Evaluating Mobility Service Providers' Strategies in an Activity-Based Supernetwork

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SHORT SUMMARY

A Mathematical Problem with Equilibrium Constraints (MPEC) is formulated to capture the relationships between multiple Mobility Service Providers (MSPs) and the users of a multimodal transport network. The network supply structure is represented as a supernetwork where users' daily activity chains are represented sequentially and their modal choices to reach different destinations are based on the mobility services active in each connection. At the upper level, a profit maximization formulation is introduced to describe MSPs' behaviour. At the lower level, groups of users choose the routes with the lowest cost, according to Wardrop's first equilibrium principle. Due to non-separable interactions between supernetwork links, the equilibrium conditions defining users travel behaviour are written as Variational Inequality (VI). Finally, a numerical example is presented in order to show the characteristics of the model when car-sharing, bus and private car are available in the network.

Keywords: Multimodal transportation, Network Design Problem, Profit Maximization, Supernetwork, Variational Inequality

1. INTRODUCTION

Multimodal network design problems (MNDP) focus on the coexistence of several modes of transport in the same network, in which a Mobility Service Provider (MSP) changes strategy in order to optimize their objective function, while users respond by modifying their choices to maximize their utility.

Analytically, the MNDP is typically a non-convex problem that can be formulated either as a bi-level optimization problem, or as a Mathematical Problem with Equilibrium Constraints (MPEC); the latter admits the lower level equilibrium in the form of a Variational Inequality (VI) or Non-linear Complementarity Problem. A variety of solution algorithms have been used to solve these problems (Sinha et al., 2018) (Luo et al. 1996).

In literature it is possible to find different approaches to model these problems, and more recently some works try to capture the interactions between classical transport services and shared ser-

vices. Nair et al. (2014) designed a shared vehicle system integrated with an existing transit network formulated as a mixed-integer bi-level program. The decisional variables of the upper level sharing system are: the decision to have a service in a certain node, the capacity of the node, and the fleet size. The lower level decisional variables are the flow that maximize the travel utility in the network. The original optimization problem is translated into the Karush–Kuhn–Tucker (KKT) conditions, and the problem becomes an MPEC. Fu et al. (2020), instead, developed an extended supernetwork in which the time and space coordination are taken into account together with activity and travel choices, subject to congestion effects. They formulate a bi-level problem in which at the upper level they maximize the accessibility of the activity locations, and at the lower level users maximize their utility. The lower level user equilibrium condition are then formulated as VI, and a heuristic algorithm is used to solve the general model. Nguyen et al. (2022) elaborated a bi-level problem considering at the upper level a car sharing company that seeks to maximize their profit changing parking spaces, prices and vehicle relocation. Users are assigned in a multimodal network in which they can choose between car-sharing, public transport and private car. Travellers can choose mode of transport, activities and routes. The lower level formulation takes the form of a VI. The general model is solved through a link-based approach.
In this context, the model proposed in this paper is based on an activity-based supernetwork in which all mobility services can be included. The problem is formulated as an MPEC where the upper level profit maximization ca be applied to different mobility services of the transport network. At the lower level users are divided in classes, and their formulation takes the form of a path-based VI.

2. METHODOLOGY

The methodological approach used in this paper is based on the construction of a supernetwork. A supernetwork (Sheffi and Daganzo, 1978) is an expanded network where the different choices that travellers encounter are represented as route choices. This network illustration permits to represent an area where several MSPs offer mobility services to users, who perform their activities in different locations during a typical weekday.
Travellers are divided into K classes based on their personal attributes and daily trip chains. For each class, their sequence of activities is modelled as a graph (Figure 1), where a node n indicates an activity location and a link a indicates a trip to move from one activity location to the next. Through this graph we do not explicitly represent the real underlying transport infrastructure, and in the same activity location different classes of users can perform different activities.

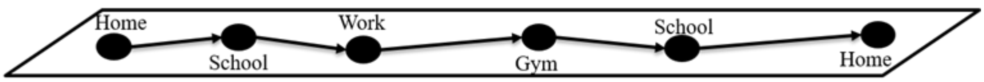

Figure 1: Network with activities for user class *k*

In order to move from a single network to a multimodal supernetwork, the other fundamental input needed is based on MSPs' information. More specifically, once the network with activities is defined, it is then divided in several supplier-specific layers, each of which represents a unimodal network operated by a single MSP, as shown in Figure 2. Furthermore, in this representation we considered that travellers can also opt to make their trip with their proper private vehicle, i.e. car, bike, scooter, etc.

