Mobility Service Providers' Equilibrium Strategies in Multi-modal Networks

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SHORT SUMMARY

Due to the increasing introduction of new mobility solutions in the transport offers, the market equilibrium among Mobility Service Providers (MSPs) has become more complex. The focus of this paper is to develop a novel analytical approach to study competition and/or cooperation between multiple MSPs within a multi-modal network system. We formulate a novel Equilibrium Problem with Equilibrium Constraints (EPEC), where each MSP seeks to maximize their own profits at the upper level. At the lower level, users are divided into classes that capture their heterogeneity in terms of socioeconomic characteristics and activity-travel behaviour. We consider the multi-modal network link costs to be non-separable, therefore the lower-level equilibrium is formulated as a Variational Inequality (VI) problem. A solution approach is proposed and illustrated, based on a relaxation of the Diagonalization method. Finally, we apply the described methodology to a small example to illustrate some key properties of the proposed approach.

Keywords: EPEC, Multi-modal Network, Supernetwork, Variational Inequality

1 INTRODUCTION

In recent years, transportation systems have been offering travellers an increasing number of multi-modal options thanks to the introduction of new mobility solutions, such as ride hailing, shared and pooled mobility services, micromobility, on-demand services, etc. Consequently, the market equilibrium among Mobility Service Providers (MSPs) has become more complex, with different competitive or cooperative strategies being observed with the aim of attracting a sufficient share of customers and hence sustain a profitable business. In such scenario, users' modal choices are crucial in determining the durability of mobility services within the transportation system.

In Transport Network Design, the problem of studying the relationships between MSPs and users has been traditionally focused on uni-modal networks. Reflecting the complexity found in real transportation systems and individuals' mobility, more recently attention has been focused on multi-modal networks (Zhang et al., 2014). Although these works developed relatively complex models, there is limited research that includes multiple leaders, coexisting, competing or cooperating. From an economic and strategic point of view, it is essential to model the interactions between MSPs and travellers of the transport network to predict the response of these actors as a consequence of the variation in strategies of the entire system. In particular, scenarios offering a new transport service, introducing new regulations/incentives, or increasing users' heterogeneity, could substantially change the equilibrium of the whole network.

In the literature, few works have analyzed this type of problems. In the context of fast charging stations for electric vehicles, Guo et al. (2016) developed a Multi-agent Optimization Problem with Equilibrium Constraints (MOPEC)-based model to study interactions between multiple competitive investors and travellers assigned to a congested transport network. In cordon toll competition, Watling et al. (2015) formulated an Equilibrium Problem with Equilibrium Constraints (EPEC), through which they study the competing behaviour of two different public authorities from two cites aiming to maximize the social welfare of the corresponding residents. Yang et al. (2022), instead, defined a bi-level model to optimize pricing and relocation in a competitive one-way car-sharing market.

Albeit the above-cited works establish important developments in their respective area of application, all of them consider that users are homogeneous and assigned to networks in which the competition between suppliers is limited to uni-modal markets.

In this paper we developed a novel analytical approach to study competition and/or cooperation between multiple MSPs, while users with heterogeneous characteristics are assigned to a multi-modal network. We formulate a novel EPEC, where each MSP seeks to maximize their own profits at the upper level; the objective functions include MSP-specific costs and revenues. At the lower level, users are divided into classes that capture their heterogeneity in terms of socioeconomic characteristics and activity-travel behaviour, resulting in different daily trip chains. Due to the non-separability of the link cost functions, the lower-level equilibrium assignment is formulated as a Variational Inequality (VI) problem. The proposed methodology is applied to a small example to show key properties of the model, using an iterative solution approach, based on a relaxation of the Diagionalization method.

2 Methodology

This study aims to develop models for the economic assessment of different suppliers' strategies in a multi-modal network. Hence, the interactions between MSPs and users are modelled and illustrated using the concept of supernetworks (Sheffi, 1985). This representation has been adopted in the literature to tackle the complexity of multi-modal networks, being further expanded to include a time component connected to users' activity-based trip chains (Fu & Lam, 2014). Our methodology is applied to a static system in which time of departure/arrival from/to a location or duration of the activities performed at each destination are not considered in full detail, given the strategic, economical purpose of the developed model.

