Energy-efficient timetabling for urban rail transit network considering passenger path choice behaviors

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Abstract

The high energy consumption of urban rail transit (URT) in metropolitan areas becomes a hotspot problem due to the ever-increasing operation mileages and pressing agendas of carbon neutralization. The majority of energy-efficient timetabling studies focus on a single URT line and are insufficient for a URT network with multiple interlinked lines. We propose a general model framework including timetabling and passenger path choice behaviors to optimize energy consumption of a URT network under passenger travel time constraints. A novel dynamic programming and heuristic search method for determining travel time components are incorporated in an iterative solution algorithm to find energy-efficient timetables. The proposed model and solution algorithm are validated with a real-world URT network under various scenarios.

Keywords: urban rail transit; energy-efficient timetabling; travel time; dynamic programming

1. Introduction

Urban rail transit (URT) is considered a low-carbon transport mode compared with other transport modes for serving the same passenger demand. However, URT is a major contributor of energy consumption due to the ever-increasing operation mileages in metropolitan areas (see energy consumption of URT in Beijing, China in Fig. 1 for example). Thus, energy-efficiency of URT becomes a significant issue for the pressure of carbon neutralization initiative.

Many studies have addressed the energy efficiency in the URT system in recent years. A number of studies focus on the speed profile optimization of one train running on a track between two stations (Howlett, 1996; Albrecht et al., 2013; Scheepmaker et al., 2017). Comparatively, increasing studies concern the energy-efficient timetabling of multiple trains running a URT line (Gupta et al., 2016, Canca and Zarzo, 2017, Wang and Goverde 2019, Xu et al., 2020, Yang et al., 2017 Wang et al., 2021). Energy-efficient timetabling aims to find the optimal timetable solution involving, for example, departure time, running time, and dwell time. Computationally, timetabling for a single URT line is an NP-hard problem (Cai and Goh, 1994). With the assumption of fixed passenger arrival rates and the absence of path choices, a few studies (e.g., Yin et al., 2017) proposed mixed-integer linear programming (MILP) formulations for energy-efficient timetabling of single URT lines.

The energy-efficiency timetabling problem is even harder when considering passenger path choices in a URT network of multiple interconnected lines, which hold for most real-world URT systems. Huang et al.

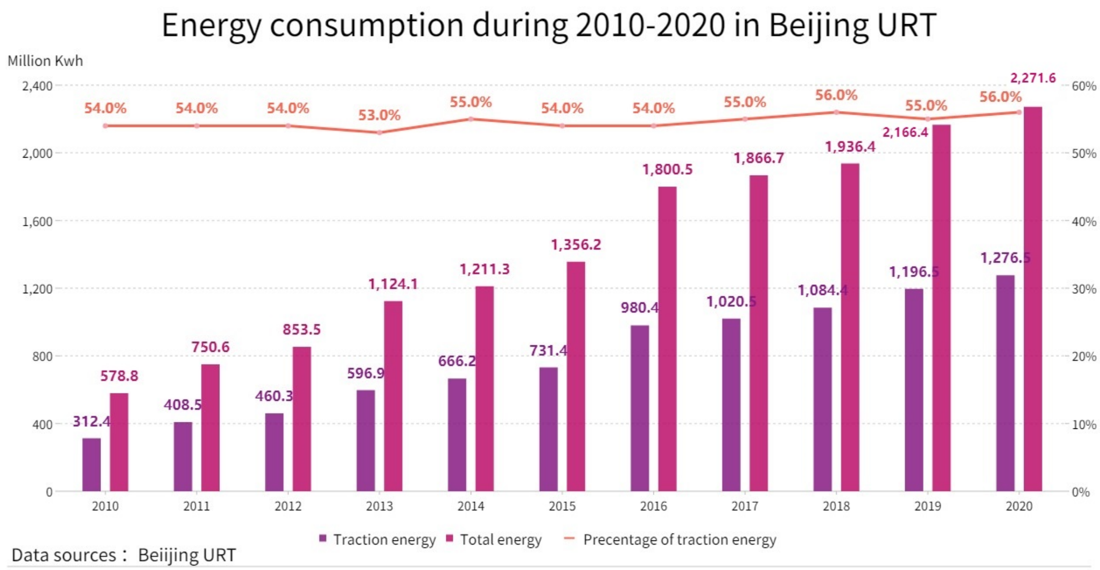


Fig. 1. Energy consumption increases in Beijing during 2010-2020.

