The sensitivity of transportation networks criticality to demand variation, level of supply degradation, and network abstraction

Chrysostomos Mylonas*[1], Konstantinos Kepaptsoglou[2], Evangelos Mitsakis[3]

[1] PhD Candidate, School of Rural and Surveying Engineering, National Technical University of Athens, Greece
[2] Assistant Professor, School of Rural and Surveying Engineering, National Technical University of Athens, Greece
[3] Associate Researcher, Hellenic Institute of Transport, Centre for Research and Technology Hellas, Greece

SHORT SUMMARY

Criticality assessment constitutes a major step towards the vulnerability analysis of transportation networks, enabling the identification of crucial infrastructure elements the unavailability of which provokes significant impacts. This paper aims to investigate the extent to which criticality in transportation networks is sensitive to several factors, including demand variation, level of capacity degradation, and network abstraction. A case study takes place in Sioux Falls, assuming that road users follow the user equilibrium routing behavior. Results showcase that criticality is sensitive to all mentioned factors, highlighting the need for transportation planning authorities to incorporate these factors in the preparation of adaptation plans to unexpected events and the enhancement of road networks robustness.

Keywords: Criticality, demand variation, capacity degradation, network abstraction

1. INTRODUCTION

Transportation networks constitute fundamental components of contemporary societies given their significance for the movement of people and goods. Therefore, their disruption caused by natural and man-made disasters, accidents, or public works provokes several impacts to human mobility and supply chains. Due to the necessity for handling such disruptions within transportation networks an entire class of studies has focused on the analysis of concepts, including vulnerability, robustness, and criticality. The first two concepts focus, in line with the most cited definition of the former (Berdica, 2002), on the assessment of the susceptibility (or resistance) of a transportation network to incidents that can considerably reduce its serviceability. A typical metric for estimating serviceability loss is the increase in the generalized travel cost. The main objective of the concept of criticality, is to identify those components (i.e., nodes or links) of a transportation network the unavailability of which mostly reduces its serviceability. This is achieved by calculating the difference or relevant difference in the generalized travel cost considering a transportation network in its original state and various instances of the same network in which one-by-one each of its components is successively removed. Subsequently, these components are ranked by assessing the impact of their removal from the network. Such an approach is termed as "full-scan approach" (Chen et al., 2012). However, several variations of the framework presented above have been suggested to quantify criticality in transportation networks. A useful measure for the assessment of criticality in transportation networks has been proposed by Nagurney and Qiang (2012). This measure termed as "Unified Network Efficiency" captures simultaneously flows, travelers' behavior, and travel costs. It is mathematically defined as follows:

$$E(G,d)=\frac{\sum_{w\in W}\frac{d_w}{\lambda_w}}{n_W} \quad (1)$$

where G denotes the topology of the network, d a demand vector, d_w the demand corresponding to OD pair w, λ_w the disutility of OD pair w, and n_W the number of OD pairs in the network. Accordingly, the importance of each component is ranked through the following metric:

$$I(g)=\frac{E(G,d)-E(G-g,d)}{E(G,d)} \quad (2)$$

A powerful feature of the measure proposed by Nagurney and Qiang (2012) is its independence from network connectivity loss due to the removal of a component g. This is enabled by the possibility to assign the demand corresponding to a disconnected OD pair to an abstract path at a cost of infinity. Therefore, the need for utilizing a different measure for capturing cases of unsatisfied demand becomes unnecessary. A systematic overview of the frameworks suggested in the relevant literature for assessing criticality fairly exceeds the scope of the current paper. Its main objective is to assess whether and how the results of criticality assessment are influenced by several factors, reflected by the structural members of Equation 1. The first factor constitutes demand variation. Higher values of travel demand may be the case when a transportation network is used for evacuation operations, while lower values of travel demand may be the case when a transportation network is affected by extreme weather events during which a considerable number of travelers may postpone or cancel their planned trips. The second factor constitutes the level of capacity degradation, i.e., whether the components of a network become partially or completely unavailable. The former most closely resembles cases in which a component of transportation network is affected by localized incidents, e.g., accidents or roadworks. On the other hand, the latter most closely resembles cases in which a component of a transportation network is affected by widespread events, e.g., natural disasters. The last factor is stemming from the fact that criticality assessment is executed through traffic assignment models. In such models, the level of network abstraction is decided upfront having in mind road hierarchy and may lead to the inclusion or exclusion from the analysis of secondary arterials or local roads. In this respect, the extent to which criticality results is affected is not readily assessable. It should be noted that the computational framework for estimating travel disutility is another influential factor. However, this factor has been already assessed by Nagurney and Qiang (2012), who compare the difference in criticality when travel times are estimated assuming user equilibrium and system optimum routing behaviors. Such an analysis can be extended to also include additional frameworks, such as static traffic assignment based on stochastic user equilibrium routing behavior or dynamic traffic assignment, feeding the scope of future research.

2. METHODOLOGY

The current paper assess criticality within Sioux Falls test network (Figure 1), based on the framework suggested by Nagurney and Qiang (2012). Travel disutility is limited to travel time, which is estimated based on static traffic assignment assuming that road users are routed based on the user equilibrium behavior. Link travel times are calculated through the function suggested by the Bureau of Public Roads:

$$t_a(x_a) = t_a^f + 0.15(\frac{x_a}{C_a})^4 \tag{3}$$

where t_a^f denotes the free flow travel time of a link a, x_a the flow passing through link a, and C_a the capacity of link a. Calculations are executed by using the Frank-Wolfe algorithm implemented in AequilibraE (v.0.6.2) Python package for transportation modeling. The maximum number of iterations was set to 1000 and the relative convergence gap was set to of 10^{-3}. Disruptions are modeled by assigning a very high value to the free flow time of the corresponding links.

