

# An optimized driver repositioning strategy in ridesplitting with earning estimates: a two-layer dynamic model and control

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## SHORT SUMMARY

Positioning of ride-sourcing drivers may improve vacant travel times, waiting times and matching opportunities. Herein, we develop a ride-sourcing fare optimizer which, in parallel with a revenue estimator, aids drivers' repositioning decisions to minimize customer abandonments in a system offering ride-hailing (solo) and ridesplitting (shared) rides. A Markov chain estimates near-future individual revenues and forms a second layer in a MFD-based model to predict system conditions. We applied the proposed model in a simulation of the central business district of Shenzhen, China. Our results show that repositioning with fare control can decrease the number of abandonments by 98%. In the other hand, controlling fares decreased travelling speeds in the busiest periods of each area of the system, without entering a hyper-congested regime. These findings expand the literature on fare optimization including ridesplitting operations and providing a tool to estimate near-future earnings.

**Keywords:** Ride-sourcing, Shared mobility, Repositioning, Model Predictive Control (MPC), Traffic Flow Theory.

## 1. INTRODUCTION

Every day ride-sourcing services grow and make more service options available at the same time that regulators try to identify and mitigate the negative externalities of these operations to the society (Rayle, Dai, Chan, Cervero, & Shaheen, 2016; Beojone & Geroliminis, 2021b). In a daily basis, geographical variations on demand can create an imbalance between service demand and driver supply to serve it. In this direction, having a strategy that keeps drivers well positioned is of vital importance to maintain a satisfactory service quality. However, drivers in ride-sourcing services are autonomous to a series of decisions, specially for this case, including where they want to wait for new assignments.

Therefore, relocating driver requires a direct influence over their own decisions. Lu, Frazier, and Kislev (2018) and Sadeghi and Smith (2019) did not force drivers to relocate but rather incentivized drivers to decide on the best location for the next assignment through mechanisms that control the supply of drivers and the demand of passengers, such as surge pricing (Castillo, Knoepfle, & Weyl, 2018). Other strategies of repositioning include Alonso-Mora, Samaranayake, Wallar, Frazzoli, and Rus (2017), Wallar, Van Der Zee, Alonso-Mora, and Rus (2018), Simonetto, Monteil, and Gambella (2019), and Liu and Samaranayake (2020) who sent empty vehicles to the location of recently unsatisfied customers. The issue with these strategies is their reactive nature, in the sense that they mainly accounted for past events (i.e., lost demand) or current conditions to make the decisions. For instance, if an area faces recurrent losses of requests, customers will likely change their travel option to a more reliable transportation mode.

Hence, the forecast of near-future conditions is of vital importance in the development of proactive repositioning strategies. MFD-based models can describe the dynamics of state evolution in urban networks partitioned in a number of homogeneously congested regions. One of its advantages is that it does not require information about the exact route of individuals. Since the first work on MFD-based perimeter control for single region (Daganzo, 2007), many control methods have been developed, such as perimeter control, congestion pricing, route guidance or a combination of these. Extending the formulation to include ride-sourcing services and drivers' repositioning behavior can create a powerful tool for network-level control in areas with this mode of transportation, allowing for near-future predictions.

Ramezani and Nourinejad (2018) used an MFD-based model to develop a relocation strategy of taxi vehicles. Nourinejad and Ramezani (2020) further developed the problem to handle pricing schemes in ride-hailing services where drivers and riders can decide whether to join or leave the service. Beojone and Geroliminis (2021a) used a multi-region formulation to develop an accurate dynamic model with ride-sourcing services allowing riders to choose among ride-hailing and ridesplitting service options. However, the studies still lack the effects of ridesplitting on repositioning strategies and the advantages of a non-accumulation-based model in forecasting the system's evolution coupled with precise revenue estimation.

