Routing Passengers while Timetabling Based on Promises from Line Planning: A Logic-Based Benders Approach

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Short summary

Effective line planning and timetabling are critical for enhancing public transport efficiency and passenger satisfaction. We propose a Logic-Based Benders decomposition approach to optimise a timetable for a passenger railway system based on the promises made in earlier planning stages. Our approach ensures that the promised travel times and transfers are available and passenger routes are chosen according to the shortest available path. We test this approach on real-world data from the Rhätische Bahn railway system, demonstrating promising results. The proposed approach has shown to be valuable for optimising transfers, improving efficiency and passenger satisfaction, and reducing travel times. The method has limitations, including the inability to consider multiple connections per origin-destination pair, adaptation time at the origin station, and crowding. Further research can focus on improving and extending the model's performance to include these factors.
Keywords: Logic-Based Benders Decomposition, Integrated Public Transport Planning, Railway Timetabling, Periodic Railway Timetabling.

1 Introduction

Railways play a vital role in the transport system; however, public transport planning is complex and includes several stages. From these stages, line planning and timetabling are two essential steps in planning public transport systems (Schiewe, 2020). Line planning involves selecting the routes and services to operate on each route while timetabling involves assigning the trips to specific times. While we assume that transfers are possible in the line planning, it might be possible that such a transfer is then not given in the subsequent timetable. This can result in passengers being unable to make their intended connections, leading to longer travel times and reduced passenger satisfaction. Combining the planning stages of line planning and timetabling is a promising step to overcome this issue. It enables finding a schedule that minimises total travel time while ensuring that all passengers have a connection and that the following timetable is feasible. However, the timetabling problem is already complicated, and combining it with line planning further increases the complexity.

Various approaches have been proposed to integrate line planning and timetabling, including integrated optimisation methods (Schiewe, 2020) and combining passenger routing with periodic timetabling (Schmidt & Schöbel, 2015; Borndörfer et al., 2017; Robenek et al., 2016). However, these methods can be computationally intensive and may need improvements to scale better for more extensive networks. In response to these challenges, Polinder et al. (2021) proposed a scalable timetabling approach that considers the passenger perspective and aims to determine a high-quality timetable outline in the strategic planning phase. In addition, the method includes adaptation time, i.e., the waiting time at the origin station, to ensure regular connections between passengers' origins and destinations.

In conclusion, effective line planning and timetabling are critical for enhancing public transport efficiency and passenger satisfaction. While the line planning problem and periodic timetabling have established methods, recent research on timetabling with passenger routing looks promising. Integrated optimisation and passenger routing approaches have shown promising results but require significant computational resources. Therefore, developing efficient and scalable solutions for integrated planning and timetabling is crucial for public transport planning.

Our approach aims to address this gap by creating an optimal timetable for a passenger railway exploiting the fact that the timetable usually stems from a line plan (Bull et al., 2019). Proposed approaches to finding optimal line plans typically minimise total travel time by assuming specific in-vehicle and transfer times. We interpret these assumptions in the line plan as promises made to passengers and use them to formulate an optimisation model. Our model ensures that the promised travel times and transfers are available and that passenger routes are chosen according to the shortest available path. We aim to provide an optimal timetable for passengers by keeping these promises and optimising travel efficiency.
This leads to the following contributions of this work:

1. We propose a logic-based Benders decomposition approach that optimises a timetable for a passenger railway system based on the promises made in earlier planning stages.
2. We test this approach on real-world data from the Rhätische Bahn railway system, demonstrating promising results.

Developing efficient and scalable solutions for integrated line planning and timetabling is crucial for enhancing public transport efficiency, reducing travel times, and improving passenger satisfaction.

2 Methodology

This section summarises our mixed integer linear programming model before we outline the Logic-Based Benders Decomposition approach.

Baseline model

We propose routing passengers and scheduling trains on a network using mixed integer linear programming (`MILP`) to minimise travel time while adhering to established schedules. Time windows are added for each commercial train arrival and departure to prevent schedule violations. Passenger expectations are met by enforcing pre-established in-vehicle travel and transfer times based on the assumptions taken during line planning. Time windows also aid in identifying optimal transfer connections. While we aim for all transfers to occur within the time limit, it may not always be possible. We allow multiple time limits per transfer to address this, which relaxes the transfer constraint. Our formulation seeks to optimise connections for all passengers within the constraints to minimise the total travel time.
To model the demand routing through the network, we use the Passenger Graph (`PG`), a directed graph representing the transportation network and consisting of *trip/dwell/transfer* arcs. We add all *trip/dwell* arcs to the `PG` for each train specified by the line plan. We then connect the arrival/departure nodes at stations according to the connections given in the line plan and add transfer arcs for each potential *transfer* between two connected stations. We add multiple *transfer* arcs between the exact arrival and departure nodes if a *transfer* has multiple duration limits.
To integrate the timetabling part into the transfer scheduling problem, we use the periodic event scheduling problem (`PESP`) formulation Serafini & Ukovich (1989), which is suitable for periodic timetabling. Furthermore, we use the Cycle Periodic Formulation (CPF) of Peeters (2003) for the `PESP`.