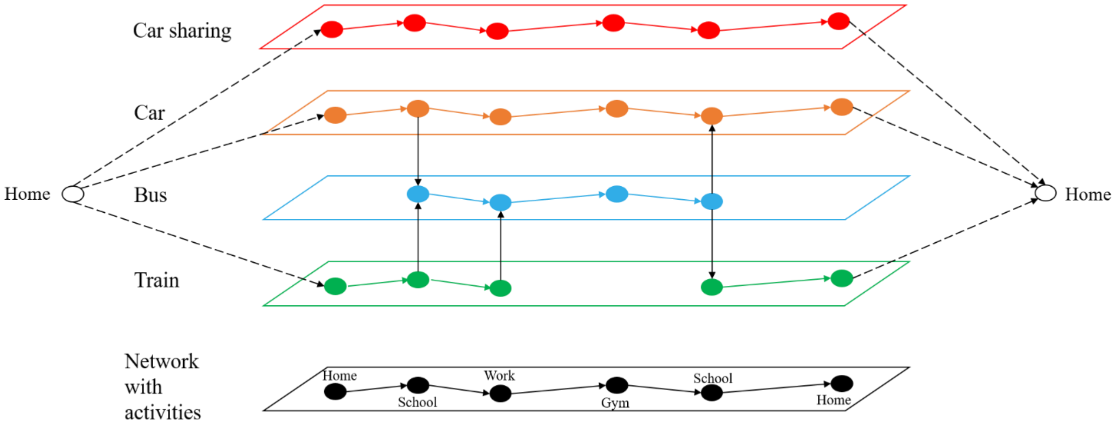

Figure 2: Multimodal supernetwork (adaptation from Carlier et al., 2003)

According to the network representation illustrated in Figure 2, each traveller must choose a path through the multimodal network in order to access their sequence of activity locations. A path (trip chain) can comprise three different types of links. Access links (black dashed lines) allow users to access a mode of transport from their origin (Home), and egress from a mode of transport to reach their final destination (Home). Mode-specific links (horizontal link) indicate trips made from one activity location to another using a specific mode of transport (designated by colour). Interchange links (vertical black link) allow users to move from one mode of transport to another. A modal link includes in turn three stages of a trip. In the first stage, the users leave the activity location and reach the mode of transport. Subsequently, the travellers use the specific mode of transport in order to reach the following destination. Finally, the users leave the mode of transport and arrive at the second activity location.
We assume that each MSP seeks to maximize the profit arising from their service. This profit is calculated considering that each MSP (j) owns a specific and uniquely defined layer of the super-network, through which they collect revenues based on how many travellers use the service and face costs that depend on how many vehicles are provided. In particular, each MSP (j) owns a fleet of vehicles (v^j), and as part of their choice, at equilibrium, they will strategically distribute these vehicles on the links (v_a) contained in their layer. The number of vehicles assigned at equilibrium to a specific link will be considered available only for that trip connection.
The lower level equilibrium decision variables are the vector of path flows ($\boldsymbol{x}$) ($\boldsymbol{f}$ for the vector of link flow).

Notation

In Table 1 we listed the general components of the notation that will not be explicitly introduced in the paper.

Table 1: General notation

Sets/Indices		**Parameters**	
J/j	Mobility Service Provider	δ	Incidence matrix
K/k	User class	c	Unitary monetary cost/Cost function
S/s	Mobility subscription	C	Total monetary cost
A/a	Arc	t	Time function
N/n	Node	d_w^k	demand of class k for w
W/w	*OD* pairs	y^j	relocation factor for j

P/p	Allowed paths in the network	l_a	length of link *a*
P_w	Allowed paths between w		
D	Total travel demand of the network		

The different parameters or functions present in the paper can contain the superscript k, when they are connected to a user class. When there is a superscript j, it means that they are connected to the mobility service owned by supplier j. Unitary costs, cost and time functions are associated to an aspect that will be directly specified in the text (e.g. leasing, access, egress, etc.). Finally, the different components are expressed in hours (h), kilometers (km), euro per hour (€/hour), euro per km (€/km), euro per day (€/day). The calculation is made considering an operative weekday (€/day), in which the distribution of the flow is spread during the entire day. Time and cost functions can depend on one of the decisional variables or both of them.