Herein, we develop relocation strategies for ride-sourcing vehicles using a fare optimization controller which also provides drivers with an estimate for their earnings for their repositioning decisions. Drivers base these decisions on their earning expectations for two possibilities, remaining idle in the current region or repositioning to a neighboring region. The operator uses a continuous time Markov chain to estimate earnings for a given decision in the short-term. We also extend the model of Beojone and Geroliminis (2021a) to cope with the dynamics of repositioning idle ride-sourcing vehicles. To the best of our knowledge, this is the first attempt to integrate fare optimization and proactive repositioning in a scenario with ridesplitting and revenue estimation.

This short paper is structured as follows. Section 2 briefly presents the MFD-based model, based on Beojone and Geroliminis (2021a), including the Markov chain for earning estimates. Later, Section 3 depicts the controller interface used to optimize the fares based on a Model Predictive Controller (MPC). Section 4 shows a summary of the computational results for the proposed problem in a prototype application. Finally, Section 5 presents some final considerations with a small discussion on the previous findings and also further research directions.

## 2. MODEL DESCRIPTION

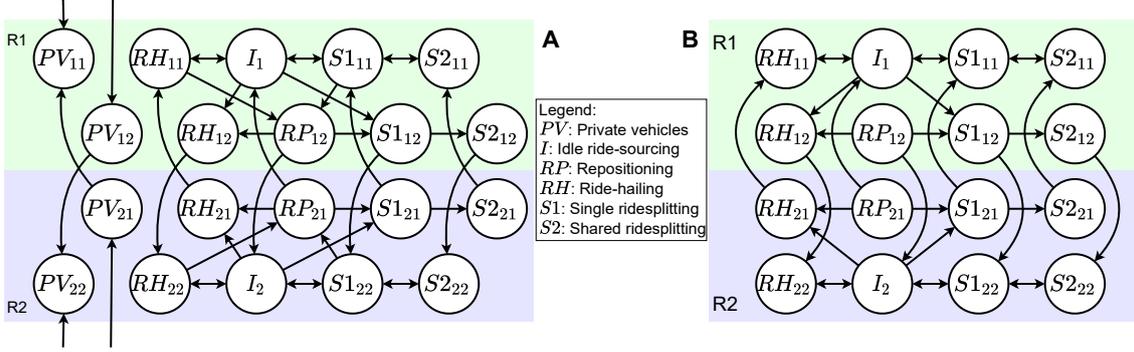
The list of states in Table 1 is derived from the list of activities. Note that pick-up and drop-off activities were aggregated to form a single state in all ride-sourcing services. Vehicles in states *RH* and *S2* are assumed completely busy and cannot receive new assignments. At the same time, vehicles in states *I* and *S1* are considered available for new assignments (*S1* only for ridesplitting assignments). For simplicity, we assume that private vehicles actions besides driving are negligible and that unserved passengers of ride-sourcing will use private vehicles.

**Table 1: List of states based on the activities vehicles perform.**

Activity	State	Accumulation	Activity	State	Accumulation
Idle	<i>I</i>	$n_o^I(t)$	Single ridesplitting	<i>S1</i>	$n_{od}^{S1}(t)$
Repositioning	<i>RP</i>	$n_{od}^{RP}(t)$	Shared ridesplitting	<i>S2</i>	$n_{od}^{S2}(t)$
Ride-hailing	<i>RH</i>	$n_{od}^{RH}(t)$	Private vehicle	<i>PV</i>	$n_{od}^{PV}(t)$

To summarize the description states and activities of ride-sourcing and private vehicles, Figures

1A and 1B present the state space with their respective transitions in the MFD-based model and the Markov Chain, respectively. Note that in Figure 1A shows states of vehicles (including private vehicles), whereas Figure 1B depicts states of individual ride-sourcing drivers. The model assumes that the TNC provides two service options that idle drivers can serve: 1) ride-hailing with higher fares to ride alone; and 2) ridesplitting, accepts some detour and longer travel times in exchange for cheaper fares. Another important assumption is that ridesplitting is limited to serve two passengers simultaneously per vehicle, which can comply with most of the current data on ridesplitting. Pricing strategies and effects of surge pricing on these services (on drivers and riders) are outside the scope the paper, and directions for further research.



