Optimal bicycle network expansions with endogenous demand

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Short summary

The challenge of identifying the ideal spatial and temporal prioritization for long-term expansions of bicycle networks is a complex undertaking. Our objective in this research is to determine the most beneficial expansions of bicycle networks for society, while considering the impact of level-of-service effects and induced demand throughout the evaluation period. While the effects of constant demand can be approximated through a sequence of linear binary mathematical programs (Paulsen & Rich, 2023), accommodating induced demand necessitates a different optimization approach that accounts for the likelihood of various segments being integrated in the infrastructure in future years during the optimization process. We put this approach to the test by applying it to the Greater Copenhagen Cycle Superhighway network. It is demonstrated that the optimized infrastructure render benefit-cost ratios exceeding 10, and that accounting for demand effects, significantly increases the societal return and changes the geographical structure of optimal investments.

Keywords: Bicycle network design; Bicycle traffic; Induced demand; Socioeconomic assessment; Dynamic optimization

1 Introduction

There is considerable evidence that bicycle demand is impacted by the presence of bicycle infrastructure, as demonstrated in several studies including van Goeverden et al. (2015) and Rich et al. (2021). The implementation of bicycle infrastructure not only affects travel time benefits resulting from route choice substitution as demonstrated in Paulsen & Rich (2023), but also the number of bicycle trips in the network (Hallberg et al., 2021). This is a result of mode substitution effects and potentially induced traffic. The societal value of increasing bicycle mileage is evidenced in Breda et al. (2018) and Martin et al. (2006), who study the external health benefit of one kilometer of cycling. The cost-benefit performance of bicycle infrastructure is studied in Rich et al. (2021), who finds that bicycle infrastructure is highly beneficial.

Optimal design of bicycle networks has been studied within operation research since early work by Smith & Haghani (2012) and Mesbah et al. (2012). The objective functions and constraints vary largely across studies, from approaches who i) minimize investment cost constrained by a minimum level-of service (Duthie & Unnikrishnan, 2014), ii) minimize local detours (Lim et al., 2021), iii) maximize cyclists on links where the stress-level is low (Chan et al., 2022; Ospina et al., 2022), iv) minimize generalized costs (Mauttone et al., 2017; Liu et al., 2019), or v) consider multi-objective costs (Lin & Yu, 2013; Lin & Liao, 2016; Liaw & Lin, 2022) subject to budget constraints. Although the studies have considered a large variety of performance measures, none of the studies calculate societal cost-benefit performance of the resulting bicycle network investment plans. Furthermore, many of these studies embed the route choice of cyclists directly in the optimization model, which becomes computationally intractable when considering large-scale applications with very large networks and many origin-destination pairs.

A recent study (Paulsen & Rich, 2023) shows that the consumer surplus of existing users can be approximated closely through sequences of linear binary mathematical programs at the level of the links. The approach is based on a method where OD-benefits are assigned to the network. The study show – under the assumption of constant demand – how optimal infrastructure expansions can be derived from a series of binary linear programs. However, as the study assumes the demand to be exogenous, it is not able to include health benefits that arise from increased bicycle demand. As demonstrated in previous research (Breda et al., 2018; Rich et al., 2021), this is the single most important factor when calculating the societal net present value of large bicycle network expansions.

This study extends Paulsen & Rich (2023), and the literature by and large, by determining the societal optimal expansions with endogenous demand. Methodologically, the approach is based on a dynamic optimization framework, within which the expected level-of-service and induced demand are approximated forward in time, to then approximate the expected accumulated benefit of selecting a given segment at a given time. By applying the algorithm to the entire evaluation period, we identify a bicycle infrastructure plan that render a solution worth 16 billion DKK in net present value terms. This correspond to a solution that is 81%–417% better than the considered reference strategies and with a benefit-cost ratio exceeding 10.

2 Methodology

Approximation of net present value

The overall aim is to provide a reasonable and computationally feasible approximation to the net present value at time t of any given existing network configuration $\boldsymbol{u}^{t-1}$ and investment action $\Delta\boldsymbol{u}^t$. Here, $\boldsymbol{u}^{t-1}$ is a binary vector, which has value 1 for *link segments* that were constructed at or before $t-1$, and $\Delta\boldsymbol{u}^t$ being a binary vector, which is 1 for the segments being constructed exactly at time t. Link segments are natural bundles of links (chunks of routes) along the same corridor, see Figure 3 for an example.

In Paulsen & Rich (2023) it is shown that the travel time function X_ω can be approximated very accurately for any configuration of $\boldsymbol{u}^{t-1}$ by assigning the OD-level travel time savings back onto the network, and by taking into account the travel time savings. That is,

$$X_\omega(\boldsymbol{u}^{t-1}) \simeq x_\omega^0 - (\Delta\boldsymbol{x}_\omega)^\intercal\, \boldsymbol{u}^{t-1}, \tag{1}$$

$\omega \in \Omega$ represents combinations of OD and traveler type, and x_ω^0 is the baseline travel time for ω without any network upgrades. $\Delta\boldsymbol{x}_\omega = \begin{bmatrix}\Delta x_{1,\omega} & \Delta x_{2,\omega} & \cdots & \Delta x_{|\mathcal{B}|,\omega}\end{bmatrix}^\intercal$ is the approximated vector of linear travel reductions from Paulsen & Rich (2023),

$$\Delta\boldsymbol{x}_{b,\omega} = \frac{\sum_{l\in b} L_l}{\sum_{l\in q_\omega^{\mathcal{L}}\cap\mathcal{L}} L_l}\left(\sum_{l\in q_\omega^0}\tau_{l,\omega} - \sum_{l\in q_\omega^{\mathcal{L}}}\hat{\tau}_{l,\omega}\right), \qquad b\in\mathcal{B},\omega\in\Omega. \tag{2}$$

Here, L_l is the length of link l, $\tau_{l,\omega}$ is the non-upgraded travel time on link l for the traveler type associated with ω, and $\hat{\tau}_{l,\omega}$ is the corresponding upgraded travel time. $q_{omega,0}$ and $q_{omega,0}$ denotes the shortest paths for OD and traveler type ω in the non-upgraded network and the fully upgraded network, respectively.