Mobility service providers' formulation

In this section, we introduce the formulation describing the MSPs behaviour in the form of a profit (G) maximization problem. Accordingly, at equilibrium, a MSP j will maximize their objective function as difference between revenues $R^j(\boldsymbol{x})$ and costs $C^j(v^j, \boldsymbol{x})$ derived from their mobility business, as described by Equation 1.

$$\max_{v^j} G^j(v^j, \boldsymbol{x}) = \max_{v^j} \sum R^j(\boldsymbol{x}) - \sum C^j(v^j, \boldsymbol{x}) \\ = \max_{v^j} FR^j(\boldsymbol{x}) + VR^j(\boldsymbol{x}) - FC^j(v^j) - VC^j(\boldsymbol{x}) \tag{1}$$

where the revenues can be split into fixed $FR^j(\boldsymbol{x})$ and variable $VR^j(\boldsymbol{x})$. Fixed revenues arise from users mobility package subscriptions:

$$FR^j(\boldsymbol{x}) = \sum_p \sum_{s^j} x_p \delta_{s^j p} c_s \tag{2}$$

Variable revenues depend on service patronage:

$$VR^j(\boldsymbol{x}) = \sum_p \sum_{a^j} (c_{a,h} t_{a,main}(\boldsymbol{f}) + c_{a,km} l_a + c_{a,ticket}) x_p \delta_{a^j p} \tag{3}$$

Costs in (1) comprise fixed $FC^j(v^j)$ and variable $VC^j(\boldsymbol{x})$. Fixed costs do not change with the number of customers served, but depend on fleet size:

$$FC^j(v^j) = c^j_{leasing}(v^j) \tag{4}$$

Here we equate fixed costs with leasing costs, including procurement of vehicles, parking spaces and stations. Meanwhile, variable costs depend on the number of users with costs relating to fuel (or electricity) consumed and relocation of vehicles at the end of the day:

$$VC^j(\boldsymbol{x}) = \sum_p \sum_{a^j} (c^j_{a,fuel} l_a \, x_p \delta_{a^j p})(1 + y^j) \tag{5}$$

Equation 1 is subject to the following constraints:

$$v^j = \sum_a v_a \, \delta_{aj} \quad \forall j \tag{6}$$

$$v_a \geq 0 \quad \forall a \tag{7}$$

where (6) indicates that the total number of vehicles of MSP *j* have to be equal to the sum of all vehicles located on the links owned, and (7) is constrains the number of vehicle in each link to be nonnegative.

The innovative aspect of this formulation consists on its possible application to calculate the profit for different mobility services, such as car-sharing, bike-sharing, bus, train, e-scooter and taxi. Due to its structure it can be naturally extended to an Equilibrium Problem with Equilibrium Constraints (EPEC), where at the upper level different MSPs compete and cooperate in order to have profit.

User classes' formulation

Each user class is assigned to the multimodal network following fixed demand traffic equilibrium, using a path-based adaptation of the multi-class and multicriteria network equilibrium model (Nagurney, 2000). Explicit enumeration of paths is used in this study.

The link flow and the path flow for class *k* are connected through the Equation (8):

$$f_a^k = \sum_{p \in P} x_p^k \, \delta_{ap} \tag{8}$$

Total flow on link *a* is:

$$f_a = \sum_k f_a^k \tag{9}$$

The class *k* demand on *OD w* must be:

$$d_w^k = \sum_{p \in P_w} x_p^k \tag{10}$$

Each mode-specific link is characterised by two components of unitary costs: real monetary costs faced by users in order to use the service associated to supplier *j*, and class-dependent perceived costs connected to access time, waiting time, congestion, etc. The latter costs are associated to flow and capacity dependent functions. In the separable case, these functions depend on the user flow (f_a) and/or the number of vehicles (v_a) on that specific link. While considering non-separable link costs, some components are function of the total flow ($\boldsymbol{f}$), the sum of the flows of all the modal links connecting the same activity locations. Hence users of different classes and different modes can influence each other on that link.

