Modelling Travel Time Anticipation Under Rational Inattention and Endogenous Information Constraints

Dimitrios Pappelis^{*1}, Emmanouil Chaniotakis², Tim Hillel³, Maria Kamargianni⁴

¹ Research Assistant, BSEER Energy Institute, University College London, UK, WC1H0NN

² Lecturer, BSEER Energy Institute, University College London, UK, WC1H0NN

³ Lecturer, Civil Engineering, University College London, UK, WC1H0NN

⁴ Professor, BSEER Energy Institute, University College London, UK, WC1H0NN

SHORT SUMMARY

Transportation research has been traditionally grounded on the economic theory of Rational Expectations, assuming that individuals are fully informed, optimizing, and self-interested decision makers. However, this assumption fails to sufficiently explain the inertia that characterizes travellers' behaviour in face of uncertainty. In recent years, there has been a rising interest in the theory of Rational Inattention, arguing that individuals choose to make seemingly suboptimal choices due to the cost of acquiring and processing available information. In this paper, we present a continuous quadratic Rational Inattention model of travel time anticipation. We showcase that its properties satisfy behavioural hypotheses derived from data collected through a case study in the city of Turin on within-day travel re-evaluation. We conduct simulation experiments and propose an alternative 2-stage framework for enhancing existing neoclassical travel behaviour models, indicating potential biases and discrepancies in the forecasted market shares, specifically with regards to rare travel time occurrences.

Keywords: Choice modelling, Dynamic travel behaviour, Inertia, Rational inattention, Traffic information

1. INTRODUCTION

Transportation planning and policy making rely on models to predict and explain the behavior of travellers. Traditionally, research on this front has been based on the economic theory of Rational Expectations, assuming that individuals are fully informed, optimizing, and self-interested decision makers. However, this assumption fails to sufficiently explain the resistance to change that characterizes travellers' behaviour in face of uncertainty. In recent years, there has been a rising interest in the Rational Inattention (RI) theory, originally developed by Christopher Sims (2003). The argument is that individuals consciously choose to make seemingly suboptimal choices due to the cost of acquiring and processing available information. In recent years, Matejka and McKay (2015) expanded the theory for discrete choice under imperfect information and cognitive capacity constraints. As such, RI has emerged as a compelling and neat framework for further understanding the behavior of decision makers in complex and dynamic environments.

In the context of transport modelling, Rational Inattention is still relatively unexplored. Fosgerau et al. (2019) and Jiang et al. (2020) defined the problems of route and departure time choice under

RI and provided simulation findings. Fosgerau et al. (2020) established the general equivalence between discrete choice and RI models, providing an alternate point of view in the interpretation of typical RUM models. From an application perspective, Habib (2022) investigated empirical use-cases and focused on estimable specifications of discrete choice RI models.

In this paper, we present a continuous-quadratic RI model of travel time anticipation. We showcase that its properties satisfy our behavioural hypotheses derived from data collected from a case study in the city of Turin on within-day travel demand shift choices. We proceed to assess the model capabilities through numerical experiments and then propose a 2-stage framework for enhancing existing neoclassical models of travel behaviour, given the open challenges associated with data collection for RI phenomena. We indicate how ignorance of the priors and information capacity constraints could lead to potential biases and discrepancies in the forecasted market shares, especially with regards to rare travel time occurrences.

2. METHODOLOGY

Data Collection

The motivation of this paper originates in the investigation of within-day re-evaluation and dayto-day learning as described by Pappelis et al. (2022). In that study, a joint Revealed Preference and Stated Preference (RP-SP) experiment was applied to collect "pseudo" panel data on withinday demand shift choices. The primary objective was to investigate individuals' adaptation strategies when faced with travel time fluctuations on their habitual schedule, and how the accumulated experience affects their future actions. Participants, whose travel patterns were initially recorded using a smartphone tracking application, were provided with travel information for an upcoming habitual trip, either during an activity or en-route to their destination. Given this information, they were asked to record their response in the form of an adaptation strategy. The strategy could involve modifying trip characteristics such as departure time, mode, or route, or changing the target activity through replacement or cancellation. At the end of each day, participants updated their anticipation of travel time for the following day based on accumulated experience and reported whether they would consider long-term adjustments to their habitual schedule.