Equality is guaranteed in Eq. (1) at the two extrema $\boldsymbol{u}^{t-1} = \boldsymbol{0}$ and $\boldsymbol{u}^{t-1} = \boldsymbol{1}$. For this study, analogously, we introduce the approximated vector of linear travel distance extension $\Delta\lambda_\omega = \begin{bmatrix}\Delta\lambda_{1,\omega} & \Delta\lambda_{2,\omega} & \cdots & \Delta\lambda_{|\mathcal{B}|,\omega}\end{bmatrix}^\intercal$, which allows approximating the traveled distance for each ω,

$$\Lambda_\omega(\boldsymbol{u}^{t-1}) \simeq \lambda_\omega^0 + (\Delta\boldsymbol{\lambda}_\omega)^\intercal\, \boldsymbol{u}^{t-1}. \tag{3}$$

Again with guarantee for equality for $\boldsymbol{u}^{t-1} \in \{\boldsymbol{0},\boldsymbol{1}\}$. Here $\Delta\boldsymbol{\lambda}_\omega$ has elements,

$$\Delta\lambda_{b,\omega} = \frac{\sum_{l\in b} L_l}{\sum_{l\in q_\omega^{\mathcal{L}}\cap\mathcal{L}} L_l}\left(\sum_{l\in q_\omega^{\mathcal{L}}} L_l - \sum_{l\in q_\omega^0} L_l\right), \qquad b\in\mathcal{B},\omega\in\Omega. \tag{4}$$

Finally, the demand function D_ω does not require searching through the network, why it can be evaluated sufficiently quickly to be used as it is. In the study we use a simple logit-based mode choice model based on parameters from Hallberg et al. (2021). More advanced models can easily be considered within the framework we propose, for instance models that include destination choice or forecasts demand according the development in GDP.

All in all, this suggest that the net present value can be approximated reasonably and computationally efficient by $\text{NPV}^t(\Delta\boldsymbol{u}^t;\boldsymbol{u}^{t-1})$ in the equation below,

$$\mathrm{NPV}^t(\Delta\boldsymbol{u}^t;\boldsymbol{u}^{t-1}) = \overbrace{\kappa^t \sum_{\omega\in\Omega} \zeta_\omega \frac{D_\omega(\boldsymbol{u}^{t-1}) + d_\omega^0}{2} \left(x_\omega^0 - X_\omega(\boldsymbol{u}^{t-1})\right)}^{\text{Consumer surplus}} + \overbrace{\kappa^t \sum_{\omega\in\Omega} \xi_\omega \left(D_\omega(\boldsymbol{u}^{t-1})\Lambda_\omega(\boldsymbol{u}^{t-1}) - d_\omega^0\lambda_\omega^0\right)}^{\text{Health benefits}} + \overbrace{\kappa^{|\mathcal{T}|} \sum_{b\in\mathcal{B}} c_b \Delta u_b^t}^{\text{Scrap value}} - \overbrace{\kappa^t \sum_{b\in\mathcal{B}} c_b \Delta u_b^t}^{\text{Construction costs}} - \overbrace{\kappa^t \sum_{b\in\mathcal{B}} m_b u_b^{t-1}}^{\text{Maintenance costs}}. \tag{5}$$

The used notation is summarized in Table 1.

$b \in \mathcal{B}$	A link segment b (containing links $l \in b$ along the same corridor), within the set of all link segments $\mathcal{B}$. $\mathcal{B}$ partitions $\mathcal{L}$, such that the each link of $\mathcal{L}$ belongs to exactly one link segment $b \in \mathcal{B}$.
c_b	Construction cost of segment b. From Incentive (2018).
d_ω^0	Baseline demand for ω, i.e. $D_\omega(X(\mathbf{0}))$.
D_ω	Demand function for ω. Parameters adopted from Hallberg et al. (2021).
ζ_ω	Value of time for the traveler type of ω. Value of 91 DKK per hour (Technical University of Denmark, 2022).
κ^t	Discounting factor for time t. From Technical University of Denmark (2022).
λ_ω^0	Baseline travel distance of ω, i.e. $\Lambda_\omega(\mathbf{0})$.
Λ_ω	Travel distance function for ω.
m_b	Annual maintenance cost of segment b. From Incentive (2018).
$\Delta\boldsymbol{u}^t$	The decision variable vector at time t, $[\Delta u_1^t \quad \Delta u_2^t \quad \dots \quad \Delta u_{\lvert\mathcal{B}\rvert}^t]^\intercal$, which is 1 for segments being chosen at time t, and 0 otherwise.
$\boldsymbol{u}^{t-1}$	The vector $[u_1^{t-1} \quad u_2^{t-1} \quad \dots \quad u_{\lvert\mathcal{B}\rvert}^{t-1}]^\intercal$ containing ones for all segments that have been selected at time $t-1$ or before, and zeroes elsewhere. That is, $\boldsymbol{u}^{t-1} = \sum_{k\leq t-1} \Delta u^k$.
x_ω^0	Baseline travel time for ω, i.e. $X_\omega(\mathbf{0})$
X_ω	Travel time function for ω. From Hallberg et al. (2021).
ξ_ω	Health benefit factor per km for ω (subtracted the corresponding accident factor). Value of 7.11 DKK per km (Technical University of Denmark, 2022).
$\omega \in \Omega$	Considered OD-pair and traveler type combinations. From Hallberg et al. (2021).

Table 1: Notation overview for the net present value calculation (Eq. (5))

Optimization framework

The idea is then to embed this expression into an optimization scheme that optimizes $\Delta\boldsymbol{u}^t$ for all t in the 50 year evaluation period $\mathcal{T}$, as stated in Problem 1.