Based on the mode-specific link structure described in Section 2, the total cost on a generic link *a* associated to a class *k* can be expressed as:

$$C_a^k(\boldsymbol{f}, v_a) = C_{a,access}^k(f_a, v_a) + C_{a,main}^k(\boldsymbol{f}\,, v_a) + C_{a,egress}^k(f_a, v_a) \tag{11}$$

where the first term represents the access cost:

$$C_{a,access}^k(f_a, v_a) = c_{a,access}^k \, t_{a,access}(f_a\,, v_a) + c_{a,wait}^k \, t_{a,wait}(f_a\,, v_a) \tag{12}$$

The second term is the cost on the main mode of transport:

$$C_{a,main}^k(\boldsymbol{f}\,, v_a) = c_{a,fuel} \, l_a + c_{a,main}^k \, t_{a,main}(\boldsymbol{f}) + c_{a,h} \, t_{a,main}(\boldsymbol{f}) + c_{a,km} \, l_a + c_{a,ticket} \tag{13}$$

The third term is the egress cost:

$$C_{a,egress}^k(f_a, v_a) = c_{a,park}^k \, t_{a,park}(f_a\,, v_a) + c_{a,egress}^k \, t_{a,egress}(f_a\,, v_a) \tag{14}$$

When access links and transfer links connect to mobility services characterised by a fixed subscription, the cost perceived by users is equal to a constant value (c_s) ; otherwise, the cost on that link is equal to zero.
In this context, the supernetwork is built with link-additive costs; therefore, the path cost on each path of the network equals the sum of its constituent link costs. The path cost for class k can be written as:

$$C_p^k(\boldsymbol{f},\boldsymbol{v}) = \sum_s c_s \delta_{sp} + \sum_a C_a^k(\boldsymbol{f}, v_a)\delta_{ap} \tag{15}$$

For each class, for all *OD* pairs and for all paths, the path flow x^* is said to be in equilibrium if the conditions (16) holds:

$$\boldsymbol{C_p^k}(\boldsymbol{f}^*,\boldsymbol{v})\begin{cases} = \rho_w^k & x_p{}^* > 0 \\ \geq \rho_w^k & x_p{}^* = 0 \end{cases} \tag{16}$$

with

$$x_p \geq 0 \ \forall p \tag{17}$$

where (18) is the path flow nonnegativity constraint.
One aspect that increases the complexity of the problem is the interdependency between flows on parallel links of the supernetwork that represent the same real transport link of the underlying infrastructure network. Consequently, the link costs are non-separable. The users' equilibrium is therefore formulated as a VI, as follows:

$$\sum_{k\in K}\sum_{w\in W}\sum_{p\in P_w} C_p^k(\boldsymbol{f}^*,\boldsymbol{v}) \ x\,(x_p - x_p{}^*) \geq 0 \tag{18}$$

which can be solved using an Extragradient method (Nagurney, 1999).

3. EXAMPLE

In this section, we show the application of the described methodology solving the users' equilibrium at the lower level for two classes of users. The upper level equilibrium for different MSPs defined in our formulation it is illustrated below, but not explicitly solved it in this paper.
In this example, the first class of users performs the daily tour Home-Work-Home with three modes available: private car, bus and one-way car-sharing service, while the second performs the tour Home-Work-Leisure-Home with bus or private car (Figure 3).
Congestion effects will affect users that decide to choose a mode of transport between car and carsharing, due to the fact that they share the same link on the base network. Bus, instead, is considered to have a dedicated line in the same connection. In this case, the attractiveness of the service will mainly decrease based on the waiting time at the bus stop.
In Table 2 are listed all the parameters used to calculate the equilibrium of the multimodal supernetwork. The elements that are not listed were not considered relevant for the specific mobility services taken into account. We considered that the two classes perceived the link costs on the same way, with the exception of Home-Work links for class 2 where the cost to access and egress are lower than class 1. The link costs functions can take the form of the conventional Bureau of Public Roads (BPR) function (Equation 19) or they can be constant. The parameters used are listed in Table 3.