(2021) pioneered an extension of energy-efficient timetabling from a single line to multiple lines in a URT network in bi-level programming framework with a simplified objective and model setup. First, the trade-off between energy consumption and passenger travel time was overlooked in Huang et al., (2021), which would sacrifice travel time for energy consumption reduction. Second, with time-invariant passenger demand, the timetable was optimized periodically with the same headways for each line, which may restrict reduction in both energy consumption and passenger travel time. Third, the formulated bi-level programming did not explore the relationship between passenger travel time and energy consumption, and the heuristic solution algorithm was limited to a uniform timetable structure for computational considerations.

In view of the limitations of Huang et al. (2021), this paper develops a more general model framework for energy-efficient timetabling for multiple lines with a nonuniform timetable structure. The time horizon of timetabling focuses on non-peak hours, during which energy-efficiency is a prioritized objective compared. Accordingly, different behavioral mechanisms are incorporated to realistically capture passenger path choice behavior in the URT network. Based on the explored relationship between energy consumption and travel time, we suggest a novel solution algorithm of dynamic programming and iterative passenger flow adjustment to decompose the model framework. The proposed model framework and solution algorithm are validated with a real-world URT network under various scenarios.

The remainder of this paper is organized as follows. Section 2 presents the problem description and modeling for the energy-efficient timetabling for a URT network. Section 3 discusses the solution algorithm for the proposed model framework. Section 4 presents a comprehensive case study in the URT system in the city of Xi’an (China) to verify the model and algorithm. Finally, Section 5 concludes the main contributions.

2. Problem description and modeling

Energy-efficient timetabling aims to determine a timetable with the minimum energy consumption while satisfying the needs of passenger mobility. The URT trains of each line are operated from one terminal station to the other terminal station in the up or down direction. The essential difference between timetabling for single-line and multi-line resides in the consideration of transfer and path choice. With different passenger path choice behavior, the energy-efficient timetables may vary. For example, we use η to denote the probability of a path being selected by the time-dependent passenger OD demand following a certain passenger path choice behavior. Given the speed profile of a track, the total weight of a train, including empty train mass and passenger loading weight, determines the energy consumption. Since passenger loading weight is the outcome of path choice, it is crucial that energy-efficient timetabling for a URT network should incorporate passenger path choice behavior. Hence, timetabling and passenger loading are two key aspects energy-efficient timetabling for a URT network that are coupled by passenger path choice behavior.

Fig. 2 shows a typical URT network with two crossed lines to illustrate the timetable elements of a URT network. Suppose there are N_l and $N_{l'}$ stations, $2N_l$ and $2N_{l'}$ platforms, and $2N_l - 2$ and $2N_{l'} - 2$ tracks (one track between two neighboring platforms) on line l and l', respectively.