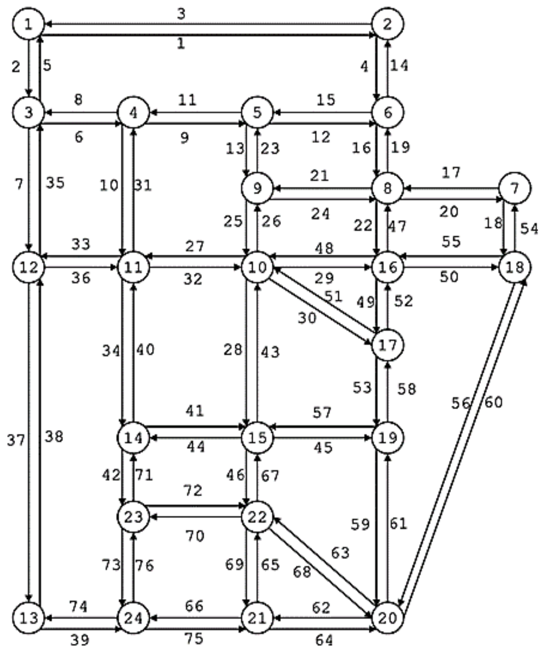

Figure 1: Sioux Falls

Demand variations are modelled by repeating the analysis for 15 Origin Destination (OD) tables, including the original scaled from 80% to 120% with a 10% step and ten more tables including different demand profiles also from 80% to 120%. Different demand profiles are constructed by keeping the total vehicle trips constant (360.000 trips) and randomly generating the number of vehicle trips corresponding to each OD pair. Therefore, different flow patterns are introduced, resembling unexpected circumstances, such as emergencies and evacuations. Moreover, different levels of capacity degradation are modelled by repeating the analysis without assigning a very high value to the free flow time of the network's links but through the successive decrease of their capacity by 25%, 50%, 75%, and 100%. Finally, with the aim of modelling the effect of excluding secondary arterials and local roads from the analysis, several bidirectional links connecting existing nodes and defining new ones at their intersections are added in the network. This addition (Figure 2) is based on the map of the actual road network that is made available by the Transportation Networks for Research Core Team (2019). A low value is assigned to the capacity of all additional links, i.e., 2000 vehicles/hour. The actual length of added links is utilized as an estimator of their free flow time. The following values are utilized:

- 10 (0.01 hours) when *length>1 km*
- 15 (0.01 hours) when *1 km<length<2 km*
- 20 (0.01 hours) when *2 km<length<3 km*
- 25 (0.01 hours) when *3 km<length<4 km*

- 30 (0.01 hours) when $4\ km < length < 5\ km$

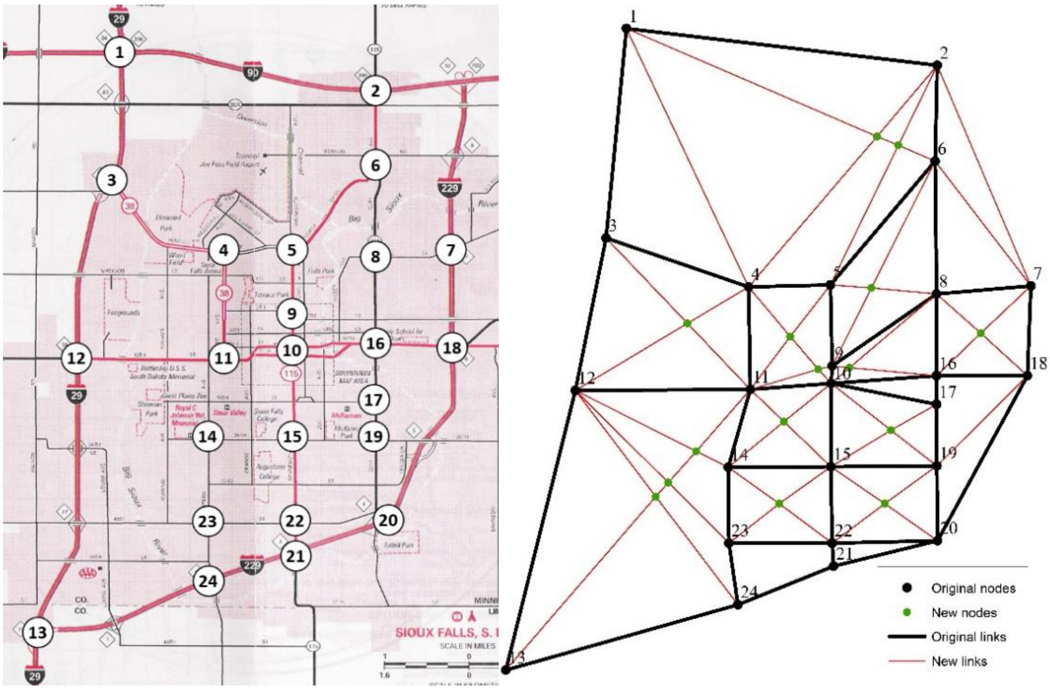

Figure 2: Map of the actual road network (left) and augmented network (right)

For the above cases, specific diagrams are presented indicating both the variability of the minimum, average, and maximum value of link importance as well as the variability of the ranking of links that are considered of low, medium, and high importance without considering the effect of any of the factors discussed in Section 1 (hereafter referred to as "initial analysis"). Correlation indices are reported to investigate the linear relationship between parameters of interest. The Pearson's and Spearman's correlation index is used for evaluating the relationship between continuous and ordinal variables, respectively.