In order to build this supernetwork, we use endogenous information regarding users and MSPs. To represent a more realistic transportation system, travellers are divided into classes based on their socio-economic attributes and the trips' purpose; considering that based on the socio-economic characteristics users associate different costs to the travel time experienced in the network. The sequence of trips made in a day, their purpose and locations, instead, are included to explicitly represent the link between daily modal choices of users. It is here argued that each travel choice made by users is influenced by earlier decisions as well as by planned later trips during a day. We therefore define users of a certain class those performing a type of activity sequence (e.g. home-work-leisure-home) in the same sequence of zone(s). Assuming that, during an ordinary weekday, users of the same class $k \in K$ perform the same sequence of trips. This sequence is then modelled as a directed graph, where a node $n \in N$ corresponds to a zone and a link $a \in A$ indicates a trip from one zone to the next (top Figure 1). The first and last location visited by a class of users represent that class's origin (O) and destination in the network (D). We assume that a single link can directly symbolize a trip without the need of defining all the different path alternatives present in the real network. Considering that, after computing a traffic assignment process, the network reaches equilibrium for which all used routes have the same generalised cost (Wardrop, 1952). We use then the information regarding MSPs to take into account the modes of transport available at each trip connection, and we expand the network into uni-modal layers (colored parallel networks in Figure 1). Each layer is owned by a specific MSP $j \in J$ that collects revenues based on how many travellers use their service (i.e. link flows), and accrue costs primarily depending on the size of their vehicle fleet v_j (capacity). We formulate the MSP profit objective function to be sufficiently general to describe different mobility services, such as car-sharing, bike-sharing, bus, train, e-scooter and taxi. The continuous upper level decision variables are controlled by the MSP fleet sizes $\mathbf{v} \in V \subseteq R^{|J|}$. MSPs decide how to strategically distribute these vehicles amongst the links of their network layer, with the purpose of maximizing their profit.

We consider each user class to be assigned to the multi-modal network following a fixed demand-based traffic equilibrium, using a path-based adaptation of the multi-class and multicriteria network equilibrium model (Nagurney, 2000). In particular, there will be cost (and revenue) components that vary with the usage of the service, and class-based cost components associated to the time spent using the service). Please see previous works (Bandiera et al., 2022a,b) for further details concerning the mathematical representation of MSPs and users, and previous results.

Given the non-separable nature of the network link cost functions, explicit path enumeration is used in this paper. While this approach is not yet readily applicable for large-scale networks, our current focus is to develop and understand the methodology presented here, for which small networks involving a limited number of multi-modal options for each class are sufficient. Undoubtedly, the

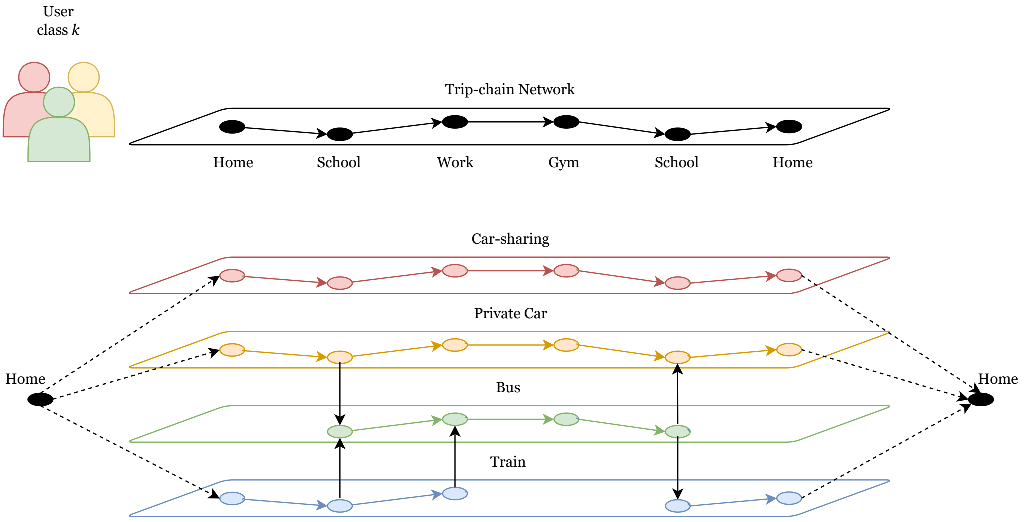


Figure 1: Multi-modal Trip-chain Supernetwork

choice of dividing users into classes increases the complexity of the model. However, the socio-economic characteristics are only affecting the cost perceived by users of a specific class and not the network expansion. On the other hand, defining users' classes based on the combination of daily trip chains and activity locations could be a non-trivial problem when considering large-scale networks. However, literature on the topic (Axhausen et al., 2002) shows that during typical weekdays the majority of travellers tend to perform home-work-home tours when using public transport or add an additional activity before/after work when travelling with private vehicles. Moreover, the combination of trip chains, activity sequences and locations are spatially limited and rather repetitive. Therefore, focusing on the most frequent tours, we cover most of travel the demand of an area.