2.1 Passenger travel time in a URT

For a parallel and nonuniform timetable, the time schedules of the first train are repeated cyclically with different headways of the ensuing trains on the same URT line. Denote the headway of train k on line l by

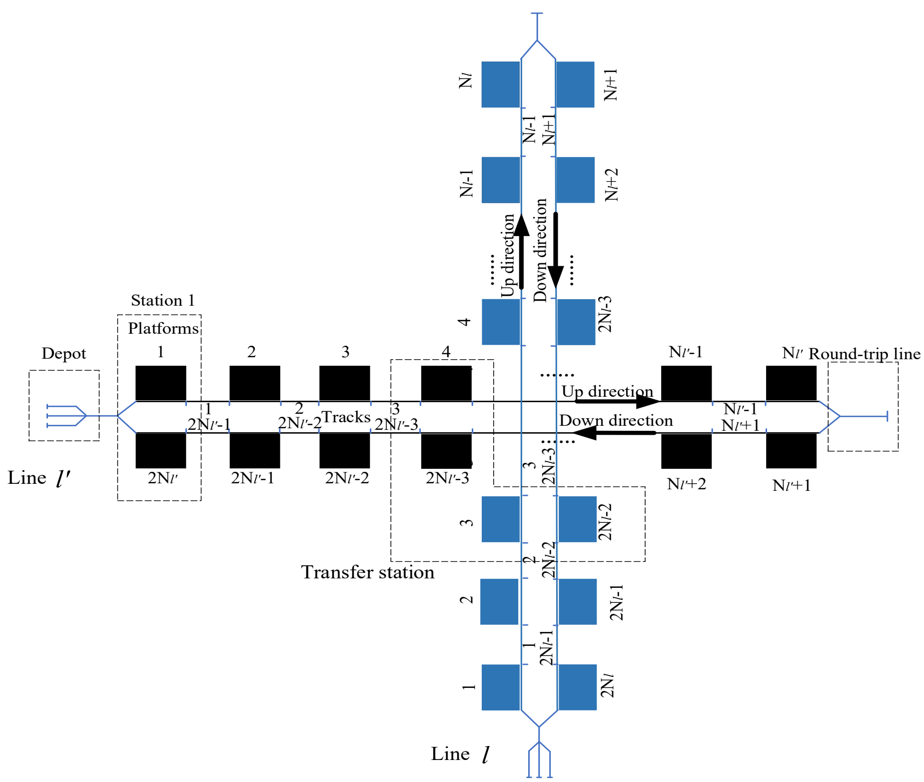


Fig. 2. Illustration of a URT network with two bidirectional lines.

h_{lk}; thus, h_{lk} may differ across k. For the dwell time on platform p ($\forall p \in P_l, k \in K_l, \forall l \in L$) on line l, the relationship between arrival time a_{lkp} and departure time d_{lkp} is formulated as Eqs. (1)-(5).

$$a_{lkp} + w_{lp} = d_{lkp}, \qquad \forall p \in P_l, k \in [1, K_l], \forall l \in L \tag{1}$$
$$w^{\mathrm{O}} \le w_{lp} \le w^{\mathrm{E}}, \qquad \forall p \in P_l, \forall l \in L \tag{2}$$
$$d_{lkp} + v_{lp} = a_{lk\overline{p}}, \qquad \forall p \in P_l, k \in [1, K_l], \forall l \in L \tag{3}$$
$$d_{lkp} + h_{lk} = d_{l\overline{k}p}, \qquad \forall p \in P_l, k \in [1, K_l - 1], \forall l \in L \tag{4}$$
$$h^{\mathrm{O}} \le h_{lk} \le h^{\mathrm{E}}, \qquad \forall k \in K_l, \forall l \in L \tag{5}$$

where K_l is train fleet on line l, and P_l is set of platforms on line l.

The travel time of a path may involve three parts, i.e., waiting at the starting platform, transfer, and in-train (including running time on the tracks and dwell time on the platforms). A 0-1 variable ξ_{urk} is introduced to denote if a passenger can board a train as

$$\xi_{urk} = \begin{cases} 1 & \text{if } d_{l\underline{k}p} < u \le d_{lkp}, \forall l = \bar{r}_1, p = \bar{r}_2, k \in [1, K_l] \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