3. RESULTS AND DISCUSSION

Criticality assessment

The results of the initial analysis are presented in Figure 3. The unified efficiency of the network is found to be $E(G,d)=45.568$. Moreover, the most important links are links 56, 60, 37, and 38, while the less important links are links 4, 14, 3, and 1. By comparing these results with link flows (Figure 4) it seems that highly important links serve varying amounts of vehicle traffic. However, as the value of the Pearson's correlation index suggests ($r_{importance,flow}=0.768$), there is a moderate correlation between these two parameters. Deviations can be translated through the conceptual framework suggested by Jenelius (2010), involving network redundancy. According to this framework, a link is critical not only when it serves a considerable amount of traffic but also when it does not possess efficient rerouting alternatives. This explains, for instance, why links 37 and 38 are of high importance, while not serving a high amount of traffic.

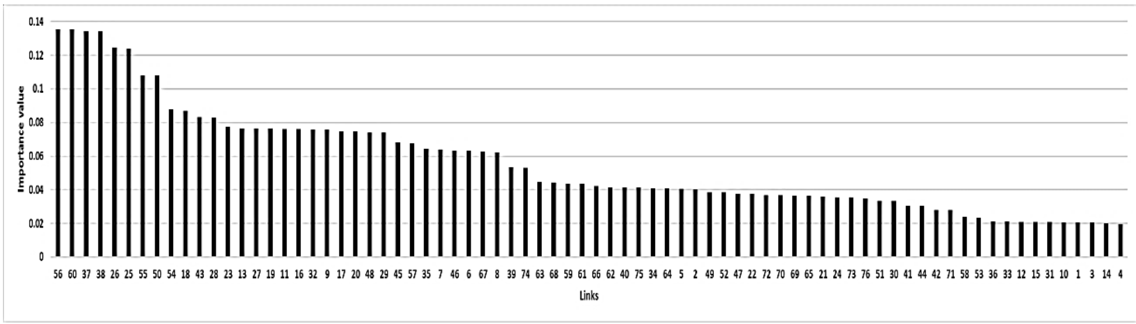

Figure 3: Link importance values and ranking

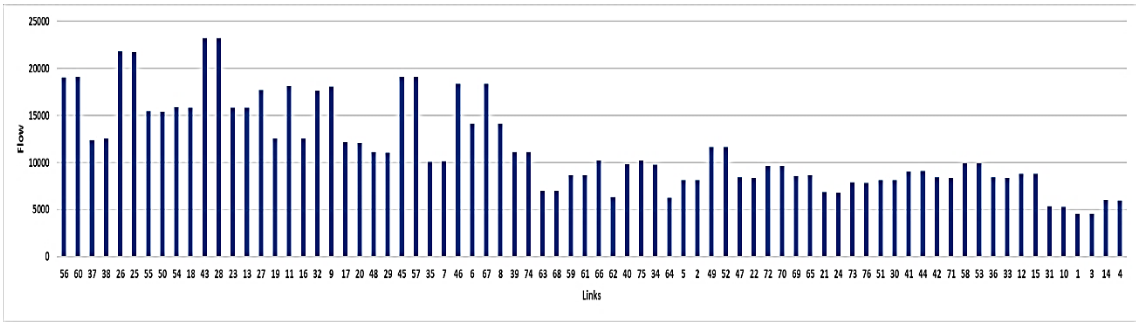

Figure 4: Link flows

Effect of demand variation

The effect of demand variation to the minimum, average, and maximum values of link importance is depicted in Figure 5. The average and minimum link importance values appear to be less sensitive. This is not the case for the maximum value of link importance, which seem to be moderately sensitive to the variation of the total number of trips and highly sensitive to the analyzed demand profile.

However, the results included in Figure 5 do not provide enough information for the sensitivity of the link rankings, considering that it is unknown to which links the presented values belong. For this reason, in Figures 6-7 the variation of the ranking of two lowly, moderately, and highly ranked links according to the results of the initial analysis are presented. While the maximum importance value appears to be highly sensitive to demand variation, this is not the case for the rankings of the most important links, which appear to be stable against both the increase and decrease of the total number of trips and the emergence of new flow patterns. Similarly, while the minimum and average importance values appear to be low sensitive, the opposite holds true for the rankings of low and medium important links. Moreover, Figures 6-7 reveal that there is tendency for low important links to get more important with the increase of the demand factor. With the aim of assessing whether the above findings can be generalized, the variation of the average ranking of the 25 most highly, 26 moderately, and 25 most lowly important links is calculated. The derived results are presented in Figure 8. It can be observed that the average ranking of the most highly and lowly important links is sensitive to demand variation, while the average ranking of the medium important links appears stable.

