# The spatial variation of travel time valuations: A general equilibrium model and application in project appraisal

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## Short summary

Current practices in transport policy appraisal are mostly restricted to partial equilibrium modelling, creating a natural need to explore new ways to understand the spatial general equilibrium impacts of transport interventions. The emerging literature of quantitative spatial models (QSM) offers new opportunities. However, the direct application of the QSM methodology in transport is hindered by the assumption of unidimensional 'iceberg' travel costs. Due to the presence of both temporal and pecuniary travel costs, the theoretical characterisation and empirical measurement of the monetary value of travel time savings has been a central theme of transport research for decades. We bridge a gap between spatial and transport economics by developing a quantitative spatial model with endogenous travel time valuations , revealing its previously neglected spatial heterogeneity. The model yields OD-specific values of time in spatial general equilibrium. Numerical implementation of the model highlights the relevance of our contribution in practical transport appraisal.

**Keywords**: transport appraisal; value of time; spatial general equilibrium.

## 1 Introduction

Transport appraisal models help divert heated debates on large-scale infrastructure projects to a somewhat more objective quantitative basis. Experience suggests that travel time savings is one of the biggest sources of benefits when a transport intervention reduces the distance and/or journey time between geographic locations. For this reason, the theoretical underpinning and empirical estimation of the monetary value of travel time received increased attention in transport research (Small, 2012). The standard transport appraisal methodology is often criticised, however, for its partial equilibrium approach, i.e., its inability to predict and quantify the impact of the spatial re-organisation of economic activity after the implementation of transformative transport investments (Mackie et al., 2011). Previous attempts in spatial general equilibrium transport appraisal, such as the so called 'land use-transport interaction' models, have not reached a consensual acceptance among economists, mainly due to the absence of microfoundations behind various assumptions on model specification and the arbitrary (theoretically inconsistent and/or statistically potentially biased) identification of model parameters.

This research reflects on recent developments at the crossroad between urban economics and economic geography: a new class of models often referred to as *quantitative spatial economics* (Redding & Rossi-Hansberg, 2017) seems to be more widely acknowledged as a tool for empirically relevant economic analysis in spatial general equilibrium. Quantitative spatial models (QSMs) are based on discrete-continuous demand models in which the choices of residential and workplace location are governed by Fréchet distributed multiplicative idiosyncratic shocks and, given the location choice, consumes optimise their consumption of housing, goods and services on a continuous scale. QSMs achieved a breakthrough in spatial economics due to very advantageous analytical properties: the existence and uniqueness of spatial equilibrium can be derived analytically and the model can be calibrated in a series of theoretically consistent econometrics exercises. Pioneering QSM papers, such as Allen & Arkolakis (2014), Ahlfeldt et al. (2015), Donaldson (2018), Monte et al. (2018), Heblich et al. (2020) and Allen & Arkolakis (2022), have been publishing in the leading journals of economics over recent years.

We believe that in the long run, QSMs may be suitable to replace the existing partial equilibrium-based transport appraisal methodologies in practical policy development as well. However, a prerequisite for that is to bring this new approach closer to transport research and align it more closely with advances in contemporary transport modelling.

In this research we focus on a common assumption of QSMs which limits their ability to replicate important characteristics of transport provision. QSMs capture the inconvenience of travel through the well-known *ad-valorem* or '*iceberg*' formulation. This unidimensional measure of cost is not suitable to distinguish the disutility of travel time loss from pecuniary expenditure. The practical consequence is that a very fast but relatively expensive transport service may appear identical to a slow but relatively cheap alternative. Arguably, these two types of services (e.g., an expensive high-speed rail link and an affordable commuter service) may have fundamentally different structural impacts on the spatial economy. Several QSMs are based on even simpler assumptions, completely neglecting the monetary cost of travel. Such models are even less suitable to appraise pricing policies, a set of measures frequently advocated by transport economists.