The described experiment allowed for the exploration of individuals' responses to travel time fluctuations and the implications on their future travel behavior. It was applied in the metropolitan area of Turin (IT) between February and April 2022, as part of a wider travel demand survey. Recruited individuals formed a stratified sample of the travel survey participants, which is representative of the population in the Turin region (a survey company was hired for recruitment). The RP data collection was performed using a smartphone-based travel survey tool, the MobyApp. The habitual activity and travel patterns were tracked from the application in the form of travel diaries over the course of 7 days. In total, 365 individuals accessed the experiment and 351 of them completed it, resulting in 702 tracked trips and 4212 observations.

The dataset revealed some interesting behavioural findings with regards to inertia effects of travel behaviour and the concept of false certainty adoption. For instance, Figure 1 displays the number of trips categorized by re-evaluation strategy, based on the daily fluctuations in travel time. The level of fluctuation is determined by the travel factor parameter, which is multiplied by the habitual travel time for a specific trip of the participant in each scenario. The analysis shows that for medium levels of travel time fluctuation, the dominant re-evaluation strategy is 'No change,' suggesting that many individuals may prefer to stick with their habitual option rather than make changes, even if from a utility maximization perspective this can be seen as "irrational". This finding aligns with the concept of resistance to change, a heterogeneous factor across the population. As travel time increases, schedule constraints and conflicts may increase stress, leading individuals to consider changing their travel plans (such as adjusting departure time, mode, or route). For extreme levels of travel time fluctuation, we observe the highest likelihood of cancellation or replacement of the activity.



Figure 1 Resistance to change for different levels of travel time fluctuation

It is also important to study how the prior expectation of travel time evolves with accumulated experience. Figure 2 depicts the participants' scaled anticipated travel time after each day, against the 2-day and 3-day moving average of different travel time orders used throughout the experiment. We observe significant sluggishness and inertia in the travel time anticipation of the participants, being influenced from their prior beliefs and experience. While Rational Expectations theory would imply that external stimuli would cause stronger and fast responses, we observe much milder adaptations and a "magnet effect" towards the reference level of travel time.



Figure 2 Inertia and sluggishness in travel time anticipation

Modelling Framework

Based on these behavioural observations, we proceed to define the travel anticipation problem as a static model of choice under Rational Inattention (Mackowiak et al., 2021). Consider an agent who plans to perform a daily trip and receives an information signal s, in order to set her travel time anticipation a, subject to unknown network conditions t. Let the utility have the following log-quadratic form,

$$U(a,t) = -b(a-t)^2$$

The agent is tracking the unknown random state of the network, which under perfect information would be equal to her anticipation. Naturally, this would allow the agent to construct her subsequent travel plans most accurately (e.g., departure time, mode, route). However, as the true travel time is infeasible to observe constantly and travel information comes at a perceptual cost, the agent chooses to receive noisy information that determines the posterior beliefs that she may hold. The utility parameter b is a scaler, which can account for agent's heterogeneity with regards to traffic information seeking. Under the general quadratic form, we assume that over or underestimation of travel time incurs equal losses. In many cases, delayed arrivals might incur costlier losses, so it is worth studying different variants of the utility function going forward. The objective of the agent is to maximize the expectation of her utility less the cost of information C(f), which is a function of the information strategy,

$$\max_{f} \int U(a,t)f(a,t)dtda - C(f) (1)$$

The joint probability f(a, t) is sufficient to describe the choice of information and action, as they are derived such that no two signals lead to the same action. Otherwise, the agent would be wasting attentional resources by distinguishing between signals that do not directly affect their actions. As a result, it is possible to make a one-to-one association between the signal and action and analyse the relationship between attention, allocation, information acquisition, and decision-making in a unified framework. The objective function (1) is maximized subject to the following constraints,

$$\int f(a,t)da = g(t), \,\forall t \,(2)$$

The prior belief of the agent is described by the pdf g(t). Constraint (2) ensures the consistency of the prior and posterior beliefs of the agent under Bayesian rationality.