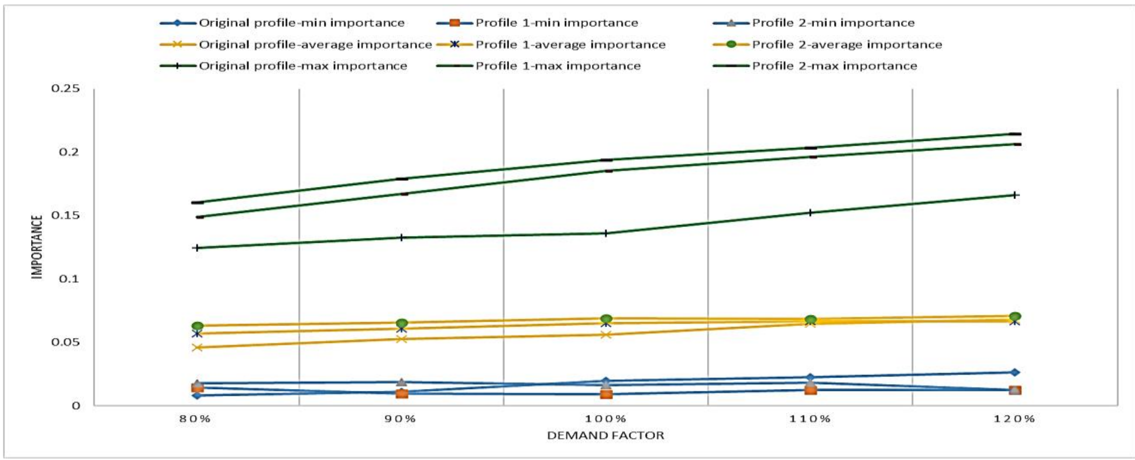

Figure 5: Variation of minimum, average, and maximum link importance values for different demand factors and profiles

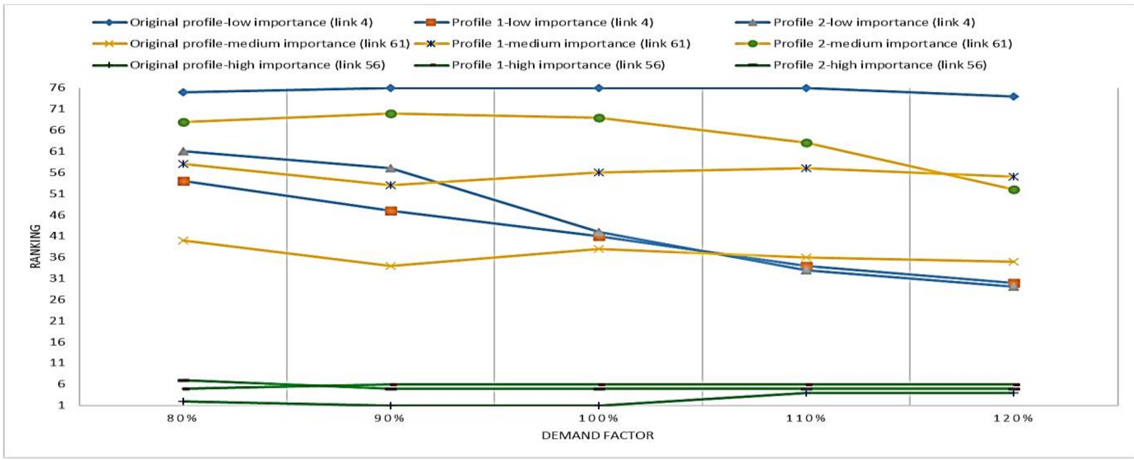

Figure 6: Ranking variation of links 4, 61, and 56 for different demand factors and profiles

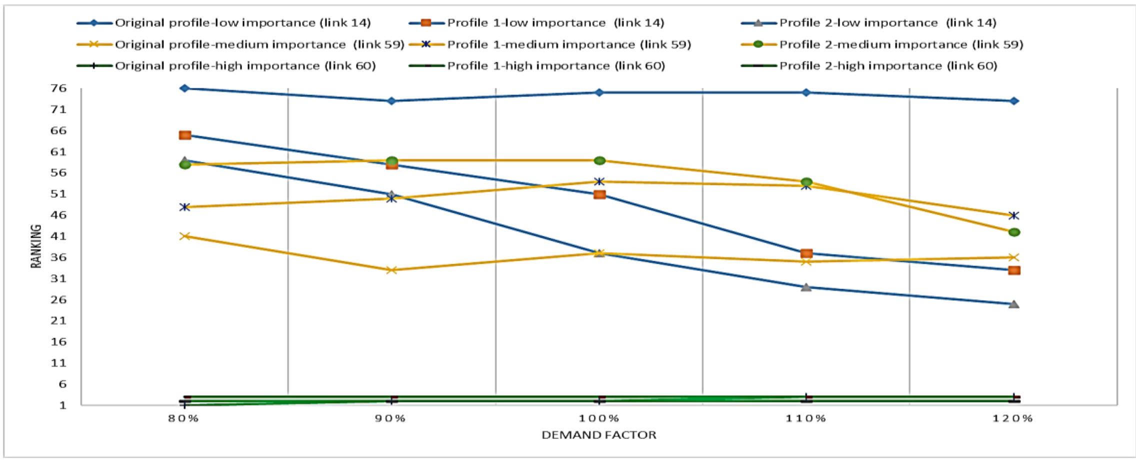

Figure 7: Ranking variation of links 14, 59, and 60 for different demand factors and profiles

