsatisfied passengers must be zero.

### *Redundancy*

The challenge in statically capturing resilience through a performance indicator is that one should account for the behaviour of the network under several disruption scenarios, as a full network scan is not computationally feasible for each candidate solution during the optimization. To that end, the number of alternative paths offers useful insights and is linked to better performance under disruptions, as route redundancy allows for flexibility to passengers (Cats, 2016). Based on this observation, we propose the use of a modified global efficiency indicator, which we name path redundancy, defined as follows:

$$p = \frac{\sum_{w \in W} d_w m_w}{\sum_{w \in W} d_w} \tag{8}$$

Where $w$ denotes an OD pair, $d_w$ the corresponding demand and $m_w$ denotes the number of distinct transit paths offered based on the physical network. This distinction is important, as, different route combinations could use the same physical path. We filter the number of available paths through the physical road network to determine the number of distinct physical paths exploited by the route network. However, physical path overlap must still be taken into account, as it may lead to overestimating the number of distinct alternatives. To account for the reduction in redundancy due to overlapping segments we scale the number of physical paths by considering an overlap coefficient. More specifically, given a set of feasible paths $L^w$ serving a specific OD pair $w$, the number of distinct physical paths is computed by considering the equivalent path index $o_l$ for each path $l$, defined as follows:

$$o_l = \frac{\sum_{u \in l_k} \frac{1}{k^w{}_u}}{|U_l|} \qquad l \in L^w \tag{9}$$

Where $u$ is an edge of path $l$, $U^l$ the set of edges comprising path $l$, $k^w{}_u$ the number of paths for the specific OD pair $w$ traversing segment $u$. Finally, the total number of distinct paths for a pair $w$ is given by summing the corresponding values for all paths $l$ in $L^w$:

$$m_w = \sum_{l \in L^w} o_l \tag{10}$$

So that $m_w$ is the number of distinct physical paths for OD pair $w$, accounting for similarities.

### *The proposed Memetic RL-enhanced PSO algorithm*

Motivated by the performance of emerging RL-enhanced PSO algorithms and Memetic PSO variants, we propose a discrete-space Memetic PSO where Q-learning is employed to select the search actions of each particle, referred to as MQLPSO. The proposed algorithm flexibly incorporates the discrete PSO operators for the TRNDP to effectively perform exploration for promising solutions within the entire region, a local-search procedure as the refinement step and a Q-learning framework as the operator selection mechanism. We employ four local search operators in total, which specifically target connectivity and thus, path redundancy, allowing the algorithm to perform exploitation for solution improvement in subregions. Each action contains a set of movements; three global search operators are used to move particles towards the global and personal best and four local search operators are used to refine individuals. The framework of the proposed algorithm is shown in Figure 1.