Eq. (6) can be linearized by introducing a large number M as

$$\begin{cases} d_{lkp} - u \ge (\xi_{urk} - 1)M, & \forall l = \bar{r}_1, p = \bar{r}_2, k \in [1, K_l] \\ u - d_{l\underline{k}p} > (\xi_{urk} - 1)M, & \forall l = \bar{r}_1, p = \bar{r}_2, k \in [1, K_l] \end{cases} \tag{7}$$

Therefore, waiting time passengers arriving at time u, c_{ur}^1, is calculated by Eq. (8).

$$c_{ur}^1 = \sum_{k \in [1, K_{\bar{r}_1}]} \xi_{urk}(d_{lkp} - u) \ , \forall p = \bar{r}_2 \tag{8}$$

For passengers arriving at time u, in-train time of path r, c_{ur}^2, is formulated as

$$c_{ur}^2 = \sum_{\forall lp} \lambda_{rlp}(v_{lp} + w_{l\overline{p}}) \tag{9}$$

where λ_{rlp} is an incidence variable: $\lambda_{rlp} = 1$ if platform p of line l belongs to path r; otherwise, $\lambda_{rlp} = 0$.

Considering the uncrowded scenario, e.g. during non-peak hours, a transfer passenger will not wait or delay for the second connecting train, in which we argue energy-efficiency is more important. As shown in Fig. 3, the time window for passengers transferring from train k on platform p of line l to train k' on platform p' of line l' should satisfy $d_{l'\underline{k'}p'} \le a_{lkp} + f_{ll'pp'} \le d_{l'k'p'}$ to reach synchronization, where $f_{ll'pp'}$ is the walking time during the transfer.

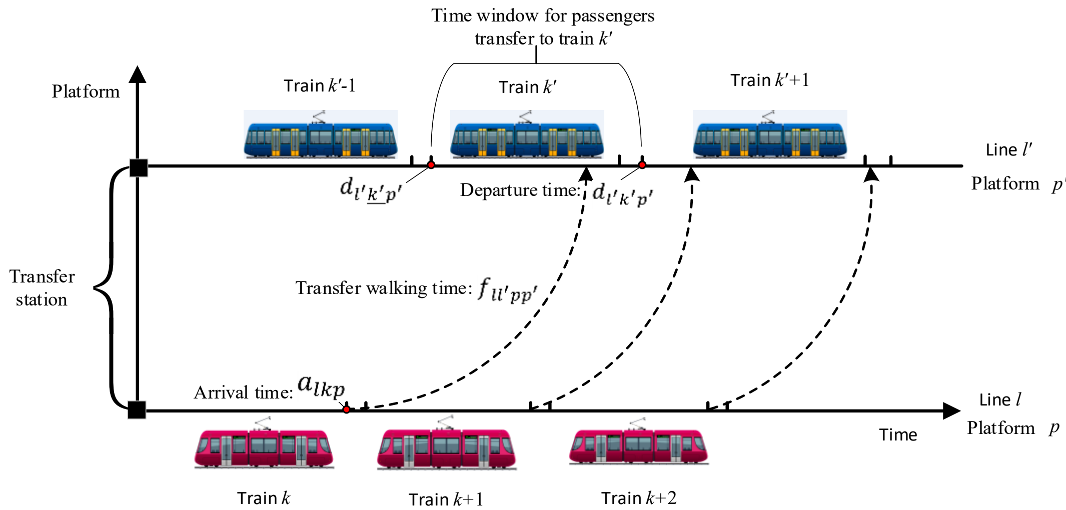


Fig. 3. An illustration of transfer synchronization.