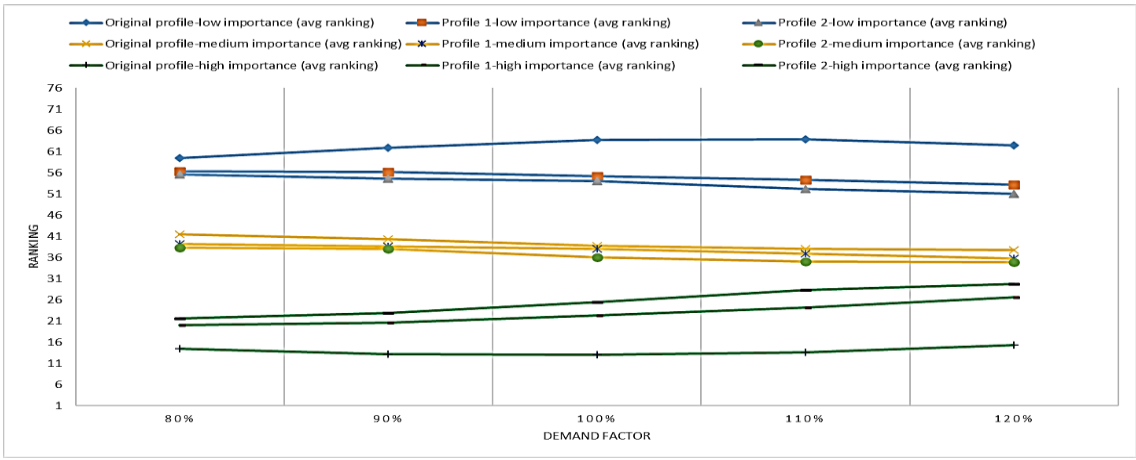

Figure 8: Average ranking variation of lowly, moderately, and highly important links for different demand factors and profiles

Such divergent results suggest the difficulty of identifying a pattern translating the relationship between criticality and demand variation. The only conclusion that can be supported, considering the values of the Spearman's correlation analysis (Table 1), is that criticality assessment is more sensitive to random flow patterns than the uniform growth or decrease of travel demand. This finding stresses the importance of demand uncertainty during the preparation of response plans to emergencies, where demand is quantitatively and spatially uncertain.

Table 1: Spearman's correlation indices between rankings derived from different demand factors and profiles

	P0 80 %	**P0 90 %**	**P0 100 %**	**P0 110 %**	**P0 120 %**	**P1 80 %**	**P1 90 %**	**P1 100 %**	**P1 110 %**	**P1 120 %**	**P2 80 %**	**P2 90 %**	**P2 100 %**	**P2 110 %**
P1 100%	0.58	0.59	0.66	0.67	0.65	0.95	0.98	1	0.98	0.92	0.89	0.92	0.89	0.84

Effect of capacity degradation

In Figure 9 the effect of capacity degradation to the minimum, average, and maximum values of link importance is presented. As it appears, lower levels of capacity degradation conclude to lower values of link importance. This seems reasonable since a uniformly lower reduction in the value of all links capacity leads to lower travel time increases and performance losses. Besides, it seems that the level of capacity degradation affects to a lesser degree the minimum and to a greater degree the maximum value of link importance.

Similar to the previous subsection, it is unknown to which links the values included in Figure 9 belong. Figures 10-11 present the variation of the ranking of two lowly, moderately, and highly ranked links according to the results of the initial analysis. It appears that the ranking of lowly and medium important links remains stable against the level of capacity degradation. In contrast, the ranking of highly important links becomes significantly lower for lower levels of capacity degradation. In Figure 12 the variation of the average ranking of the 25 most highly, 26 moderately, and 25 most lowly important links is presented.

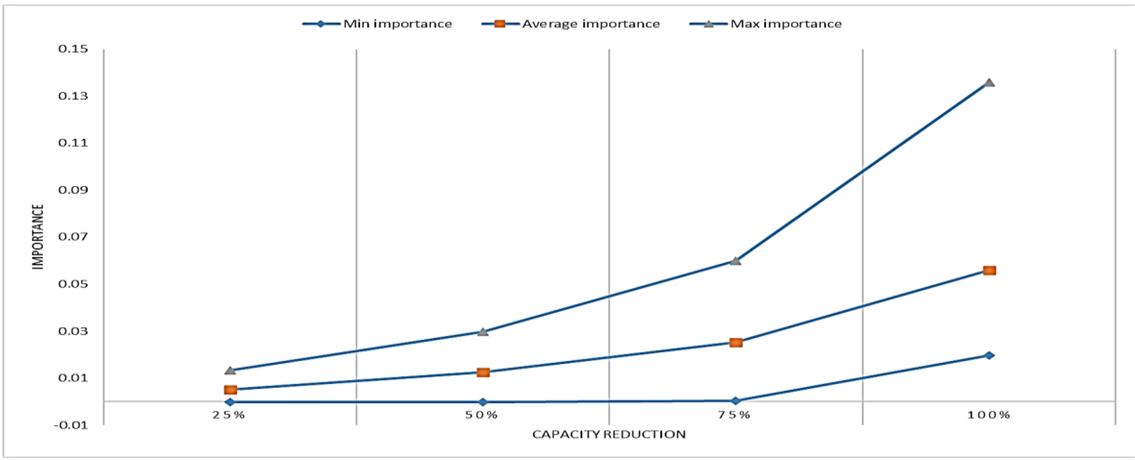

Figure 9: Variation of minimum, average, and maximum values of link importance for different levels of capacity degradation