The lower level equilibrium decision variables are the path flows represented by the vector $\mathbf{x} \in X \subseteq R^z$, with z paths and X the set of demand-feasible flows. Travellers choose a path through the multi-modal network in order to perform their sequence of trips. A path can comprise three different types of links. In line with the economic assessment purpose of the model, links represent the main mode of transport connecting two zones. Access links (black dashed lines in Figure 1) allow users to access a mode of transport from their origin (Home), and egress from a mode of transport to reach their final destination (Home). These links play an important role inside the network, capturing costs related to a monthly subscription for a single service or a package containing a combination of them. Mode-specific links (horizontal links), instead, indicate trips made from one location to another using a specific mode of transport (designated by colour). These link costs include three stages of a trip: 1) accessing the selected mode of transport from the departure node; 2) travelling using the main mode of transport; 3) egress from the selected mode of transport and reaching the destination. Finally, interchange links (vertical black links) allow users to move from one mode of transport to another.

Figure 2 shows cost components for different links, which characterize each mode of transport.

The lower level problem is complicated by the presence of multiple classes at the lower-level and the interdependency between flows on parallel links of the supernetwork. Concretely, some supernetwork links represent copies of the same real transport link of the underlying infrastructure network e.g. travel time on car-sharing links is influenced by travellers using private car and vice versa. Consequently, the corresponding link costs are non-separable. For this reason the users' equilibrium is formulated as a VI (Dafermos, 1980).

Let $C(\mathbf{x}, \mathbf{v})$ be the path cost function for the lower level problem, which depends on the capacities, $\mathbf{v}$, supplied by MSPs. Then a vector of path flows $\mathbf{x}^* \in X$ is a Wardrop equilibrium if and only if it satisfies the VI problem:

$$\langle C(\mathbf{x}^*, \mathbf{v}), \mathbf{x} - \mathbf{x}^* \rangle \geq 0 \quad \forall \mathbf{x} \in X \tag{1}$$

Given MSP fleet sizes $\mathbf{v}$, we denote the set of equilibrium solutions $X^*(\mathbf{v})$.