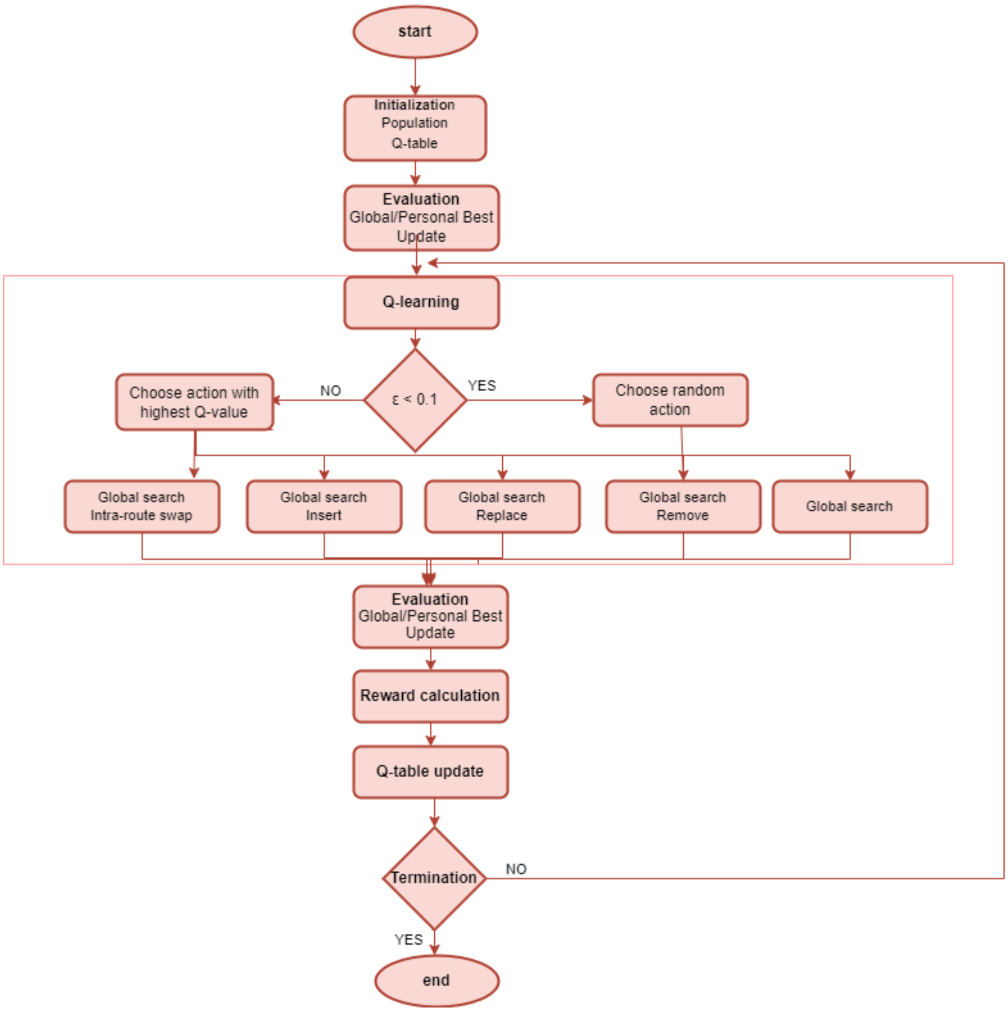


**Figure 1 MQLPSO framework**

In this specific instance, we are interested in designing a network that offers low average travel times with as much redundancy as possible. Therefore, the following three-piece reward function is defined, after experimentation:

$$r = \begin{cases} 1 & \text{, if } ATT' < ATT \text{ and } p' > p \\ 0.5 & \text{, if } ATT' < ATT \text{ or } p' > p \\ -1 & \text{, otherwise} \end{cases} \tag{11}$$

Where *ATT* and *ATT'* denote the value of *ATT* for the previous solution and the current, respectively; *p* and *p'* are the redundancy scores of the previous and current solution, respectively.

## 3. RESULTS AND DISCUSSION

The road network used as input by the proposed MQLPSO algorithm is based on a real Swiss road network (Mandl, 1980), comprised of 15 nodes and 21 links., and is the widely accepted benchmark for the TRNDP. The demand matrix is symmetric, and the routes run in both directions. The configuration is shown in Figure 2.

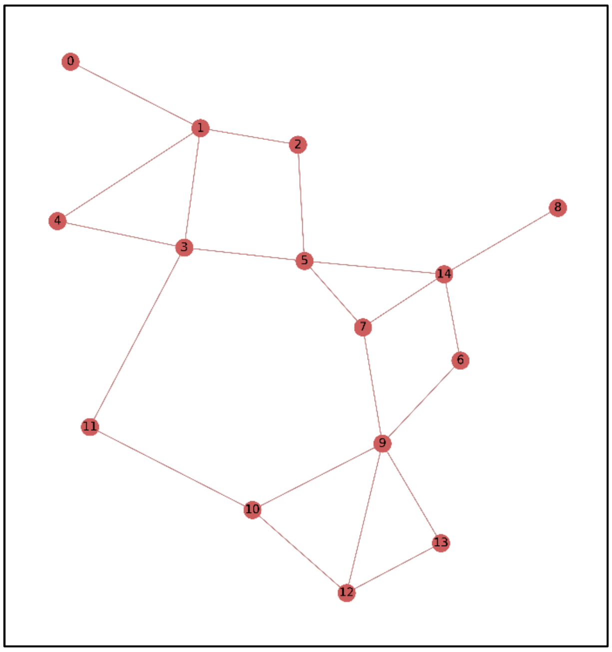


**Figure 2**. Mandl's Network configuration

In this case, we simulate link-based attacks, by removing links both at random and based on their criticality. For random attacks, we simulate the removal of 1- 10 random links and run 100 experiments per case to assess the impact on network performance to evaluate a representative set of scenarios (Matisziw et al., 2009). For targeted attacks, we consider scenarios without the most critical link and the most critical sequence of up to 10 link disruptions. This process enables the identification of links absorbing trips that have been diverted because of disruption in different scenarios. We identify critical links based on the value of the passenger betweenness centrality indicator (Cats, 2016). The measure is defined as follows:

$$PBC_u = \frac{1}{\sum_{w\in W} d_w} \sum_{w\in W} g_w(u) d_w \tag{18}$$

Where $g_w(u)$ denotes the fraction of shortest paths for OD pair $w$ traversing link $u$. We recalculate betweenness centrality after each edge removal.