**Figure 1: (A) Two-region state-space for ride-sourcing vehicles and private vehicles with their schematic state transitions in the MFD-based model. (B) Individual ride-sourcing driver state-space and transitions in the Markov chain model.**

The urban area is composed of a set  $\mathcal{R} = \{1, 2\}$  of regions. Mass conservation equations [1] keep track of the number of vehicles in each state and their remaining distance to be traveled (Sirmatel, Tsitsokas, Kouvelas, & Geroliminis, 2021). The main difference is for idle ride-sourcing vehicles, which have no trips to complete but they switch from one state to another. Instead, they cruise waiting for the assignment of a new passenger, thus, there is no need to model the remaining distance, only the number of vehicles.

$$\dot{n}_{od}^K(t) = I_{od}^K(t) - O_{od}^K(t) \quad (1a)$$

$$M_{od}^K(t) = I_{od}^K(t)L_{od}^K(t) - n_{od}^K(t)v_o(t) \quad (1b)$$

Where  $n_{od}^K(t)$  and  $M_{od}^K(t)$  are the number of drivers and the total remaining distance to be travelled for drivers at an activity  $K$  at area  $o$  to area  $d$ , respectively. Furthermore,  $I_{od}^K(t)$  and  $O_{od}^K(t)$  represent the total inflows and outflows for these drivers, respectively.  $L_{od}^K(t)$ , in the other hand, is their average trip length and  $v_o(t)$  is the current average speed at that area.

Given the large frequency of events (passenger arrivals, deliveries, transfer flows) one can approximate the revenue generation for the company by means of a continuous rate (instead of a service by service basis). Furthermore, if the regions are reasonable uniform, we can assume that drivers split earnings equally. Therefore, drivers' earning generation depends on the activity they are performing and booking and distance fares.

We assume that drivers try to maximize their earnings but they are unable to accurately estimate earnings themselves (due to limited information and rationality). To help drivers, the service provider uses a continuous-time Markov chain to depict near-future activities and earnings of individual drivers. Figure 1B depicts the transitions on the Markov chain. Note that this Markov chain is depicted on an individual level, i.e., arrival rates, trip-completion and transfer flows are

normalized based on current information of the MFD model. Moreover, it assumes that drivers will only make this decision the moment they are about to become idle, therefore, there is no transition from idle to repositioning state because the probability making this decision again in the near-future is negligible.

$$\dot{\pi}_{od}^K(t) = -\tilde{O}_{od}^K(t)\pi_{od}^K(t) + \sum_k \tilde{I}_{od}^k(t)\pi_{od}^k(t) \quad (2)$$

Where  $\pi_{od}^K(t)$  is the instantaneous probability of a driver being in state  $K_{od}$ ; and  $\tilde{O}_{od}^K$  and  $\sum_k \tilde{I}_{od}^k \pi_{od}^k(t)$  are the normalized outflows and sum of inflows towards state  $K_{od}$ . Note that, the starting solution of the Markov chain represents one of the driver's possible decisions. Therefore, the resulting probabilities are used to estimate drivers' expected earnings for a given decision. A logit model computes the ratio of drivers choosing each option, which are then translated back into the MFD-model.

### 3. CONTROL INTERFACE

In this prototype application, we consider that the main objective in the process of optimizing fares is to decrease the number of lost ride requests for ride-sourcing services (called abandonments in the remaining of the paper). Moreover, the responsible for setting fares (and supplying drivers with estimates on their near-future earnings) is the service provider, which can control the fares concerning booking a ride and the ride distance for both service options independently. However, the fares are homogeneous across different regions, i.e., fares are independent of the regions and unique for the entire simulated area. To approximate the number of unserved requests, we must solve an integral of the instantaneous rate that the system loses customers. The design of the fare optimization problem of the ride-sourcing service, subject to the dynamics introduced in Section 2 and the constraints for fares is formulated in Equation [3].