This paper documents an initial attempt to capture temporal and monetary costs through separate time and money constraints facing households in the QSM framework. We derive an analytical expression of endogenous travel time valuations for each residence–workplace combination of the spatial model. This generates spatial heterogeneity in the valuation of travel time savings which is mostly neglected in the current transport appraisal practice. In a numerical implementation of the model, we provide an illustration of how this novel approach can be applied in transport policy evaluation and project ranking.

## 2 Methodology

The paper's key contributions lie in the we model household preferences. Therefore, in the present short paper, we detail the demand side of the methodology, suppressing other components of the spatial model into a brief description.

### *Household preferences*

Let us define the utility of a representative worker who resides in location $i$ and commutes to location $j$ as

$$U_{ij} = \left(\frac{C_{ij}}{\beta}\right)^{\beta} \left(\frac{L_{ij}}{\gamma}\right)^{\gamma} \cdot z_{ij}. \tag{1}$$

In this specification, $C_{ij}$ denotes consumption of a variety of goods, $L_{ij}$ is a measure of leisure time, $\beta$ and $\gamma$ are structural parameters, and $z_{ij}$ is an idiosyncratic taste shock associated with the combination of locations $i$ and $j$. Households are ex-post heterogeneous in their location preferences. In our notation we suppress the unique identifier of households; note, however, that $z_{ij}$ takes a different value for each household.

Commuters face two constraints through which individual labour supply $x_{ij}$ will affect utility. This way we adopt the modelling approach of Arnott (2007) and Hörcher et al. (2020) in a spatial setting. First, wage $w_j$ at workplace $j$ and the monetary price of commuting $\tau_{ij}$ determine the budget available for consumption, where $P_i$ is the price index of the consumption variety.

$$x_{ij}\left(w_j - \tau_{ij}\right) = P_i \cdot C_{ij} \quad [\kappa] \tag{2}$$

Second, leisure time $L_{ij}$, time spent at work ($T$, exogenous), and commuting time $t_{ij}$ cannot exceed $\bar{L}$, the daily time endowment of households.

$$\bar{L} = L_{ij} + x_{ij}\left(T + t_{ij}\right) \quad [\mu] \tag{3}$$

With Lagrange multipliers $\kappa$ and $\mu$, first-order condition of the optimal choice of individual labour supply implies

$$\kappa(\tau_{ij} - w_j) + \mu(T + t_{ij}) = 0, \tag{4}$$

which equates the monetary benefit of the marginal trip to work with its monetary as well as time cost. Rearrangement leads to an expression of the ratio of the marginal utilities of time and money:

$$\frac{\mu}{\kappa} = \frac{w_j - \tau_{ij}}{T + t_{ij}} = v_{ij}. \tag{5}$$

We interpret this ratio as a monetary valuation of the incremental relaxation of the worker's time endowment. We call it the (marginal) *value of time* and denote by $\upsilon_{ij}$. This quantity has been one of the key variables of the literature of transport economics since its emergence (DeSerpa, 1971; Small, 2012; Jara-Díaz, 2020). The value of time provides a suitable exchange rate between travel time savings and monetary expenditures, thus allowing the analyst to quantify in monetary terms the benefit of journey time reduction after transport improvements. As one would expect, the worker's wage is among the determinants of the value of time, as foregone time could always be used to earn income through work. Equation (5) also reveals that the value of time depends on the monetary and time cost of commuting as well. The core consequence from a spatial economic point of view is that the value of time will likely differ between commuters by the place of residence and work, which is often neglected in mainstream transport policy appraisal. To emphasise this feature of the model, we keep the subscripts of $\upsilon_{ij}$ throughout the forthcoming analysis.

After simple algebraic manipulations, first-order conditions lead to the following expressions for the optimal consumption, labour supply and leisure quantities.

$$C_{ij} = \xi \frac{\upsilon_{ij}}{P_i}, \tag{6}$$

where $\xi = \left(1 + \frac{\gamma}{\beta}\right)^{-1}$. Naturally, consumption decreases with the price index at the residential location ($P_i$) and increases with the $\beta$ parameter of our direct utility function. More surprisingly, $\upsilon_{ij}$ enters this formula directly. That is, someone with a high value of time is expected to consume more. It may be more appropriate in the present context to interpret $\upsilon_{ij}$ as a *net hourly wage* instead of a travel time valuation, where both the money cost and the temporal duration of commuting is part of the net wage. With this interpretation, it is more convincing that consumption increases with the net hourly wage, indeed.