$$C(f) = \lambda \cdot I(a; t) = \lambda \cdot [H[g(t)] - E[H[t|a]] (3)$$

The cost function (3) is defined in terms of the mutual information between the agent's anticipation and the actual travel time. It is based on the difference between the entropy of the prior distribution of travel times and the conditional entropy of the distribution of travel times given the agent's prediction. The parameter λ typically referred to as the "attention cost" or "information cost" reflects the required effort of acquiring and processing the information.

$$H[g(t)] = -\int g(t)\log g(t)dt (4)$$

Entropy (4) is quantified using Shannon's definition, which measures the amount of information present in the probability distribution of travel time. The cost function penalizes travel time predictions that require more attention to achieve a specific level of accuracy. By minimizing the difference between the prior and conditional entropy based on the prediction, the cost function encourages accurate predictions that require less attention. The solution to the agent's problem for

an unknown network state t has a probabilistic logit form. The solution of the agent's problem for an unknown state of the network t is has the following probabilistic logit form.

$$f(a|t) = \frac{p(a)e^{U(a,t)/\lambda}}{\int_z p(z)e^{U(z,t)/\lambda} dz}$$

In most cases, RI problems do require numerical solution methods. A well-studied exception is the case of quadratic utility, Gaussian prior uncertainty, and an unbounded action space, where Gaussian signals are optimal Interestingly, for a bounded or truncated action space, the solution of the continuous problem is discrete, indicating that the agent contemplates only specific levels for a given choice, a phenomenon commonly observed in the stickiness of product prices. In the context of travel time, this would imply that travellers choose from a finite set of levels when updating their anticipation and might, for instance, set a regular departure time and standard "safety" departure when expecting a range of potential delays.

3. RESULTS AND DISCUSSION

The collection of data for the practical estimation of RI models is challenging, mainly because the concept of cognitive capacity constraints is abstract and difficult to measure. In the context of travel time anticipation and travel behavior, an ideal dataset would need to capture multiple factors simultaneously, including the agent's beliefs (i.e., their prior perception of the probability distribution of travel times), the world (i.e., network conditions such as travel time), attention allocation (i.e., the choice of signal or level of information), and action (i.e., the agent's choice). The design of such sophisticated experiments is an ongoing task in economics research. In absence of this complete dataset, we proceed to perform numerical experiments on the travel time anticipation model and then propose a 2-stage approach to enhance traditional neoclassical models of travel behaviour.

Numerical Experiment

To assess and showcase the properties of the modelling framework, we perform numerical experiments that justify our behavioural hypotheses derived from the data analysis. A triangular prior distribution is assumed for the agents' belief, a common approach in related studies (Figure 1).



Figure 3 Triangular prior anticipation of trip travel time distribution

We then proceed to solve the RI problem (Eq.1-4) for two different levels of the marginal cost of information λ . The optimization problem was solved using the GAP-SQP geometric algorithm proposed by Armenter et al. (2021). Figure 2 presents the joint probability of anticipated travel times, as well as the conditional probability of the non-zero solutions (discrete choice set). It is apparent that the responsiveness of an action to a given state can be increased by altering the stakes or reducing the cost of information. When the stakes are high or the cost of information is low, individuals are more motivated to make accurate predictions of the travel time and allocate their attention accordingly, thus the plurality in possible actions. This increased attention leads to greater responsiveness of the action to the state, as individuals are more likely to adjust based on the information available to them. On the contrary, for lower stakes or high values of the information constraints, the agent might only consider few alternatives and apply them over a range of states of the network.



Figure 4 Simulated joint and conditional probabilities for higher (λ =0.03, left) and lower (λ =0.005, right) values of marginal information cost

Empirical Findings

The theory of Rational Inattention and the endogenous processing of information raise important questions about what traditional empirical methods, such as controlled experiments, capture in a transportation setting. This is particularly relevant for travel re-evaluation behavior, where it is most often assumed that individuals are fully aware and process all available advanced information. Furthermore, in a revealed preference setting, such effects might already be captured in the data, thus there is a need to not only disentangle preferences, but also consider their equilibrium relationships with the supply side.