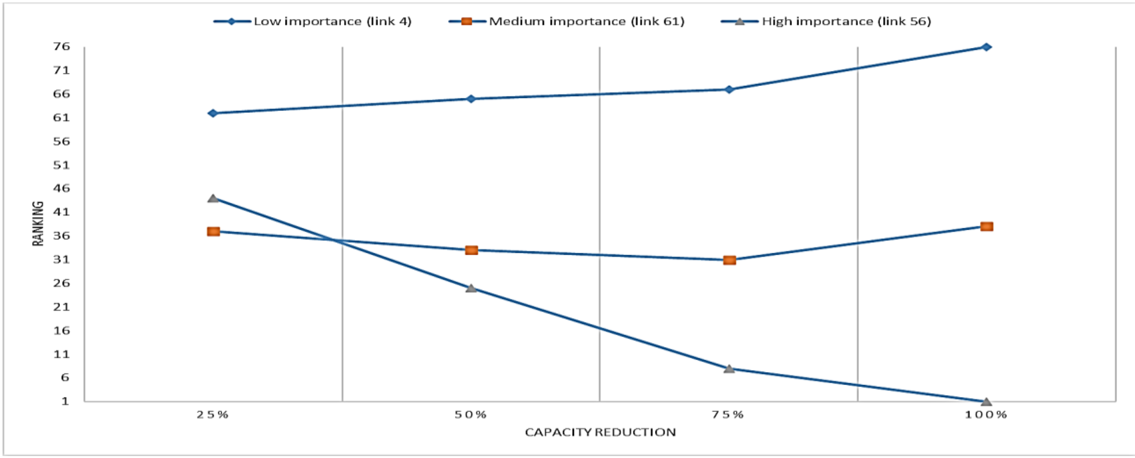

Figure 10: Variation of the ranking of links 4, 61, and 56 for different levels of capacity degradation

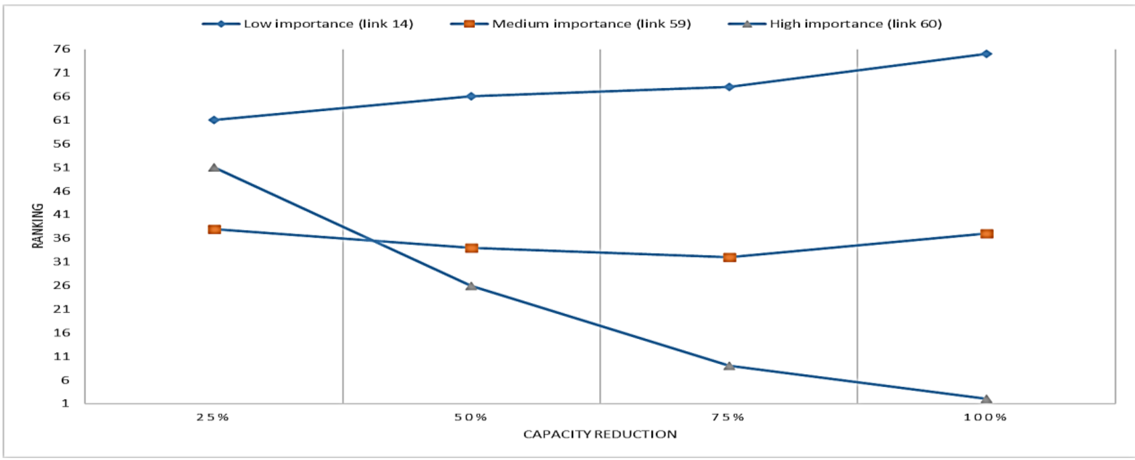

Figure 11: Variation of the ranking of links 14, 59, and 60 for different levels of capacity degradation

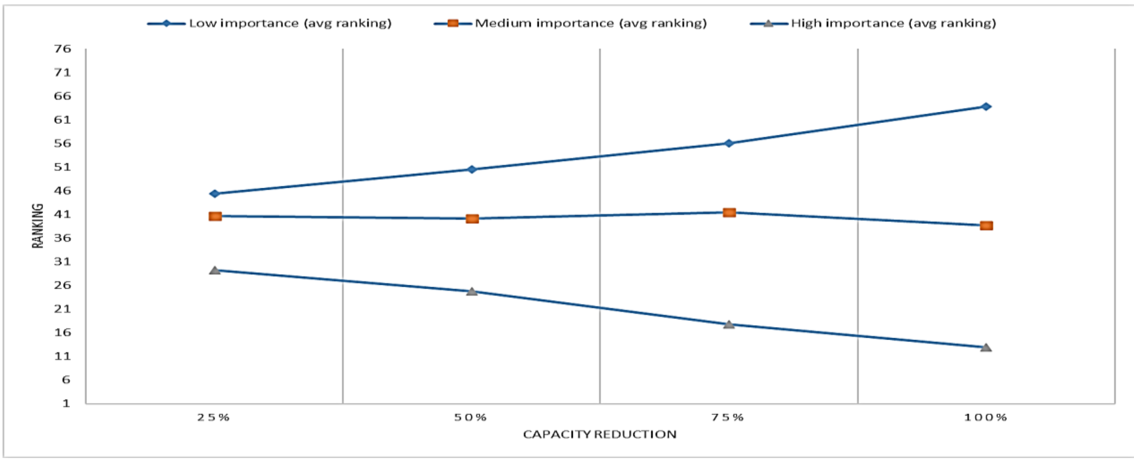

Figure 12: Variation of the average ranking of lowly, moderately, and highly important links for different levels of capacity degradation

Through the results included in Figure 12, it seems that the average ranking of only medium important links remains stable against the level of capacity degradation, while the ranking of lowly important links is considerably sensitive. Moreover, it is confirmed that the ranking of highly important links becomes significantly lower for lower levels of capacity degradation. Figure 12 also reveals that the variation of link rankings is gradually increasing when the level of capacity degradation gets lower.