Combining the first-order conditions with respect to $C_{ij}$ and $L_{ij}$ with the monetary budget constraint in (2) and (5), we find

$$x_{ij} = \xi (T + t_{ij})^{-1}. \tag{7}$$

This rule suggests that individual labour supply increases with the utility of consumption through $\beta$ and decreases with the utility of leisure time through $\gamma$. Commuting time has a negative impact on $x_{ij}$. Interestingly, the gross wage cancels out in this formula, so it has no direct impact on individual labour supply under the present assumptions. The consumer problem's solution with respect to leisure time is

$$L_{ij} = \left(1 + \frac{\beta}{\gamma}\right)^{-1}. \tag{8}$$

In this simple formula, $\beta$ and $\gamma$ have the expected impact on leisure time, and all the remaining endogenous variables cancel.

The last four results yield the following indirect utility function for a given combination of residential and working locations.

$$u_{ij} = \left(\frac{\upsilon_{ij}/P_i}{\gamma + \beta}\right)^{\beta} \left(\frac{1}{\gamma + \beta}\right)^{\gamma} \tag{9}$$

In the specific case $\beta + \gamma = 1$, indirect utility simplifies to

$$u_{ij} = \left(\frac{\upsilon_{ij}}{P_i}\right)^{\beta} z_{ij} = \left[\frac{w_j - \tau_{ij}}{P_i(T + t_{ij})}\right]^{\beta} z_{ij}. \tag{10}$$

That is, the net hourly wage (or in a different interpretation, the marginal value of time), the local price index, the $\beta$ parameter, and idiosyncratic taste are the only determinants of a residence–workplace combination's attractiveness to households.

### *Spatial equilibrium*

In the rest of this modelling exercise, we follow standard practices in the quantitative spatial economics literature with some adjustments necessitated by the methodology introduced in the previous subsection. In particular, household heterogeneity is represented *à la* Eaton & Kortum (2002), the production side of the model follows Monte et al. (2018) while the present version of the model neglects competition in the housing market, just as in Hayakawa et al. (2021). The main differences are routed in (i) the endogeneity of individual labour supply and (ii) the unique specification of indirect utility, as derived above.

The idiosyncratic utility shock is specified as a draw from a Fréchet distribution:

$$F_{ij}(z) = \exp(-A_i B_j z^{-\epsilon}), \tag{11}$$

where the average amenity (i.e. the scale parameter) is defined as the product of residence and workplace dependent local fundamentals $A_i$ and $B_j$, and $\epsilon$ governs the spread of individual preferences. These assumptions lead to location choice probabilities that take the form of a commuting gravity equation.

$$\lambda_{ij} = \frac{A_i B_j \left[\frac{w_j - \tau_{ij}}{P_i(T+t_{ij})}\right]^{\beta\epsilon}}{\sum_r \sum_s A_r B_s \left[\frac{w_s - \tau_{rs}}{P_r(T+t_{rs})}\right]^{\beta\epsilon}} \tag{12}$$

To model the production side of the economy we follow a conventional appraoch in new economic geography and Monte et al. (2018) more closely. Varieties of the consumption good are produced under monopolistic competition, using labour as the sole input. A fixed factor of production and the constant marginal cost of producing a unit of one of the symmetric varieties in location $j$ imply increasing returns to scale. Productivity is an exogenous characteristic of each location. Following the usual derivations, profit maximisation and the zero profit assumption yield an equilibrium unit price for a variety produced in $j$ and sold in $i$ under *ad valorem* trade cost. This setup provides a separate gravity equation of trade flows, measuring the fraction of spending in $i$ on goods produced in $j$. We use the trade gravity equation to compute the vector of equilibrium wages. Finally, CES preferences and monopolistic competition yield a price index for each location that we use in the commuting gravity equation above. Unfortunately, the length limit of this short paper does not allow for a more detailed elaboration of the model.