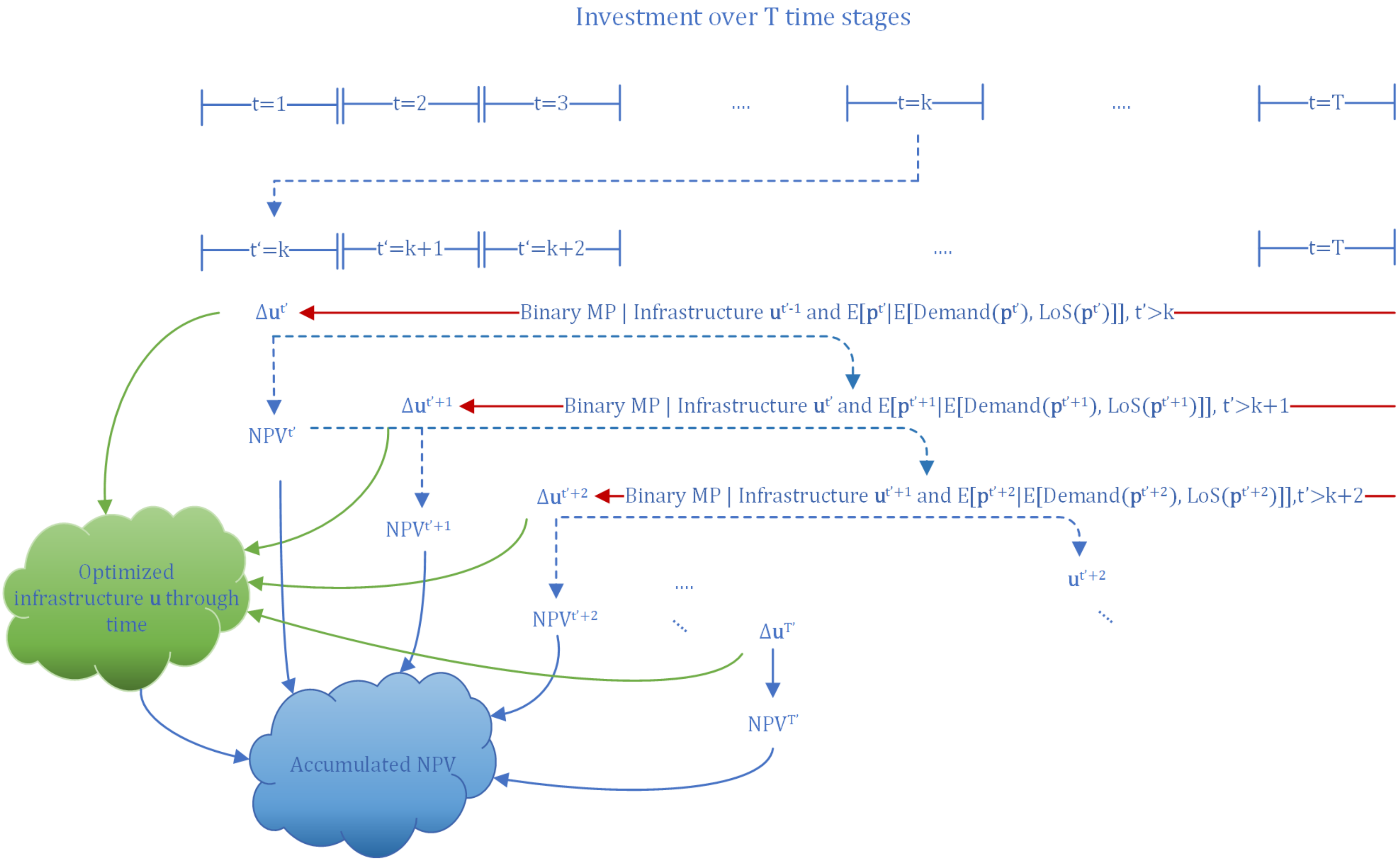


Figure 1: Flow chart of the proposed optimization scheme. $\Delta u^{t'}$ are the optimal strategies at each time step, whereas $\boldsymbol{p}^{t'}$ represents the expected infrastructure composition at future time steps. The binary mathematical program (Binary MP) is Problem 2. $E\left[\boldsymbol{p}^{t'}|E\left[Demand(\boldsymbol{p}^{t'}), LoS(\boldsymbol{p}^{t'})\right]\right] = E\left[\boldsymbol{p}^{t'}|E\left[D(\boldsymbol{p}^{t'}), X(\boldsymbol{p}^{t'})\right]\right]$ forms a fixed point problem across all future $t' > k$, which leads to the vector $\boldsymbol{S}^t$. The calculation of $\boldsymbol{S}^t$ is further detailed in Figure 2, and constitutes the coefficients for the linear objective function of Binary MP used to determine $\Delta u^{t'}$.

$$
\begin{aligned}
\max_{\Delta \boldsymbol{u}^t} Z &= \sum_{t \in \mathcal{T}} \mathrm{NPV}^t(\Delta \boldsymbol{u}^t, \boldsymbol{u}^{t-1}) && s.t. && \text{(P1a)}\\
B^t &\geq \sum_{k \in \mathcal{T}: k \leq t} \kappa^k \left(\boldsymbol{c}^\intercal \Delta \boldsymbol{u}^k + \boldsymbol{m}^\intercal \boldsymbol{u}^{k-1} \right), && \forall t \in \mathcal{T} && \text{(P1b)}\\
u_b^t &\geq u_b^{t-1}, && \forall b \in \mathcal{B}, \forall t \in \mathcal{T} && \text{(P1c)}\\
\Delta u_b^t &\in \{0, 1\}, && \forall b \in \mathcal{B}, \forall t \in \mathcal{T} && \text{(P1d)}
\end{aligned}
$$

Problem 1: NPV-model with flexible demand

Here, B^t is the cumulative budget for time t, whereas $\boldsymbol{c}$ and $\boldsymbol{m}$ are construction costs and maintenance costs vectors defined by $\boldsymbol{c} = [c_1 \quad c_2 \quad \cdots \quad c_{\mathcal{B}}]^\intercal$ and $\boldsymbol{m} = [m_1 \quad m_2 \quad \cdots \quad m_{\mathcal{B}}]^\intercal$, respectively. A sketch of the overall idea is outlined in Figure 1.

So far, the approach is more or less similar to that of Paulsen & Rich (2023). That is, given previous decisions $\boldsymbol{u}^{t-1}$, we determine the optimal composition of the segments to select at time t (Δu^t) subject to budget constraints, such that the expected future net present value is maximized. Once the optimal $\Delta \boldsymbol{u}^t$ has been found, $\boldsymbol{u}^t$ can be updated to $\boldsymbol{u}^t \leftarrow \boldsymbol{u}^t + \Delta \boldsymbol{u}^{t-1}$, and we can consider the next choice situation at time $t \leftarrow t+1$. The basic idea is that at any given decision time t, we can assume that we already know what has happened in the past, i.e. $\Delta \boldsymbol{u}^{t'}, \forall t' < t$.