Logic-Based Benders decomposition

The `MILP` is challenging due to the computational complexity of the timetabling part, as shown by (Borndörfer et al., 2020). To address this issue, we propose a Logic-Based Benders Decomposition approach that separates the routing and timetabling problems to improve solution efficiency (Hooker, 2007). Specifically, we use SAT-based approach for the timetabling subproblem, adapted from Großmann (2016), allowing for faster feasibility determination and the generation of combinatorial Benders cuts from unsatisfiable cores (Codato & Fischetti, 2006). Classical Benders Decomposition is a two-stage method that optimises the master problem and checks the feasibility of the subproblem. In contrast, the Branch-and-Cut Benders method combines branch-and-cut algorithms with Benders Decomposition to reduce computational time (Rahmaniani et al., 2017). We implement each method and a hybrid approach that uses both, allowing for information exchange in the form of feasible solutions to improve efficiency further. We employ two strategies to improve the cut generation process, relaxing requirements and expanding conflicting transfer cuts.

3 Results and discussion

We evaluate the performance and suitability of our decompositions using real-world data from the Swiss railway company, `RhB`. We compare four different approaches to solving the problem: `Baseline` (`MILP` implementation), `Classical` (Classic Logic-Based Benders decomposition), `Modern` (Logic-Based Branch and Benders Cut), and `Hybrid` (`Modern` and `Classical` parallel). We use an instance of the `RhB` network with today's line plan of ten lines and a real-life dataset for demand, which includes 1747 origin/destination locations. The line plan is depicted in Figure 1.

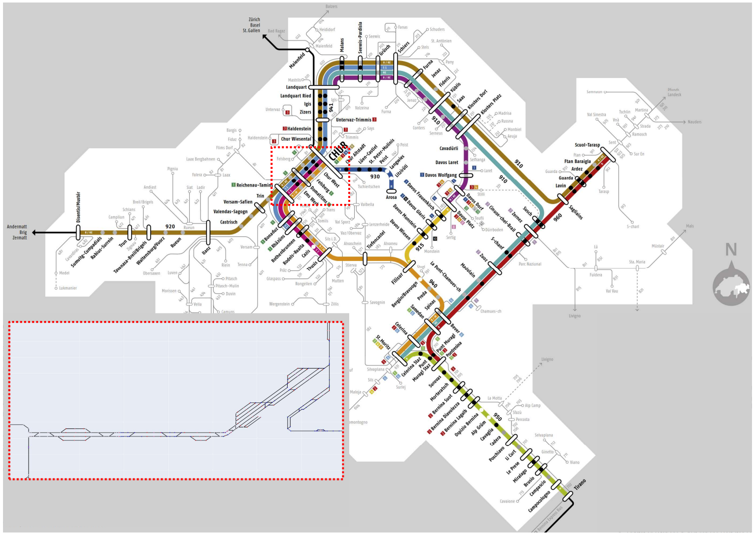

Figure 1: Current line plan of `RhB` (RhB, 2023).

We run each model with eight different random number seeds in the `MILP` solver. We report the best solution found in the form of total travel time and the remaining optimality gap expressed as a percentage. We are implementing the approach using Python 3.10 and the OpenBus Toolbox (Fuchs & Corman, 2019) and solving the `MILP` formulation using GUROBI 10.0.1 (Gurobi Optimization, LLC, 2023). In addition, we are solving the sat instances using the Glucose 4.1 sat solver (Audemard & Simon, 2018) with pySAT (Ignatiev et al., 2018). We allow GUROBI to use up to four threads for each run, while Glucose is single-threaded.

Table 1: Results for a time of three hours. A cell shows the objective (total travel time) and optimality gap.

Seed	`Baseline`	`Hybrid`	`Modern`	`Classic`
0	-, -	323.62 h, 38.76 %	323.54 h, 91.57 %	323.58 h, 37.09 %
1	-, -	323.55 h, 39.55 %	323.53 h, 79.74 %	324.24 h, 27.34 %
2	-, -	323.58 h, 38.38 %	323.55 h, 89.16 %	324.23 h, 19.57 %
3	-, -	324.23 h, 27.80 %	323.53 h, 79.13 %	324.23 h, 17.84 %
4	-, -	324.24 h, 27.81 %	323.54 h, 75.70 %	324.23 h, 20.64 %
5	-, -	323.56 h, 38.77 %	323.54 h, 90.21 %	323.57 h, 38.58 %
6	-, -	323.53 h, 38.61 %	323.54 h, 82.15 %	323.57 h, 39.09 %
7	-, -	323.57 h, 38.63 %	323.56 h, 76.57 %	324.23 h, 17.40 %

The results presented in Table 1 demonstrate that our proposed decomposition approach outperforms the `Baseline` approach. Notably, all runs of the `Baseline` implementation failed to yield any results, in stark contrast to all the decomposition approaches. Our comparison of the `Classical`, `Modern`, and `Hybrid` approaches revealed that they all produce similar objective function results. However, we observe differences in their ability to close the optimality gap, which remains a challenging problem. In particular, the `Classic` approach exhibited superior performance on a one-hour time limit, while on an extended three-hour time limit, `Classic` and `Hybrid` approaches performed comparably.

4 Conclusions

The proposed approach is valuable for optimising transfers, improving efficiency and passenger satisfaction, and reducing travel times. The `Classic` approach outperformed the other two decomposition approaches due to its ability to fix infeasible instances using a heuristic. However, the optimality gap remains a challenge. Solving the problem as a MILP, as in the Baseline approach, is not viable. Furthermore, the proposed method has limitations, including the inability to consider multiple connections per origin-destination pair, adaptation time at the origin station, and crowding. Further research can focus on improving and extending the model's performance to include these factors. Overall, the proposed approach has shown promising results on real-world data from Rhätische Bahn and can be applied to other transportation systems.

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