In the numerical implementation of the model, we solve for spatial equilibrium by iteratively re-evaluating the equilibrium conditions of the model and updating the wage and price index vectors.

## 3 Results and discussion

Figure 1 shows the layout of our simulation framework. This toy network includes 18 locations arranged in a grid. We mimic a system of two cities connected by a transport link. Each city has a central node and eight spokes around it. The local fundamentals of the locations are set to the same values except for the dark shaded ones: productivity in nodes A5 and B5 are set to $E_{A5} = 2.0$ and $E_{B5} = 1.5$, while in the remaining locations $E_j = 1$. This implies that city A is somewhat more efficient in production. Nodes A2 and B2 feature higher amenity levels than other places; $A_{B2} = B_{A2} = 2.0$ while all the remaining amenity variables are normalised to one, thus allowing us to observe the impact of amenities by comparing A2 to A8 and B2 to B8. The attribute levels of transport links between these notes are depicted in Figure 1. Commuting times and costs in city B are 10 percent higher than in city A, but there is no difference in intracity goods transport. Finally, commuting through the intercity link is significantly costlier than within the two cities, which is expected to limit the attractiveness of intercity residence–workplace combinations while the trade impedance is milder. We set the structural parameters of the model following typical values in the quantitative spatial economics literature.[1]

Figure 2 depicts the core results of this exercise: the pattern of heterogeneity in travel time valuations. Recall from (5) that $\upsilon_{ij}$ may potentially differ between each residence–workplace combination (or origin–destination pair, OD-pair, in transport terminology). In our toy network, each link can be used by commuters of multiple OD-pairs. To derive the mean value of time of each link, we take the flow-weighted average of the relevant $i$–$j$ combinations.

[1] In particular, $\sigma = 4$, $\epsilon = 6.83$, $\beta = \gamma = 0.5$, $\bar{L} = 1$, $T = 8/24$, $F = 50$, and population is $M = 10,000$.

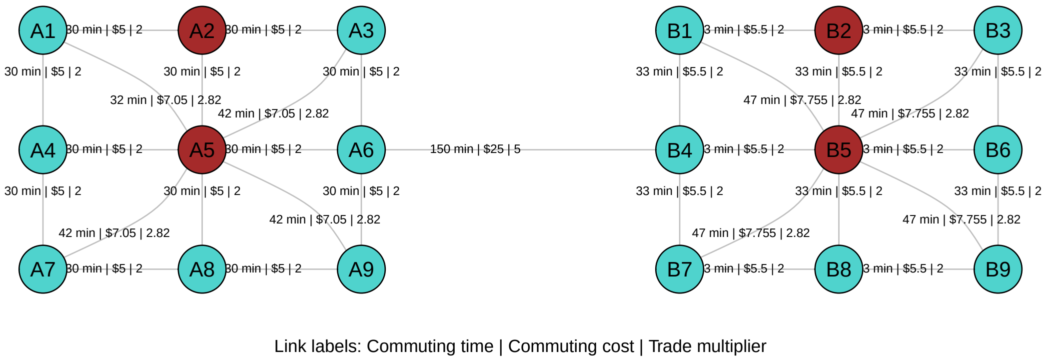


Figure 1: Network layout of the simulation framework.