However, the optimization problem is complicated by the presence of endogenous demand. A very precise approximation could be made concerning future net present values without taking into account the expectations of future investment in Paulsen & Rich (2023) under the assumption of constant demand. In that case, the effect of each segment could be linearized, fully ignoring their interaction without any notable loss in net present value precision. This would clearly be inappropriate when taking endogenous demand into account.

Thus, instead we develop a vector $\mathbf{S}^t$ that also takes the expectations of future investments $\boldsymbol{p}^{t'}$ into account for $t' > t$. It gives an approximation of how each, so far un-selected segment, contributes to the expected accumulated net present value. We use this as the coefficients for our objective function in Problem 2.

$$
\begin{aligned}
\max_{\Delta \boldsymbol{u}^t} Z &= \left(\boldsymbol{S}^t \right)^\intercal \Delta \boldsymbol{u}^t && s.t. && \text{(P2a)}\\
B^t &\geq \sum_{k \in \mathcal{T}: k \leq t} \kappa^k \left(\boldsymbol{c}^\intercal \Delta \boldsymbol{u}^k + \boldsymbol{m}^\intercal \boldsymbol{u}^{k-1} \right), && \forall t \in \mathcal{T} && \text{(P2b)}\\
u_b^t &\geq u_b^{t-1}, && \forall b \in \mathcal{B}, \forall t \in \mathcal{T} && \text{(P2c)}\\
\Delta u_b^t &\in \{0, 1\}, && \forall b \in \mathcal{B}, \forall t \in \mathcal{T} && \text{(P2d)}
\end{aligned}
$$

Problem 2: The individual binary linear problems

***Calculation of* $\mathbf{S}^t$**

The calculation $\boldsymbol{S}^t$ is a tedious and complex task not particularly suited for being explained in detail in an extended abstract. Still, we aim at outlining the key aspects in this section. A flow chart of the process of calculating $\boldsymbol{S}^t$ for a single $t \in \mathcal{T}$ is found in Figure 2.

The calculation of $\boldsymbol{S}^t$ considers future time stages $t' \in \mathcal{T} : t' > t$, in which we do not yet know which segments will be chosen. We accommodate this by loosening the restriction of binary decisions, and instead introduce a cumulative probability vector $\boldsymbol{p}^{t'}$ for future $t' > t$ for all segments that remain unchosen at time t, denoted by $\mathcal{B}^t = \{b \in \mathcal{B} : u_b^{t-1} = 0\}$. Likewise, we introduce the instantaneous probability vector $\Delta \boldsymbol{p}^{t'} = \boldsymbol{p}^{t'} - \boldsymbol{p}^{t'-1}$. Initially, we do not differentiate between the probability of various segments, i.e. assign uniform probabilities $p_1^{t'} = p_2^{t'} \ldots = p_{|\mathcal{B}|}^{t'}, \forall t' > t$ across segments, but we will later set up a fixed point problem where the probabilities feed into an expected future net present value for each $b \in \mathcal{B}^t$, and the future net present values affect the probabilities.