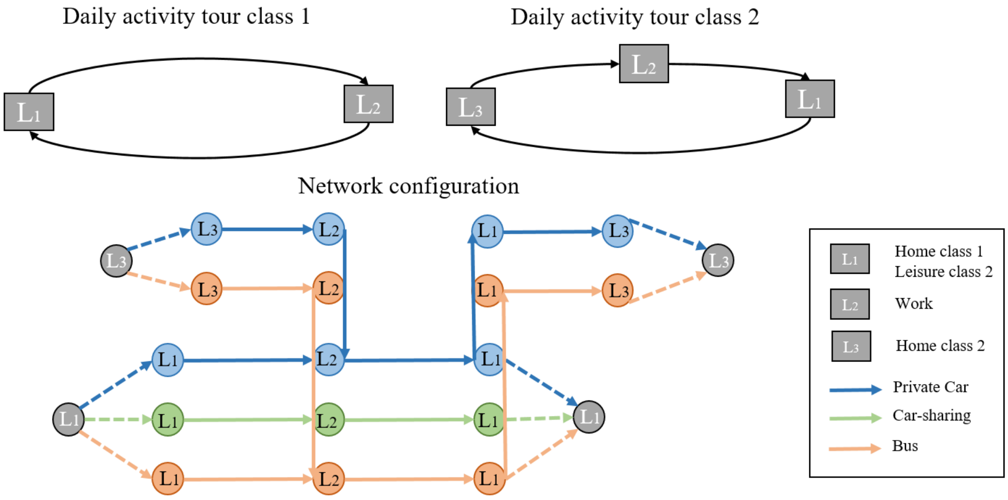

Figure 3: Example 2 for user class 1 and 2

Table 2: Parameters

Parameters	**Private car (j=1)**	**Car-sharing (j=2)**	**Bus (j=3)**
d_1^1	200		
d_2^2	100		
c_s	-	2	0.5
$c_{a,fuel}^j$	-	0.08	-
$c_{a,access}^k / c_{a,egress}^k$	8	8.5	10
$c_{a,access}^2 / c_{a,egress}^2$	8	8.5	8.8 (Home-Work)
$c_{a,wait}^k$	-	8	10
$c_{a,main}^k$	8	8	10
$c_{a,fuel}$	0.36	-	-
$c_{a,h}$	-	0.5	-
$c_{a,km}$	-	0.5	-
l_a	10	10	10
$c_{a,ticket}$	-	-	-
$c_{a,park}^k$	13	-	-

$$t = t_0 + \alpha \left(1 + \frac{f}{C}\right)^{\beta} \tag{19}$$

Table 3: Functions parameters

Function	Private car					Car-sharing					Bus				
	t_0	α	β	C	f	t_0	α	β	C	f	t_0	α	β	C	f
$t_{a,access}(f_a, v_a)$	-					0.0125	0.15	4	v_a	f_a	0.0625	-			
$t_{a,wait}(f_a, v_a)$	-					0.05	0.2	4	v_a	f_a	0.3	0.15	2	200	f_a
$t_{a,main}(f)$	0.2	2	4	200	f	0.2	2	4	200	f	0.3	-			
$t_{a,park}(f_a, v_a)$	0.1	4	4		f_a	-					-				
$t_{a,egress}(f_a, v_a)$	0.075	-				0.0125	0.15	4	v_a	f_a	0.0625	-			
$c^j_{leasing}(v^j)$	-					$3v^{j=2}$					$0.05v^{j=3}$				

In the example, we calculate the profit of the bus service (Figure 4, left side, bold line) and of the carsharing supplier (Figure 4, right side, bold line). We additionally illustrate how the profit of a MSP can change when changing the strategies of the competitors (Figure 4, dotted lines). In both cases we increased of one unit the capacity at each iteration. In solving user equilibrium, the extragradient algorithm was deemed to have converged when the standard path cost gap function arrived below 1E-4.