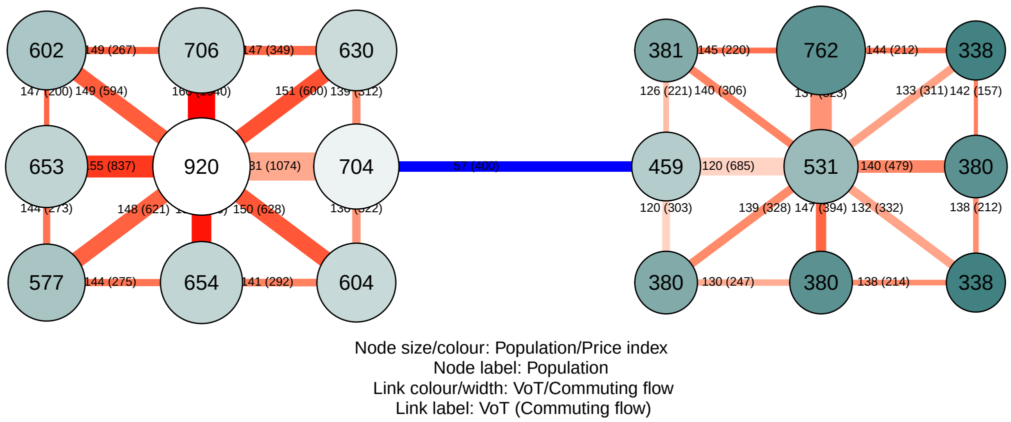


Figure 2: Travel time valuations, commuting flow, and the distribution of residential population and price indices in spatial equilibrium.

The result in Figure 2 reveals considerable heterogeneity in the average values of time by link. Time is valued generally higher in the more productive and better connected city A where the mean value of time is 157.5 as opposed to 145.4 in city B. The difference in commuting flows is even greater between the two cities. Note that the correlation between commuting flows and travel time valuation is positive but weak: 0.173 under the current set of parameters. That is, as opposed to flows, the forces behind the value of time differ from gravity between residential and workplace locations. The pattern of flows follows the regular characteristics of monocentric cities: traffic between peripheral locations is moderate due to the lower level of employment in these areas. Commuting movements are sparse between the two cities due to the relatively high travel time and cost in this market, even though the only intercity link is shared by more OD-pairs than regular intracity links. The value of time is also significantly lower for long-distance commuters who have a higher share on links A5–A6 and B4–B5.

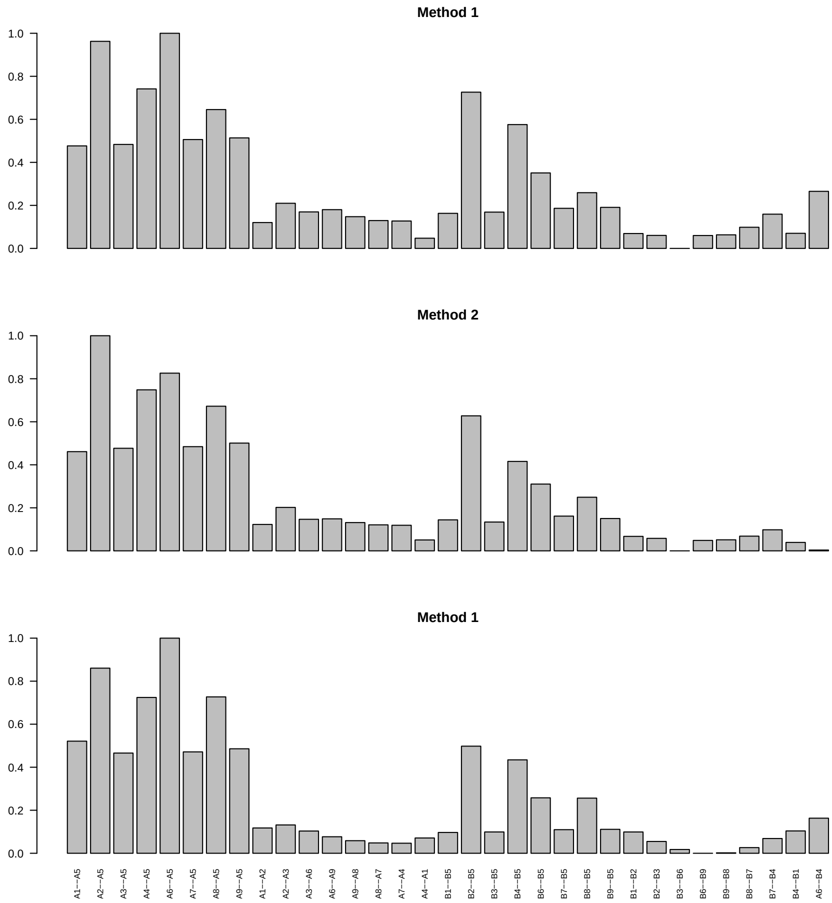


Figure 3: Relative efficiency of link-level travel time reduction according to three appraisal methods.