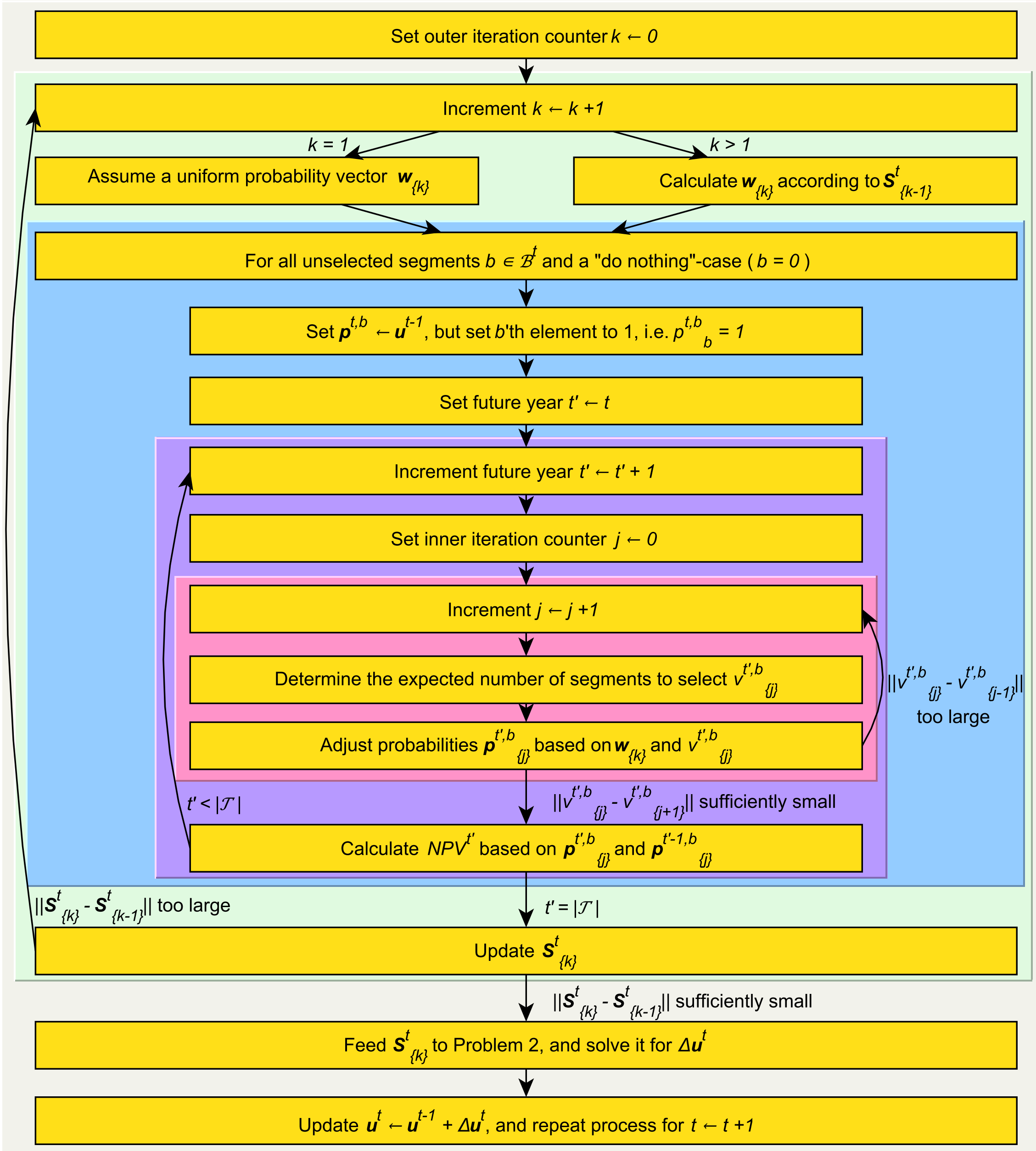


Figure 2: Flow chart of the process of calculating $\boldsymbol{S}^t$ for a single $t \in \mathcal{T}$. The calculation contains two fixed point problems. One for determining $\nu^{t',b'}$ for every future time steps $t' > t$ for every unselected segment $b \in \mathcal{B}^t$ with intermediate solutions indexed by $j \in \mathbb{N}^+$, and an overall fixed point problem for $\boldsymbol{S}^t$ with solutions indexed by $k \in \mathbb{N}^+$.

At time t we aim at evaluating the approximate effect of choosing each of the unselected segments $b \in \mathcal{B}^t$, and compare it to a situation where no action is taken. The difference between the two (Eq. (6)),

$$S^t_{b\{k+1\}} = \begin{cases} \sum\limits_{t' \geq t} NPV_{t'}\left(\Delta \boldsymbol{p}^{t',b,t}_{\{k\}}, \boldsymbol{p}^{t'-1,b,t}_{\{k\}}\right) - \sum\limits_{t' \geq t} NPV_{t'}\left(\Delta \boldsymbol{p}^{t',0,t}_{\{k\}}, \boldsymbol{p}^{t'-1,0,t}_{\{k\}}\right), & b \in \mathcal{B}^t \\ 0, & \text{otherwise} \end{cases}, \tag{6}$$

is calculated using Eq. (5) using continuous rather than binary vectors as input, and reflects an approximation of the added value of choosing segment b at time t.

However, in order to calculate $S^t_{b\{k+1\}}$ we need the probability vectors $\boldsymbol{p}^{t',b,t}_{\{k\}}$. As we will see shortly, these are mutually dependent and form a fixed point problem which is solved iteratively across an iteration counter k. An exception to this is for $t' = t$, for which we have the evaluated action at time t, that is

$$p_i^{t,b,t} = \begin{cases} 1, & i = b \\ u_i^{t-1} & \text{otherwise} \end{cases}, \forall b \in \mathcal{B}^t. \tag{7}$$

Before we can determine the actual probability vector for $t' > t$, it turns out to be relevant to determine the expected number of segments chosen in a given future timestep $t' > t$. We denote this number by $\nu^{t'}$. As it (also) forms a fixed point problem with the probability vector in all future time stages, we index it by j. For a given $t' \geq t$ and possibly an intermediate probability vector $\boldsymbol{p}_{\{j\}}^{\nu_{\{j\}},b,t}$, $\nu_{\{j\}}^{t'}$ can be determined by dividing the (expected) remaining budget $R_{t'}$ at time t' with the probability weighted average construction costs of the remaining segments $\bar{c}_{\{j\}}^{t',b,t}$,

$$\nu_{\{j+1\}}^{t'} = \frac{R_{t'}}{\bar{c}_{\{j\}}^{t',b,t}} = \begin{cases} \dfrac{B^t - \sum\limits_{k \in \mathcal{T}: k < t} \kappa^k \boldsymbol{c}^\intercal \Delta \boldsymbol{u}^k - \sum\limits_{k \in \mathcal{T}: k \leq t} \kappa^k \boldsymbol{m}^\intercal \boldsymbol{u}^{k-1}}{\kappa^{t'} \frac{1}{|\mathcal{B}^{t,b}|} \sum\limits_{b' \in \mathcal{B}} c_{b'}}, & j+1 = 0 \\ \dfrac{B^t - \sum\limits_{k \in \mathcal{T}: k < t} \kappa^k \boldsymbol{c}^\intercal \Delta \boldsymbol{u}^k - \sum\limits_{k \in \mathcal{T}: k \leq t} \kappa^k \boldsymbol{m}^\intercal \boldsymbol{u}^{k-1}}{\frac{\kappa^{t'} \sum\limits_{b' \in \mathcal{B}^{t,b}} \Delta p_{b'\{j\}}^{\nu_{\{j\}}^{t'}} c_{b'}}{\sum\limits_{b' \in \mathcal{B}^{t,b}} \Delta p_{b'\{j\}}^{\nu_{\{j\}}^{t'}}}}, & j+1 \in \mathbb{N}^+ \end{cases} \cdot \tag{8}$$

The probability vector does not only depend on the expected number of selected segments, but also on baseline probabilities $\boldsymbol{w}^t$ which takes into account the expected performance of each segment. As $\boldsymbol{S}^t$ measures exactly this, it seems reasonable to include $\boldsymbol{S}^t$ in the determination of the baseline probabilities. Furthermore, since the investments at each time step are limited by the construction costs of the segments, we also adjust the probabilities according to the construction costs, so that expected net present value increase per construction costs is used in the denominator. The suggestion that this is a good performance indicator is supported by Paulsen & Rich (2023) in which a greedy algorithm based on this ratio yields practically identical results as the optimal solution. This, leads to the following baseline probability expression $\boldsymbol{w}_{\{k\}}^t$ for Iteration k based on a based on a Multinomial Logit (McFadden, 1973) formulation:

$$w_{b\{k\}}^t = \begin{cases} \frac{1}{|\mathcal{B}^t|}, & k = 0 \\ \dfrac{\exp\left(\frac{\mu}{M^t} \frac{S_{b\{k\}}^t}{\kappa^t c_b}\right)}{\sum\limits_{i \in \mathcal{B}^t} \exp\left(\frac{\mu}{M^t} \frac{S_{i\{k\}}^t}{\kappa^t c_i}\right)}, & k \in \mathbb{N}^+ \end{cases}, \forall b \in \mathcal{B}^{t'}, \forall t \in \mathcal{T}. \tag{9}$$

Here M^t is the range between the best and worst segment in Iteration 1, i.e.

$$M^t = \max_{b \in \mathcal{B}^t} S_{b\{1\}}^t - \min_{b \in \mathcal{B}^t} S_{b\{1\}}^t, \tag{10}$$

and μ is a hyperparameter.