Given these open research challenges, we extend the travel re-evaluation framework developed by Pappelis et al. (2022). At this point, it is important to clarify that -in this context of RI- we are not referring to the cognitive constraints of the participant with regards to the experiment setting and attributes, which is also important to be controlled, but with the inattention to information (e.g., journey planners, radio) that would be observed in the transition to a real-world setting. Figure 5 illustrates a two-stage sequential framework for incorporating RI effects in the demand shift models. In the first stage, we utilize the continuous RI model to solve for travel time anticipation. In the second stage, we use the output of the RI model as a more realistic depiction of travel time when simulating dynamic demand shift decisions.



Figure 5 Sequential approach for incorporating travel time anticipation under RI

The model selected for the evaluation of the framework is the static Mixed Nested Logit, which was designed to generate the probability of specific adaptation strategies being selected, when faced with travel time fluctuation during a habitually performed trip. The nesting structure and the alternatives of the travel re-evaluation model are depicted in Figure 6 (see full paper for complete specification).



Figure 6 Demand shift model choice alternatives and nesting structure (Pappelis et al., 2022)

Applying the 2-stage framework, we proceed to perform sensitivity analysis on the information cost parameter λ of the travel time variable, maintaining the assumption of the triangular distribution, and then comparing the simulated market shares for different ranges of the travel time distribution. We observe that for severe delays (travel factor >2.5), the Rational Expectations model might overestimate the aggregate response of the travellers, especially when it comes to cancellation of a given trip. Comparing it to the extreme case of a marginal information cost above the threshold of any signal acquisition, a significant discrepancy of over 20% can be observed in

the market share of the "Habit" alternative. On the contrary, for lower levels of travel time fluctuation (travel factor <1.5), the Rational Inattentive agent might falsely overreact due to false signals, when she would be better off following her habitual schedule. Such discrepancies indicate the importance of measuring and accounting for the prior beliefs, the information processing constraints and marginal cost of information λ in travel behaviour modelling and forecasting.



Figure 7 Forecasted shares for different levels of marginal information cost and travel time fluctuation

4. CONCLUSION

In conclusion, our paper highlights the potential benefits of incorporating Rational Inattention theory into transportation modelling and travel time anticipation in particular. Future steps include the extension of the framework to a dynamic setting, allowing for individuals to acquire informative signals which can also be used as predictors of future actions. Finally, the relevance and applicability of the RI theory in transportation needs to be further examined through the design of sophisticated data collection experiments.

5. ACKNOWLEDGEMENTS

The research reported in this paper is supported by European Union's Horizon 2020 research and innovation programme under Grant Agreement No. 815269, project HARMONY.

6. REFERENCES

Armenter, R., Muller-Itten, M. and Stangebye, Z., 2021. Geometric methods for finite rational inattention.

Fosgerau, M. and Jiang, G., 2019. Travel time variability and rational inattention. *Transportation Research Part B: Methodological*, *120*, pp.1-14.

Fosgerau, M., Melo, E., De Palma, A. and Shum, M., 2020. Discrete choice and rational inattention: A general equivalence result. *International economic review*, *61*(4), pp.1569-1589.

Habib, N. and Mohammed, K., Rational Inattention in Discrete Choice Models: Estimable Specifications with Empirical Applications of Ri-Multinomial Logit (Ri-Mnl) and Ri-Nested Logit (Ri-Nl) Models. 2022. Available at SSRN 4182187.

Jiang, G., Fosgerau, M. and Lo, H.K., 2020. Route choice, travel time variability, and rational inattention. *Transportation Research Part B: Methodological*, *132*, pp.188-207.

Maćkowiak, B., Matějka, F. and Wiederholt, M., 2021. Rational inattention: A review.

Matějka, F. and McKay, A., 2015. Rational inattention to discrete choices: A new foundation for the multinomial logit model. American Economic Review, 105(1), pp.272-298.

Pappelis, D., Kamargianni, M., and Chaniotakis, E., 2022. On Modelling the Dynamics of Withinday Travel Behaviour. *Proceedings of the 102nd Annual Meeting of the Transportation Research Board*, January 2023, Washington DC, United States.

Sims, C.A., 2003. Implications of rational inattention. *Journal of monetary Economics*, 50(3), pp.665-690.