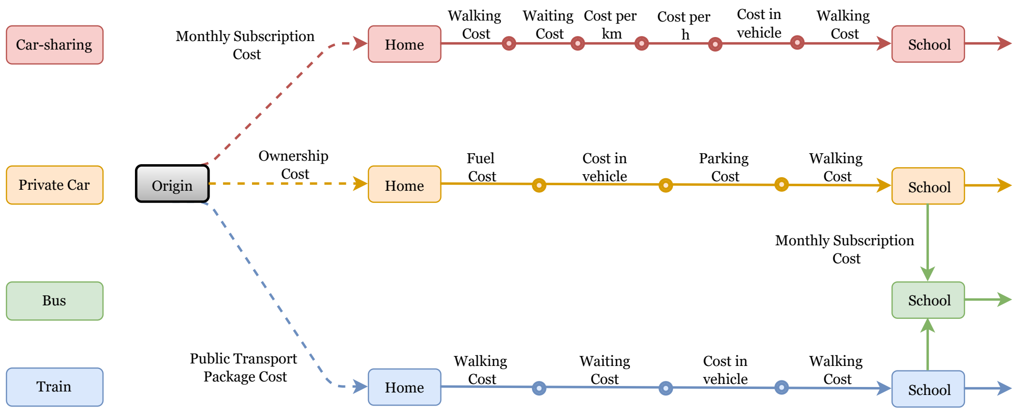


Figure 2: Example of detailed costs from Figure 1

Problem formulation and solution algorithm

We formulate the interaction between MSPs and users as an EPEC. An equilibrium at the upper level corresponds to no MSP wishing to unilaterally change their fleet size, given that the lower level satisfies the Wardrop equilibrium conditions.
Each MSP $j \in J$ seeks to maximize their profit, which is given by a continuous differentiable function, $p_j(v_j|\mathbf{x})$, depending on their fleet size, v_j, and on the path flows. The equilibrium path flows depend on the vector of MSP fleet sizes, so with the lower level constraint in place we have $p_j(v_j|\mathbf{x}^*(\mathbf{v}))$. Each MSP can change only their own fleet size: for a vector $\mathbf{v}$ we denote a change in only the j-th component by $\mathbf{v}_{[j]}$. Collecting the profit functions into $\mathbf{p} = [p_j]$ (adopting the obvious vector notation), $\mathbf{v}^*$ is an equilibrium solution if and only if

$$\mathbf{p}(\mathbf{v}^*|\mathbf{x}^*) - \mathbf{p}(\mathbf{v}^*_{[j]}|\mathbf{x}^*) \geq \mathbf{0} \quad \forall j \in J \tag{2}$$

$$\text{with} \quad \mathbf{x}^* \in X^*(\mathbf{v}^*) \tag{3}$$

where the lower level equilibrium path flows $X^*(\mathbf{v})$ are defined above (see Equation 1).
EPEC problems are well known in literature for the difficulty of finding equilibrium solutions. In this paper, we solve the EPEC using the Diagonalization Method: applying iteratively at the upper level a minimization approach while a the lower level we calculate the equilibrium solution using the Extragradient Method (EM), often used in the context of traffic assignment (Nagurney, 2000). Due to challenge of achieving convergence of the general EPEC, relaxation approaches are usually applied to a standard Diagonalization method. Here we introduce a steplength variation scheme in the iterations of the upper-level problem, based on the Self-Regulated Averaging Method proposed by Liu et al. (2007).

3 EXAMPLE AND DISCUSSION

In this section, we showcase the implementation of the methodology to a small network. We consider two classes of users performing two different trip chains, described at the top of Figure 3. The example may represent a user class performing a home-work-home tour, and a second class performing a work activity in the same zone, but also chaining another activity in another zone before returning home. We consider that each trip connection is covered by four modal options: one-way car-sharing 1, one-way car-sharing 2, a transit service, and private car is available to all users. The multi-modal network resulting graphically representing the four services and the two user classes consists of 32 nodes and 40 links (Figure 3). For each OD, users can therefore choose from four paths, based on the modal options available. For simplicity, in this example we do not allow travellers to use more than one mode of transport. However, this simplification is not a restriction of the model nor of the solution algorithm.
Congestion effects inside the network are modelled using the conventional Bureau of Public Roads (BPR) function, considering that users choosing the car-sharing services, or the private car, experience a travel time that is influenced by the presence of the other modes of transport on the same infrastructure. The transit service, instead, is considered to have a dedicated lane throughout the network, hence this service is affected only by the number of public transport users.