When evaluating the effect of choosing segment b at time t, the baseline probabilities have to be altered accordingly, such that they still sum to 1 when not taking b into account, i.e.

$$w_{b'\ \{k\}}^{t,b} = \begin{cases} \frac{w_{b'\{k\}}^t}{1 - w_{b\{k\}}^t}, & b' \in \mathcal{B}^{t,b} \\ 0, & \text{otherwise} \end{cases}, t \in \mathcal{T}. \tag{11}$$

Assume now that $\nu \in \mathbb{N}^+$ segments are to be selected among $\mathcal{B}^t$. As selected segments cannot be unselected, it follows that the segment probability is a monotonously non-decreasing function of ν. Based on the baseline probabilities $\boldsymbol{w}^t$ and ν, we propose the following recursive definition of the probability of being selected within the first ν segments (excluding b),

$$q_{b'}^{\nu,b,t} = \begin{cases} \frac{w_{b'\{k\}}^{t,b}\left(1-Q_{b'}^{\nu-1,b,t}\right)}{\sum\limits_{i\in\mathcal{B}^{t,b}} w_{i\{k\}}^{t,b}\left(1-Q_{i}^{\nu-1,b,t}\right)}, & \nu \leq \left|\mathcal{B}^{t,b}\right| \\ 1, & \text{otherwise} \end{cases}, \nu \in \mathbb{N}^+, \forall b' \in \mathcal{B}^{t,b}, \forall b \in \mathcal{B}^t, \forall t \in \mathcal{T}, \tag{12}$$

with

$$Q_{b'}^{\nu,b,t} = \begin{cases} 0, & \nu = 0 \\ \min\left\{1, \sum\limits_{n=1}^{\nu} q_{b'}^{n,b,t}\right\}, & \nu \in \mathbb{N}^+ \end{cases}, \qquad \forall b' \in \mathcal{B}^{t,b}, \forall b \in \mathcal{B}^t, \forall t \in \mathcal{T}. \tag{13}$$

This is the regular probability expression, but corrected by the cumulative probability of being selected within the first ν segments. It can be generalized for non-integer ν's as follows:

$$p_{b'\{j+1\}}^{\nu_{\{j\}},b,t} = \begin{cases} Q_{b'}^{\lfloor\nu_{\{j\}}\rfloor,b,t} + Q_{b'}^{\lfloor\nu_{\{j\}}\rfloor+1,b,t} \cdot \left(\nu_{\{j\}} - \lfloor\nu_{\{j\}}\rfloor\right), & b' \in \mathcal{B}^{t,b} \\ 1, & \text{otherwise} \end{cases}, \nu \in \mathbb{R}^+, \forall b \in \mathcal{B}^t, \forall t \in \mathcal{T}. \tag{14}$$

Since ${\nu^{t',b,t}}_{\{j\}}$ and ${\boldsymbol{p}^{\nu_{\{j-1\}},b,t'}}_{\{j\}}$ are mutually dependent, the determination of the two forms a fixed point problem across j. Empirically, since the calculations are very fast, the problem converges quickly. Once the fixed point problem has been solved, i.e. when $\left|\left|\nu_{\{j\}}^{t',b,t} - \nu_{\{j-1\}}^{t',b,t}\right|\right|_\infty < \epsilon_\nu$, the resulting $\nu_{\{j\}}^{t'}$ is denoted by $\nu^{t',b,t}$ and we assign ${\boldsymbol{p}^{t',b,t}}_{\{k\}} \leftarrow \boldsymbol{p}^{\nu^{t',b,t},b,t}$.

By doing this for all $t' > t$, the vector $\boldsymbol{S}_{\{k+1\}}^t$ can be obtained from Eq. (6) for a given k, and $k \leftarrow k+1$ can be incremented. The $\boldsymbol{S}_{\{k\}}^t$ is then used to update the baseline probabilities $\boldsymbol{w}_{\{k\}}$. By applying the Method of Successive Averages (Robbins & Monro, 1951; Sheffi, 1985) on the sequence of $\boldsymbol{S}_{\{k\}}^t$, the sequence have been found to converge in our application.

When $\left|\left|\boldsymbol{S}_{\{k\}}^t - \boldsymbol{S}_{\{k-1\}}^t\right|\right|_\infty < \epsilon_S$ for some k, the optimal strategy at time t can be determined by assuming linear independence between the elements of $\boldsymbol{S}_{\{k\}}^t$, and solving Problem 2 using $\boldsymbol{S}_{\{k\}}^t$ as $\boldsymbol{S}^t$.

3 Results and discussion

We test our proposed methodology on a large-scale network of Greater Copenhagen, where we consider the expansion of 43 proposed cycle superhighway routes divided into 202 segments (see Figure 3) over a 50 year planning period. Each of the 202 segments has specific construction and maintenance costs from Incentive (2018), and at each $t \in \{1, 2, \ldots, 50\}$ the available budget is given by $B^t = 50 \cdot t$ mill. DKK. The set of origin-destination pairs and traveler types Ω are taken from Hallberg et al. (2021) and contains the combinations of 258 origins and destinations and nine traveller types (combinations of speed preference and bicycle technology, see Hallberg et al. (2021) for details), leading to a total of 596,754 entries.

Table 2 summarizes the various costs, benefits and performance measures associated to each of the seven applied solution strategies. **W/ demand effects** is our proposed method, whereas **W/o demand effects** is the solution where demand effects is not taken into account(Paulsen & Rich, 2023). The bottom-five strategies are baseline reference strategies that are not based on optimization.