Let us now explore whether the spatial differentiation of the value of time may affect policy decisions in prioritising investments in a transport network. In a series of numerical simulations, let us reduce travel time on each link of our toy network one by one, by 10 minutes, and rank these potential improvements according to the economic benefit they generate. We approximate the welfare effect by three distinct methods:

**Method 1** – Multiply traffic flow on each link by the 10-minute savings and the mean value of time considering every trip in the network.

**Method 2** – Repeat Method 1 with the link-specific values of time derived in the previous subsection.

**Method 3** – Compute the aggregate welfare effect of a link-level improvement in general equilibrium by computing the difference of aggregate indirect utilities across all OD-pairs, before and after the improvement.

Table 1: Ranking of link-level travel time reductions according to three appraisal methods

| Link endpoints | Method 1 | Method 2 | Method 3 |
|---|---|---|---|
| A6–A5 | 1 | 2 | 1 |
| A2–A5 | 2 | 1 | 2 |
| A4–A5 | 3 | 3 | 4 |
| B2–B5 | 4 | 5 | 6 |
| A8–A5 | 5 | 4 | 3 |
| B4–B5 | 6 | 10 | 10 |
| A9–A5 | 7 | 6 | 7 |
| A7–A5 | 8 | 7 | 8 |
| A3–A5 | 9 | 8 | 9 |
| A1–A5 | 10 | 9 | 5 |

In Figure 3 we observe similarities in the three patterns of the relative performance of the link-level investment projects we simulated. Table 1 compares the ranking of ten alternative projects according to the three appraisal methods. Note that the divergence in policy recommendations is more substantial in terms of rankings. The general equilibrium approach agrees with the most naive method in that Links A6–A5 and A2–A5 should be prioritised first, but most of the remaining rankings differ between these methods. The project ranking of Methods 2 and 3 do not differ in more than 1 unit which reveals the value of the link-specific differentiation of the value of time (that we apply in Method 2). The are only two infrastructure segments from city B in this list, and Method 1 consistently overestimates their ranking relative to Methods 2 and 3. For example, link B4–B5 ranks $6^{\text{th}}$ in the first column, but when the relatively low value of time of travellers is taken into account (Method 2) or general equilibrium impacts are considered (Method 3), this project falls back to the $10^{\text{th}}$ place.

## 4 Conclusions

This paper builds on the emerging literature of quantitative spatial models with the aim of making this spatial general equilibrium approach more suitable to assess the economic impact of large-scale transport policies. We relax the assumption of '*iceberg*' commuting costs that expresses the disutility of travel as a multiplier of consumer utility. We replace this assumption by integrating a leisure–labour trade-off and distinct time and money constraints into the consumer problem of the standard QSM approach. This implies that the monetary value of travel time becomes an endogenous, heterogeneous, and location-dependent outcome of the spatial equilibrium. Numerical results showcase the degree of heterogeneity in travel time valuations and the possible bias that the use of homogeneous value of time estimates imply in practice.

This manuscript documents the first stage of a research project. Subsequent stages include (i) further theoretical work, with particular attention being paid to an adequate representation of the housing market, (ii) the adaptation of our theoretical framework to granular spatial data and the analysis of real geographies, (iii) the development of empirical identification strategies to estimate structural parameters for the new model, and (iv) an analytical exploration of the properties of equilibrium/equilibria in this framework. Naturally, the endogeneity of travel time valuation is not the only precondition of the widespread adaptation of quantitative spatial models in transport policy appraisal. Ideally, QSMs should be adopted to this purpose such that they do not lose the advantageous properties they have in terms of theoretical and empirical consistency, analytical tractability and modularity. Our hope is that this research will serve as a starting point for a more widespread awareness and use of QSMs in the transport field as well.

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