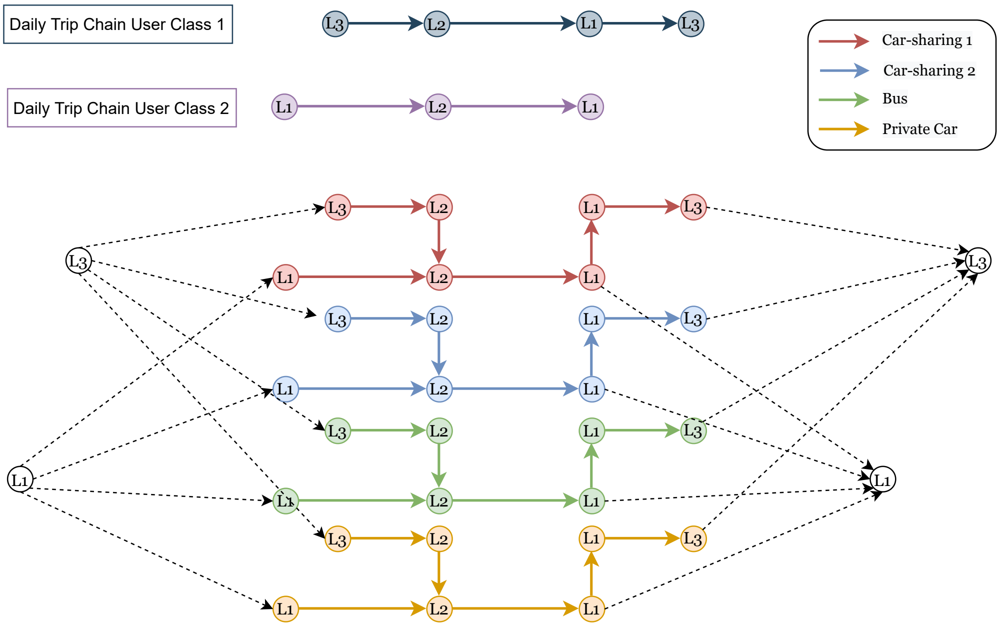


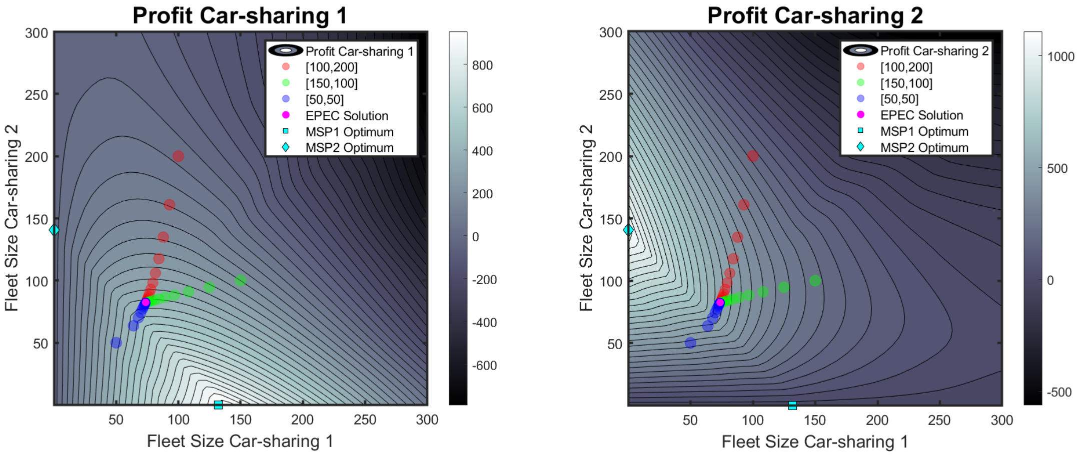
Figure 3: Example competition between MSPs

In this example, we try to understand the behaviour of the two car-sharing services competing inside the multi-modal network, applying the solution algorithm described in Section 2. We consider the two MSPs to adopt different price strategies: Car-sharing supplier 1 sells a cheaper monthly package compared to Car-sharing supplier 2, whereas the latter charges users a cheaper rate per hour and per kilometer.

Here we focus on the properties of the EPEC and the algorithm. To do so, we computed the full objective function (profit) surface for each MSP over a range of fleet sizes. This is computationally expensive and impractical in most cases, but allows us to verify solutions proposed by the algorithm. Figure 4 shows the objective functions surfaces of the two car-sharing suppliers. We examine whether the algorithm converges, and where to, starting from different initial conditions for the fleet sizes. In each case, the algorithm converges to the same point, indicated in magenta. Note that at the solution, Figure 4 left hand plot shows MSP1 profit is maximised (with v_2 fixed, varying v_1) and similarly, MSP2 profit is maximised in the right hand plot (fixing v_1). In this scenario, it seems that the car-sharing 2, offering a cheaper fixed price for the package, manages to attract more users, with a bigger feet size ($v_1 = 74; v_2 = 82$) and more profitable service ($p_1 = 421.5; p_2 = 538$). In Figure 4 we also indicate on the axis the best strategy for each MSP, which occurs in both cases when their competitor is not operating in those zones.

It is also interesting to observe the variation of the total travel cost for the lower level (Figure 5). Obviously, an increase in the fleet size of both suppliers is translated in a reduced travel cost for users, but this is not economically feasible for the MSPs. It is particularly interesting to see by looking at the total cost surface that when the fleet size is sufficiently large, only one of the two suppliers could survive in the market.

The proposed methodology shows promising results on the proposed network. The aim of future developments is to expand its application to bigger networks considering the competition and cooperation between multiple suppliers at the upper level with multiple user classes at the lower level. Through this approach it will be possible to study different dynamics that occur in the transportation network due to the presence of heterogeneous actors with diverse purposes.