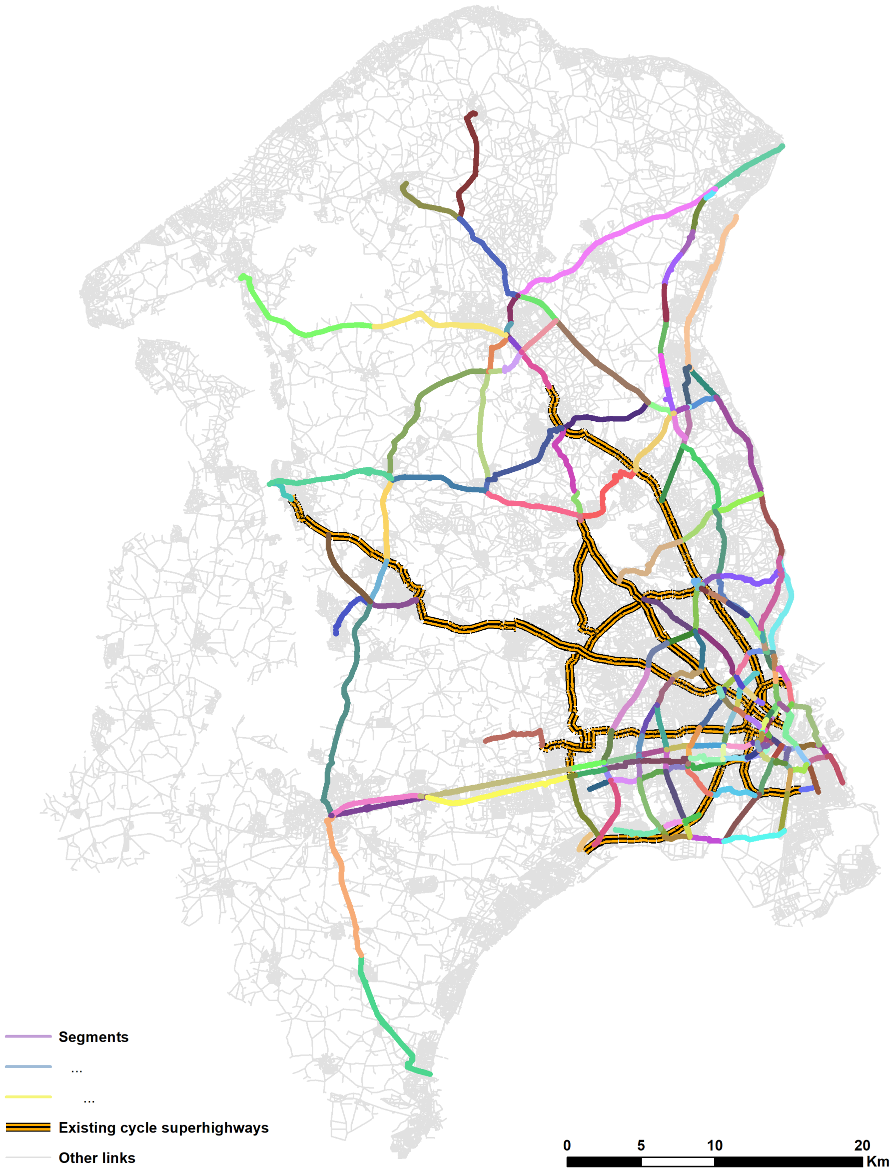


Figure 3: The 202 segments forming the 580km planned future cycle superhighway network extension for the Greater Copenhagen area (Sekretariatet for Supercykelstier, 2019) as well as the existing network.

Strategy	Construction costs	Scrap value	Maintenance costs	Consumer surplus	Health benefits	Net present value	Benefit-cost ratio*
W/ demand effects	715.8	181.9	1,080.5	2,746.3	14,890.5	16,022.4	10.9
W/o demand effects**	321.7	60.4	445.9	2,197.6	11,270.0	12,760.4	19.0
Random order	986.7	358.6	1,406.4	1,153.4	6,526.7	5,645.6	3.77
Shorter segments first	997.2	358.6	1,387.8	1,864.1	9,001.1	8,839.6	5.36
Shorter routes first	998.9	358.6	1,386.9	1,570.3	8,166.4	7,709.5	4.80
Longer segments first	983.6	358.6	1,410.4	694.4	5,187.5	3,846.4	2.89
Longer routes first	978.8	358.6	1,414.7	1,235.7	7,997.7	7,198.5	4.54

Table 2: Investment key-performance indicators [mill. DKK]. *Benefit-cost ratio is dimensionless. ** The solution found with the methodology from Paulsen & Rich (2023).

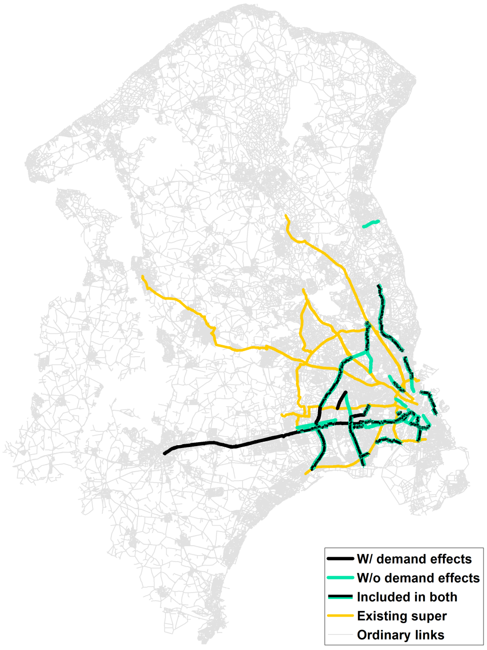
From the results of the baseline strategies it is clearly shown that the overall project portfolio is profitable, leading to net present values between 3.8 and 8.8 billion DKK – largely driven by the health benefits from added bicycle kilometers. The variation in net present value across the baseline reference strategies are substantial, underlining that the order in which segments are implemented have a large effect on the socioeconomic performance. It is also seen that taking a mathematical optimization approach leads to large net present value improvements of at least 3.9 billion DKK when using the method from Paulsen & Rich (2023) (**W/o demand effects**) and 7.2 billion DKK with the approach proposed in this study (**W/ demand effects**), when compared to the best baseline reference strategy (**Shorter segments first**). Thus, the improved methodology leads to a net present value increase that is 83% higher than that of Paulsen & Rich (2023), underlining that taking demand effects into account in the optimization is highly important. Based on the raw net present values of 16.0 billion DKK (**W/ demand effects**) and 12.8 billion DKK (**W/o demand effects**), the relative improvement is 26%.

We note that our optimization routine maximizes an approximation of the net present value, why it is not surprising that the method from Paulsen & Rich (2023) leads to a higher benefit-cost ratio. Especially since that method stops when further expansions are no longer deemed profitable without considering demand effects. When considering these effects, more segments are deemed profitable, leading to a premature stop of investments for the **W/o demand effects** strategy. Only investing in the most profitable segments naturally lead to a high benefit-cost ratio, but fails to achieve the full potential net present value.