$$\min_{f_B^s(t), f_T^s(t)} J = \int_{t_0}^{t_f} \sum_{s \in \mathcal{S}} \sum_{(o,d) \in \mathcal{R}^2} p_{od}^s(v_o(t), n_{av}^s(t)) \lambda_{od}^s(t) dt \quad n_{av}^s(t) = \sum_{k_s \in \mathcal{K}_s} n_{od}^{k_s}(t) \quad (3a)$$

$$\text{s.t.: Equation [1]} \quad t \in [t_0, t_f] \quad (3b)$$

$$f_{B,\min}^s \leq f_B^s(t) \leq f_{B,\max}^s \quad t \in [t_0, t_f] \text{ and } s \in \mathcal{S} \quad (3c)$$

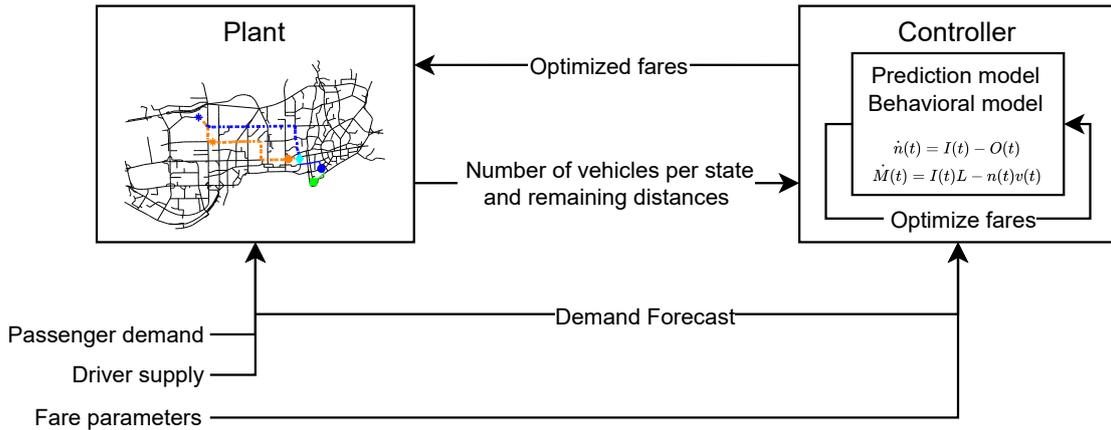
$$f_{T,\min}^s \leq f_T^s(t) \leq f_{T,\max}^s \quad t \in [t_0, t_f] \text{ and } s \in \mathcal{S} \quad (3d)$$

$$v(t), n_{od}^K(t), \dots \geq 0 \quad t \in [t_0, t_f] \quad (3e)$$

In the objective function (Equation [3a]),  $p_{od}^s(t)\lambda_{od}^s(t)$  represents the fraction of arriving riders that will not find a vehicle capable of serving them (i.e., will not enter the system, which is the complement of entering rates  $\tilde{\lambda}_{od}^s(t)$ ). The main constraint is shown in Equation [3b] that summarizes the mass conservation equations that describe the dynamic model of Section 2. Equations [3c] and [3d] constraint the values of fares related to the request booking and the trip distance. Finally, Equation [3e] guarantees that the states of the model remain non-negative. Note that we assume a given initial number of ride-sourcing vehicles and their activities in each region. Moreover, we assume that there is some knowledge of the incoming demand, in the form of arriving rates, and that the model captures drivers' attitude towards repositioning. For the sake of simplicity, in this prototype implementation, we neglect the market effects on customers and drivers decisions for joining or leaving the system. This is a direction for further research.