Profit Car-sharing 1

Profit Car-sharing 2

Figure 4: Profits variation with fleet sizes and equilibrium solution

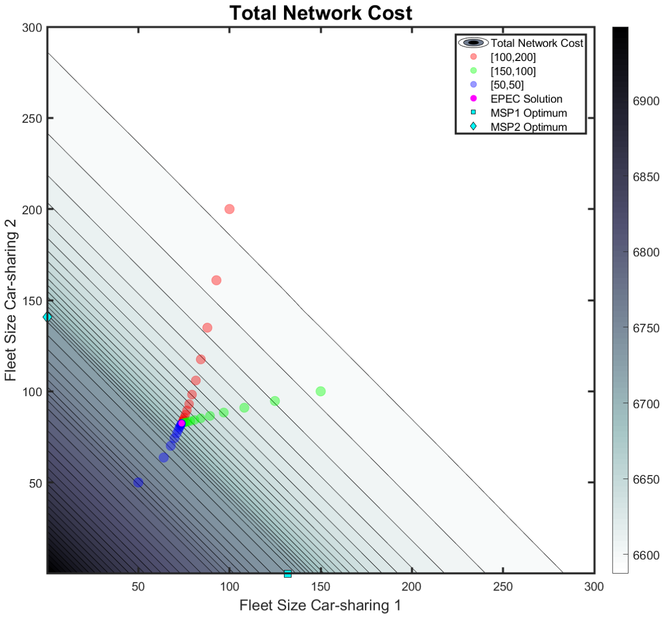

Total Network Cost

Figure 5: Total travel cost variation with fleet sizes

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REFERENCES

Axhausen, K. W., Zimmermann, A., Schönfelder, S., Rindsfüser, G., & Haupt, T. (2002). Observing the rhythms of daily life: A six-week travel diary. *Transportation*, *29*(2), 95–124.

Bandiera, C., Connors, R. D., & Viti, F. (2022a). *Competition and Cooperation between Suppliers in Multimodal Network Design Problems.* `https://tristan2022.org/Papers/TRISTAN_2022_paper_2182.pdf`. ([Online])

Bandiera, C., Connors, R. D., & Viti, F. (2022b). *Evaluating Mobility Service Providers' Strategies in an Activity-Based Supernetwork.* `https://transp-or.epfl.ch/heart/2022/abstracts/269.pdf`. ([Online])

Dafermos, S. (1980). Traffic equilibrium and variational inequalities. *Transportation Science*, *14*, 42-54.

Fu, X., & Lam, W. H. K. (2014). A network equilibrium approach for modelling activity-travel pattern scheduling problems in multi-modal transit networks with uncertainty. *Transportation*, *41*, 37-55.

Guo, Z., Deride, J., & Fan, Y. (2016). Infrastructure planning for fast charging stations in a competitive market. *Transportation Research Part C-emerging Technologies*, *68*, 215-227.

Liu, H. X., He, X., & He, B. (2007). Method of successive weighted averages (mswa) and self-regulated averaging schemes for solving stochastic user equilibrium problem. *Networks and Spatial Economics*, *9*, 485-503.

Nagurney, A. (2000). A multiclass, multicriteria traffic network equilibrium model. *Mathematical and Computer Modelling*, *32*, 393-411.

Sheffi, Y. (1985). *Urban transportation networks* (Vol. 6). Prentice-Hall, Englewood Cliffs, NJ.

Wardrop, J. G. (1952). Road paper. some theoretical aspects of road traffic research. *Proceedings of the institution of civil engineers*, *1*(3), 325–362.

Watling, D. P., Shepherd, S. P., & Koh, A. (2015). Cordon toll competition in a network of two cities: formulation and sensitivity to traveller route and demand responses. *Transportation Research Part B-methodological*, *76*, 93-116.

Yang, S., Wu, J., Sun, H., Qu, Y., & Wang, D. Z. W. (2022). Integrated optimization of pricing and relocation in the competitive carsharing market: A multi-leader-follower game model. *Transportation Research Part C: Emerging Technologies*.

Zhang, L., Yang, H., Wu, D., & Wang, D. (2014). Solving a discrete multimodal transportation network design problem. *Transportation Research Part C: Emerging Technologies*, *49*, 73–86.