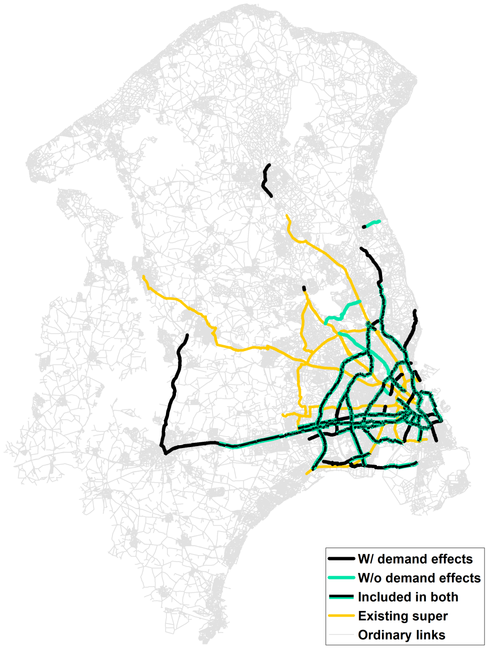
Discussion

We consider the same case study and project portfolio as in Hallberg et al. (2021); Rich et al. (2021), and Paulsen & Rich (2023). Our demand model shares many similarities with Hallberg et al. (2021) and Rich et al. (2021) in that we apply similar level-of-service data and model parameters. However, in the present study we only consider choice of mode and not choice of destination. When upgrading the entire network, we get a relative increase in the number of trips of 3.7%, which compares to an increase of 4.5% in Hallberg et al. (2021). The difference is due to not considering choice of destination. Also, the increase in average cycled trip distance of 8.3% are in line with the 7-8% of Rich et al. (2021). It suggest that our demand sensitivity are largely in line with previous findings.

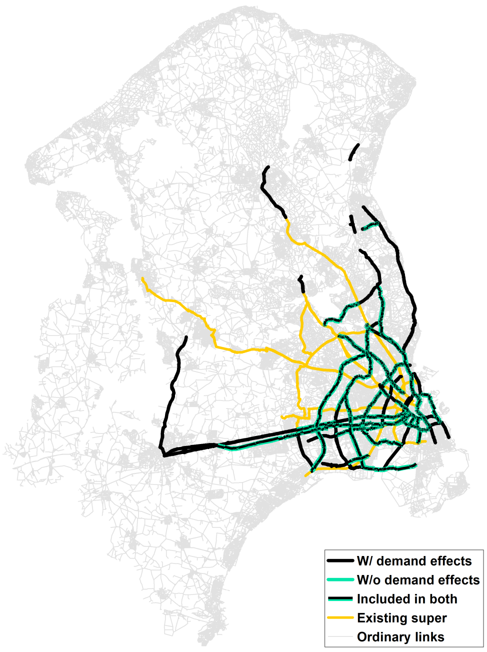
In Figure 4 we compare the solution of our proposed method with that of Paulsen & Rich (2023) that does not incorporate demand effects. Clearly, we see that including such effects encourage building longer routes further away from the city center and cause more segments to be profitable from a socioeconomic point-of-view. Hence, the integration of demand effects implies not only a sizable increase in the welfare contribution, but change the spatial investment pattern as well. The fact that the investment pattern becomes more spatially scattered have some positive indirect implications for the practical implementation of such strategies. Where the solution without demand effect is concentrated mostly the in city center, and hence discourages other municipalities from taking part in the investment scheme, the improved solution actually goes across the geography and makes it highly relevant for municipalities to collaborate when upgrading the infrastructure.



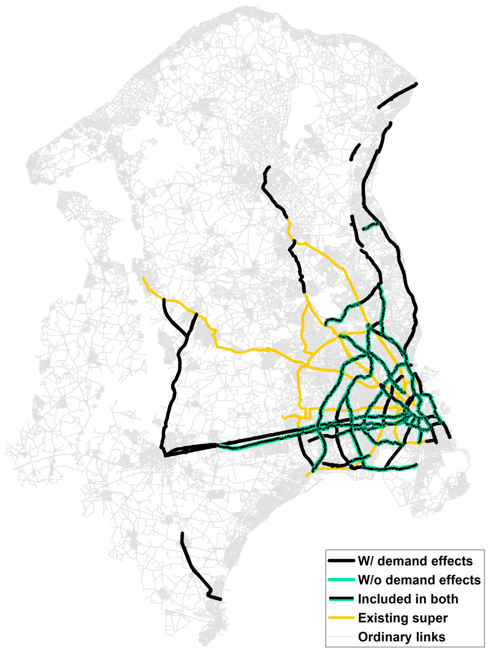
(a) Year 4, cumulative budget of 200 mill. DKK



(b) Year 12, cumulative budget of 600 mill. DKK



(c) Year 20, cumulative budget of 1,000 mill. DKK



(d) Year 28, cumulative budget of 1,400 mill. DKK

Figure 4: Spatial comparison of obtained solutions with (present study) and without (Paulsen & Rich (2023)) demand effects.

4 Conclusions

With this study we develop and show the large-scale applicability of a methodology for societally optimal expansions of bicycle networks where demand is integrated into the problem. The proposed methodology leads to massive societal benefits with a net present value exceeding 16 billion DKK, in the range of 81%–417% higher than the baseline reference strategies, and 26% higher than the solution found without taking demand effects into account as presented in Paulsen & Rich (2023).

Despite providing a significant contribution to the literature, several research avenues remain open for future research. Methodologically, it is relevant to investigate alternative ways of calculating the expected future net present value contribution of segments ($\boldsymbol{S}^t$) and compare the performance of the different variations. It will also be relevant to test the effect of a less sensitive demand response, and to incorporate more advanced demand models that allow modeling the composition of regular bicycle users versus electric bicycle users dynamically as network changes occur. As demand effects are even more pronounced for electric bicycle users that travel further (Hallberg et al., 2021), it is of particular interest to investigate if and how such dynamic modeling of the share of electric bicycle users would alter the optimal infrastructure plans. Future research also includes looking further into regional distribution effects, how to integrate regional budget constraints, and, not least, how these would affect the solution and socio-economic performance measures.

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