We prepared a rolling time horizon MPC controller seen in Figure 2. At each time step of 3

minutes (0.05 hours), the controller tries to minimize the abandonments to derive optimized fare inputs to the plant (control outputs) for the next time step. In order to make reasonable use of computational resources, the optimization procedure only considers the first 10 time steps (not the entire experiment). The feedback loop from the plant to the prediction model provides an estimation of system states, including ride-sourcing and private vehicles numbers and remaining distances. At the same time, the controller consists of a prediction model, i.e. Eq. [1] and an optimization tool that minimizes Problem [3]. The optimization output is the applied fares that ride-sourcing drivers will receive for ride-hailing and ridesplitting services, which is applied to the plant for one interval. Note that the MPC framework considers a demand prediction module that is the input to the prediction model. For the sake of simplicity, we assume that the demand prediction module is based on historical data and supplies the model with the instantaneous travellers arrival rates (assuming a Poisson arrival process).

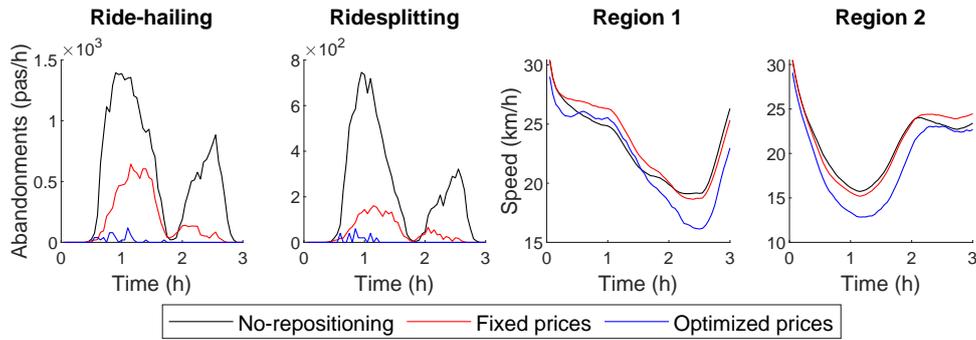


**Figure 2: Plant-Controller interface for fare optimization to reduce abandonments.**

The analysis focused on three possible strategies. The first one does not allow drivers to reposition when idle. On the second one, drivers were provided with an estimate of their earnings but prices were fixed. Finally, the last one provides optimized fares and estimates computed using the MPC controller.

#### 4. COMPUTATION RESULTS

The observation of abandonment rates shows how repositioning has potential to decrease the number of lost requests. Fixing fares and allowing drivers to move between regions according to their expectations on revenues was capable to decrease the number of lost requests by more than 70%. However, optimizing fares dynamically, brought abandonments to a near-zero situation, decreasing by 93% the number of abandonments with a fixed price scenario (98% compared to the non-repositioning scenario). It is interesting to note that it was not necessary to have different fares for each region of the system, only distinct fares for each service option, to achieve a near-zero abandonments case. However, the impacts of these decisions on traffic are secondary to the problem as shown in Figure 3, where the optimization of fares decreased the travelling speeds, in general. Furthermore, the greatest decreases in speed were in the most congested periods of each region. It indicates that such a policy (focused on minimizing abandonments) takes drivers to a region before a shortage of drivers appears there. We must point out that, in all cases, travelling speeds remained above the critical speeds, i.e., the system never entered a hyper-congested state.



**Figure 3: Comparison of realized abandonments and average travelling speed among three scenarios for (left) ride-hailing and (right) ridesplitting).**

## 5. FINAL CONSIDERATIONS

In this paper we developed relocation strategies for ride-sourcing vehicles using a fare optimization controller which also provides drivers with an estimate for their earnings for their repositioning decisions. Our main results show that repositioning could brought passenger abandonments to a near-zero scenario at the expense of decreasing travelling speeds, especially at the moments of higher demand.

Such findings are expected in a problem with a single objective (minimize abandonments). We must highlight that, in this case, the decrease in abandonments is a lot more pronounced than the decrease in speeds, which did not enter a hyper-congested state. More evidence is needed but these findings provide a likely path for testing the impacts of different regulatory strategies under a transient situation. Nevertheless, this work contributes by expanding the test of fare optimization to ride-sourcing services with the presence of ridesplitting option and providing a powerful tool to estimate near-future earnings through a Markov chain, given the additional options drivers and passengers have in these travel modes.

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