For evaluation, we will compare the following algorithmic setups using the same random seed, conducting 20 experiments in all cases:

i. MQLPSO with coefficient $\omega$=0.1 in the objective function, so that *ATT* and redundancy are incorporated both at the objective and reward functions.

ii. MQLPSO with coefficient $\omega$=0 in the objective function, so that *ATT* minimization is the optimization objective and redundancy is enforced only through the reward function.

iii. The PSO global search step combined with random selection of local search operators, referred to as Memetic PSO (MPSO) aiming to minimize only *ATT*. We use this as a benchmark reflecting the typical TRNDP design process while capturing the exploitation capabilities of the neighborhood operators to some extent.

Table 1 shows the results for transit network configurations with 4 routes.

**Table 1. Comparison of solutions generated among methods for 4-Route Case**

| Performance Criteria | Algorithm | MQLPSO best | MQLPSO best | MPSO best | MQLPSO avg | MQLPSO avg | MPSO avg |
|---|---|---|---|---|---|---|---|
| | $\omega$ | 0.1 | 0 | 0 | 0.1 | 0 | 0 |
| *ATT (min)* | | 10.6 | **10.51** | 10.54 | 10.7 | 10.68 | 10.71 |
| *p* | | 3.42 | **3.42** | 3.03 | 3.17 | 3 | 2.97 |
| $d_0$ *(%)* | | 89.21 | 90.88 | 91.59 | 89.71 | 89.94 | 89.86 |
| $d_1$ *(%)* | | 10.79 | 9.12 | 7.71 | 10.03 | 9.86 | 9.85 |
| $d_2$ *(%)* | | 0 | 0 | 0.71 | 0.26 | 0.2 | 0.29 |
| $d_{un}$ *(%)* | | 0 | 0 | 0 | 0 | 0 | 0 |
| *Run time* (s) | | | | | 72 | 60 | 78 |

As seen in Table 1, MQLPSO produces similar quality results in both cases. The best solutions produced in this case feature the same redundancy value (3.42 vs 3.03 of MPSO, i.e. a 13% improvement), yet the solution under $\omega$=0 features a lower value for *ATT* and improved direct demand coverage, which is reasonable. In the average case, higher redundancy values are generated if both criteria are enforced through the objective function (3.17 on average), with a slight improvement of passenger-related performance criteria observed under $\omega$=0. Both cases of MQLPSO produce superior solutions to MPSO with zero two-transfer shares, demonstrating that the proposed RL scheme can improve both *ATT* and redundancy at the same time. To illustrate the value of reinforcing redundancy we compare the route configuration generated by MPSO with the route configuration under $\omega$=0.1, as on average it yields improved values for redundancy. Figure 3 shows the performance decrease of the network with *p*=3.42 vs *p*=3.03 under a series of random attacks, with boxplots summarizing 100 random runs.