Effect of network abstraction

Through Figure 13 it becomes evident that either the minimum, average, or maximum value of link importance is lower in the case of the augmented network. The variation of the ranking of two lowly, moderately, and highly ranked links according to the results of the initial analysis is depicted in Figures 14-15. It appears that the ranking of the most important link is less sensitive to network abstraction, while the less and moderately important links are more sensitive. However, the variation of the average ranking of the 25 most highly, 26 moderately, and 25 most lowly important links, presented in Figure 16, suggests the opposite. Hence, it is not easy to conclude to any meaningful verdict concerning how network abstraction affects the ranking of highly, moderately, and lowly important links.

An interesting observation is that the flows of the augmented network exhibit an almost linear relationship with the flows of the original network ($r_{\text{flows-augmented,flows-original}}$= 0.999), while the same relationship is weaker for link importance values ($r_{\text{importance-augmented,importance-original}}$= 0.677). This finding suggests that a narrow network abstraction would be enough for assessing the hierarchy of the road network's links in terms of the amount of served flow but not for assessing criticality.

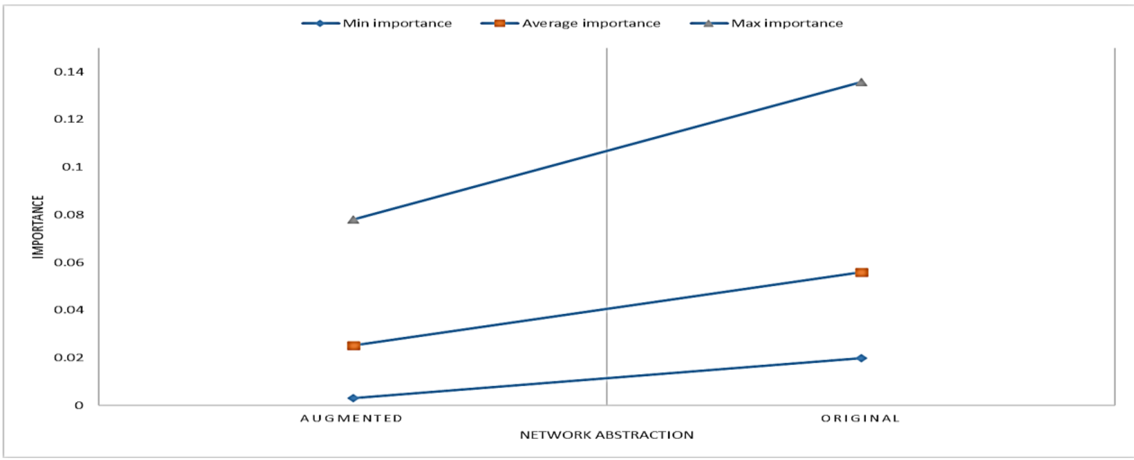

Figure 13: Variation of minimum, average, and maximum values of link importance for the two network abstractions

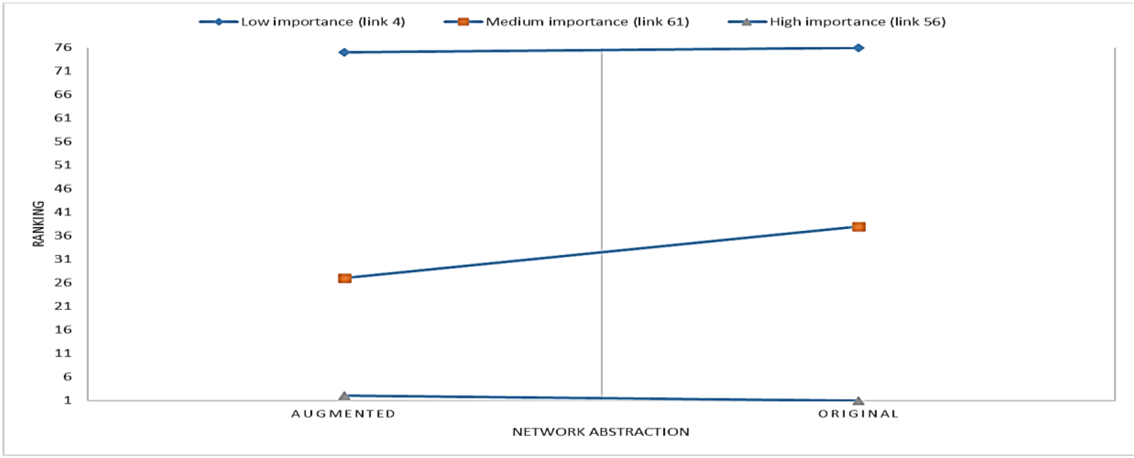

Figure 14: Variation of the ranking of links 4, 61, and 56 for the two network abstractions