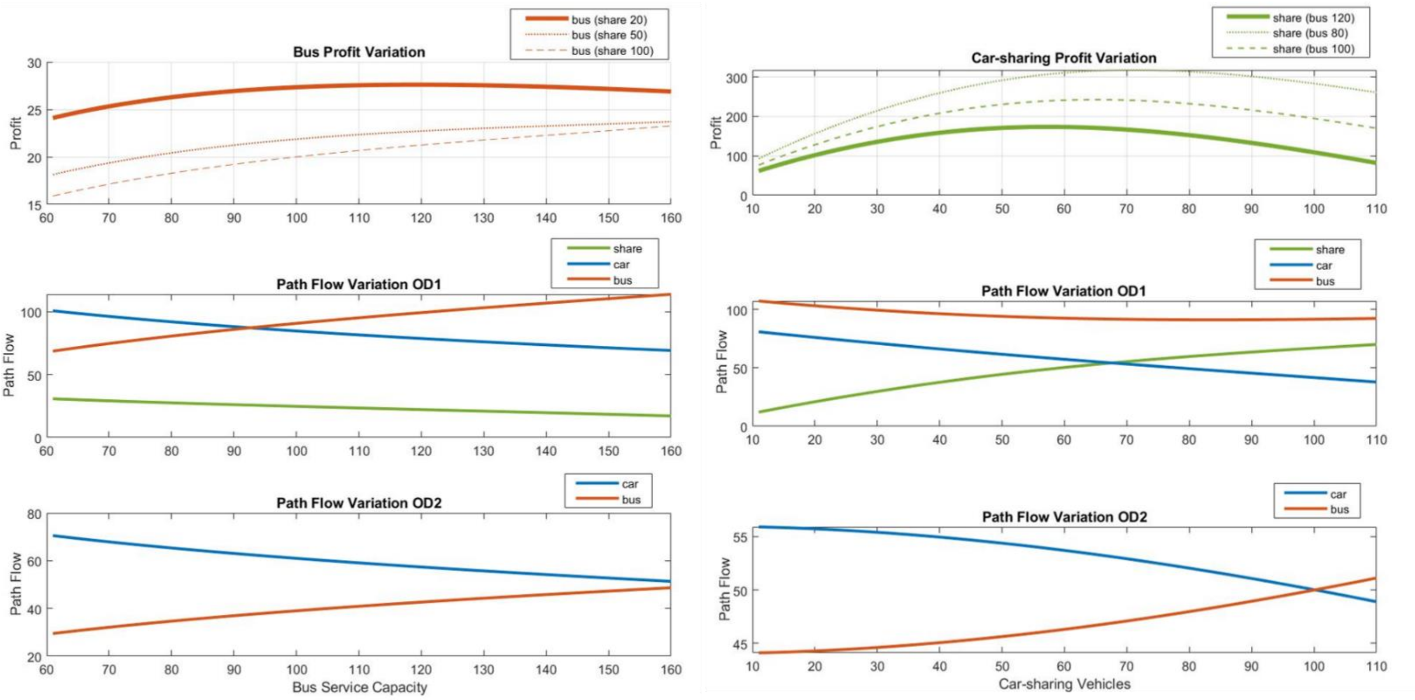

Figure 4: Bus (left) and Car-sharing (right) profit maximization

In Figure 4 the path flows for the two OD are illustrated, on the left side it is possible to see how increasing the capacity (frequency) of the service users are moving from the private car to the bus, while the car-sharing service with a constant number of vehicles works almost constantly at capacity. The scenario in which the car-sharing company is increasing the number of vehicles available (right side) shows that the flow on car and car-sharing are fully correlated, and it is clear that bus takes all the exceeding demand.

4. CONCLUSIONS

The interaction between different MSPs and multiclass users is formulated as an MPEC, where profit maximization provides the MSP objective functions and equilibrium conditions are written as a VI. The use of the VI in this type of problem helps to consider congestion effects that occur when different modes of transport interact. To present the properties of the model, a numerical example is solved in which a car-sharing provider and a bus company optimize the capacity of their service while competing with private car usage. The results clearly show the relationship between car and car-sharing, due to the BPR travel time function that considers these modes of transport sharing the same road. What it is also clear is that increasing the frequency of the bus service increases its attractiveness, as a consequence of the reduction in waiting time.
Our future work will extend this model in several ways: (i) the use of more complex functions combined to calibrated parameters will help to better understand the properties and the relationship between different actors. (ii) The variation of more decisional variables in the process, to better optimize the supplier's profit. (iii) Finally, a full study on the upper level cooperation and competition between different MSPs.

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