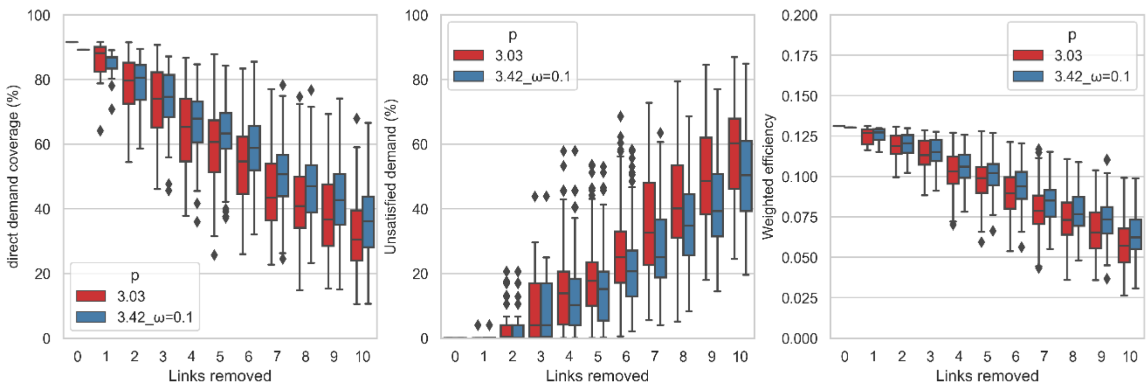


**Figure 3. Random attack impacts for the 4-route case.**

Figure 3 clearly shows the difference in performance decrease between the two network configurations, across all relevant indicators. As a general observation, a clear trend may be discerned where the route network with the smaller redundancy exhibits larger variability in terms of demand coverage under random attacks, with proportionally more scenarios resulting in worse outcomes. In fact, for the route network with the lowest path redundancy 25% of scenarios feature direct coverage between 45% and 64%, after the removal of 3 links versus 56% - 69% with *p*=3.42, besides a couple of outliers. After 4 link removals, the median value for unsatisfied demand is consistently higher in the unprotected network, with discrepancies becoming larger with the extent of disruption. The removal of a 4th link seems to be a turning point for performance loss, as a notable rise in unsatisfied demand and an abrupt decline in the efficiency of both networks is observed with consistently inferior values for *p*=3.03. For 7-link disruptions, the differences between the two networks become more pronounced, with the more redundant network retaining a larger portion of its serviceability with under 25% of unsatisfied passengers in the median case, compared to 33% for the less redundant network. Even in the case of an extended 10-link disruption, the former maintains a 10% advantage in terms of demand satisfaction over the latter.

Figure 4 shows the performance decrease under targeted attacks up to 10-edge disruptions.

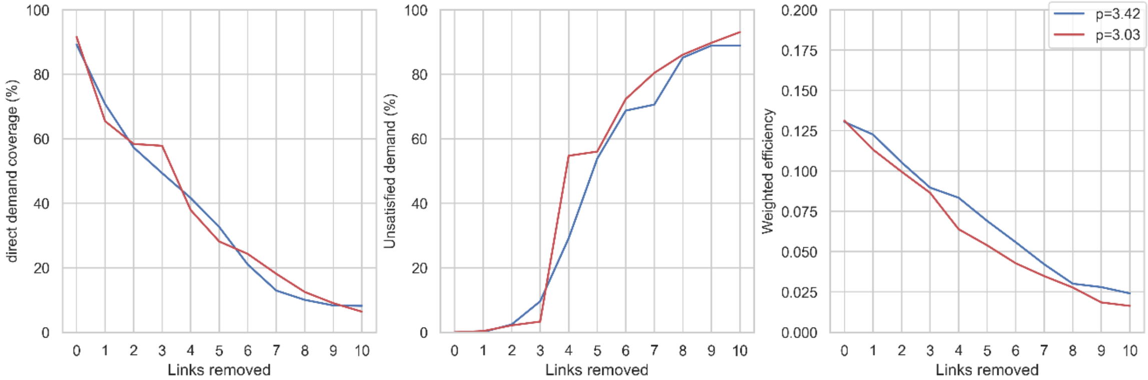


**Figure 4. Targeted attack impacts for the 4-route case.**

As seen in Figure 4, the value of redundancy becomes apparent after 3 consecutive removals, with unsatisfied demand higher across all cases after that point for the network with the lower redundancy measure. The efficiency and thus the passenger carrying capacity drops more abruptly in this case, with 4-link disruptions being the critical point. Indeed, for a sequence of 4 critical links removed, there is more than 25% difference in unsatisfied demand between the two networks and 20% in efficiency. Even though the relative gap becomes smaller with successive removals, the results for the unprotected network are consistently superior to those for the reinforced network. Even at 10 successive removals of the most critical link, the more redundant network serves 6% more passengers.

## 4. CONCLUSIONS

This study showcased a two-pronged resilience-oriented design framework: on the one hand, a redundancy indicator was developed and incorporated in the TRNDP solution process within an intelligent optimization framework and on the other hand, multiple simulation runs were performed to assess generated solutions and evaluate the design process. Results demonstrated that redundancy can reduce the impacts of link disruptions, reducing associated repercussions on demand coverage, with negligible costs in terms of average travel times. The more redundant network incurred a lower share of unsatisfied demand for multiple-link disruptions, either random or targeted, and a smoother decline in efficiency, retaining a larger portion of its serviceability. Respectively, the networks with lower redundancy exhibited more unpredictable behavior under attacks, with higher variability and more damaging worst-case scenarios. For targeted attacks, the value of redundancy is still evident, yet relatively lower compared to the case of random attacks.

This may perhaps be expected, as targeted attacks are based on the maximum weighted betweenness centrality measure, which may still be high even in redundant networks.

## ACKNOWLEDGEMENTS

This work was supported by the “Swiss Government Excellence Programme” through a Postdoctoral Fellowship (recipient: Christina Iliopoulou) under the project RESPOND.

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