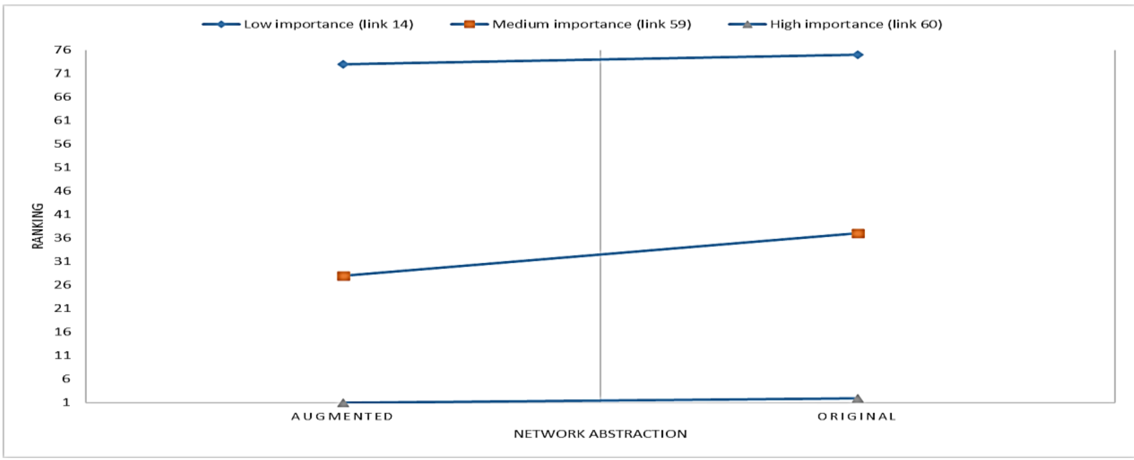

Figure 15: Variation of the ranking of links 14, 59, and 60 for the two network abstractions

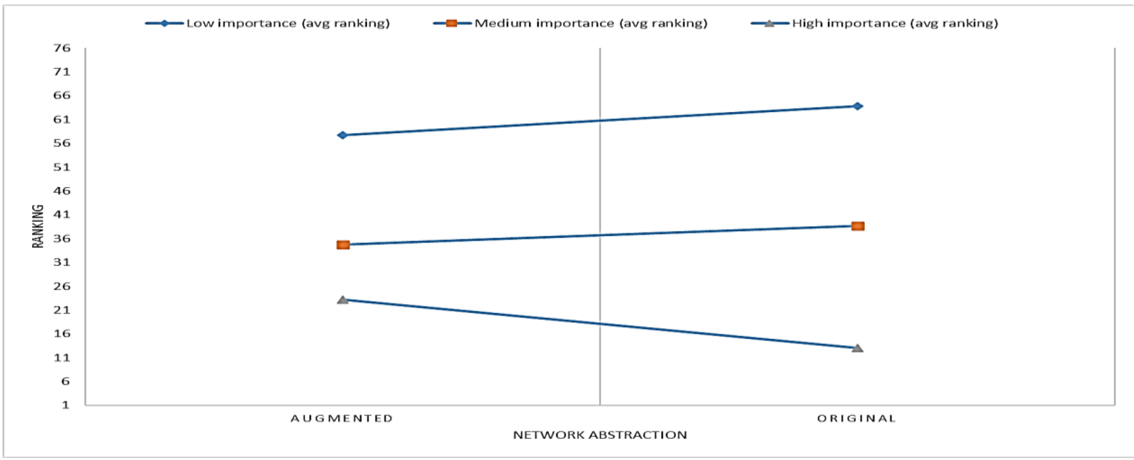

Figure 16: Variation of the average ranking of lowly, moderately, and highly important links for the two network abstractions

4. CONCLUSIONS

This paper provides an exhaustive analysis on the sensitivity of criticality assessment within road networks to demand variation, level of capacity degradation, and network abstraction. The derived results suggest that the aforementioned factors highly and unevenly affect the outputs of criticality assessment. The main conclusions that can be drawn with respect to each factor include, firstly, the high sensitivity of criticality assessment to random demand profiles. Secondly, the ranking of highly important links becomes significantly lower for lower levels of capacity degradation. Thirdly, the exclusion of lower hierarchy roads from traffic assignment models overestimates to a great extent the values of link importance. Finally, the variation of link importance is much greater than the variation of links' ranking in terms of served flow when roads of lower hierarchy are excluded from the analysis. Such findings stress the importance of incorporating demand uncertainty, the analysis of scenarios involving various capacity degradation cases, and accurate network representations in the preparation of adaptation plans to unexpected events and the allocation of scarce resources for enhancing the robustness of road networks.

REFERENCES

Berdica K. 2002. An introduction to road vulnerability: What has been done, is done and should be done. *Transp Policy*. Vol. 9, No. 2, pp. 117-127.

Chen B.Y., Lam W.H.K., Sumalee A., Li Q., Li Z.C. 2012. Vulnerability analysis for large-scale and congested road networks with demand uncertainty. *Transp Res Part A Policy Pract*, Vol. 46, pp. 501-516.

Jenelius E. 2010. Redundancy importance: Links as rerouting alternatives during road network disruptions. *Procedia Eng*, Vol. 3, pp. 129-137.

Nagurney A., Qiang Q. 2012. Fragile networks: Identifying vulnerabilities and synergies in an uncertain age. *Int Trans Oper Res*. Vol. 19, No. 1–2, pp. 123-60.

Transportation Networks for Research Core Team. 2020. *Transportation Networks for Research*. https://github.com/bstabler/TransportationNetworks. Accessed December, 20, 2021.