The transfer time of path r for passengers arriving at time u, c_{ur}^3, can be presented as

$$c_{ur}^3 = \sigma_{ur} \sum_{i \in I_r} (d_{l'k_i'p'} - a_{lk_ip}) \tag{10}$$

The travel cost of path r, c_{ur}, can be expressed as a function of the above three parts ($c_{ur}^1, c_{ur}^2, c_{ur}^3$) and passenger loading (q_{ur}) as

$$c_{ur} = C(c_{ur}^1, c_{ur}^2, c_{ur}^3, q_{ur}) \tag{11}$$

2.2 Passenger flow loading

In a parallel and nonuniform timetable, train index k is dropped in the variable of passenger volume q_{lt} on a track. For instance, in Fig. 7, for the same speed profile on a track, passengers who go through track $2 \rightarrow 3$ either in train 1 or 2 have the same effects on the total energy consumption of this track during the planning period. Therefore, the total passenger volume q_{lt} of all operational trains determines the total energy consumption on this track.

To capture the path choice behaviors on energy-efficient timetabling, we consider three passenger loading mechanisms, namely, all-or-nothing assignment, linear proportional assignment, one-off stochastic assignment below. Specifically, we use η_{ur} to denote the probability that path r is selected by the time-dependent passenger OD pair following a passenger path choice behavior.

In the all-or-nothing assignment, η_{ur} is described as

$$\eta_{ur} = \begin{cases} 1, & c_{ur} = \min \{c_{ui} | i \in R_{\bar{r}\underline{r}}\} \\ 0, & \text{otherwise} \end{cases} \tag{12-1}$$

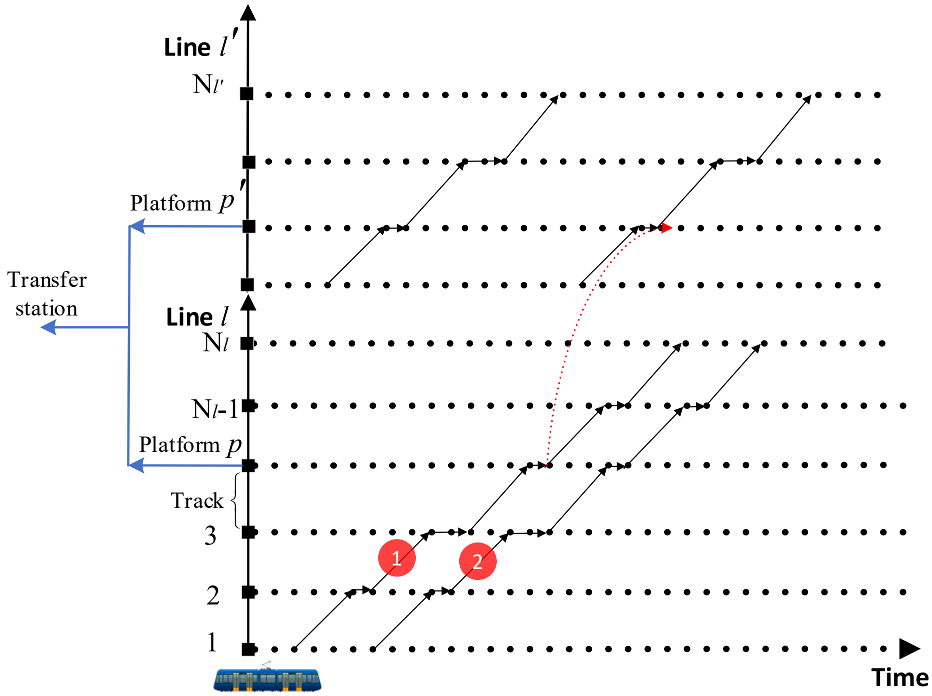


Fig. 4. Illustration for train number's ignorance in passenger loading.

In the linear proportional assignment, η_{ur} is described in a linear function as

$$\eta_{ur} = \frac{c_{ur}}{\sum_{i \in R_{\bar{r}\underline{r}}} c_{ui}} \tag{12-2}$$

In the stochastic assignment, η_{ur} for a one-ff assignment is formulated as

$$\eta_{ur} = \frac{e^{-\beta c_{ur}}}{\sum_{r \in R_{\bar{r}\underline{r}}} e^{-\beta c_{ui}}} \tag{12-3}$$

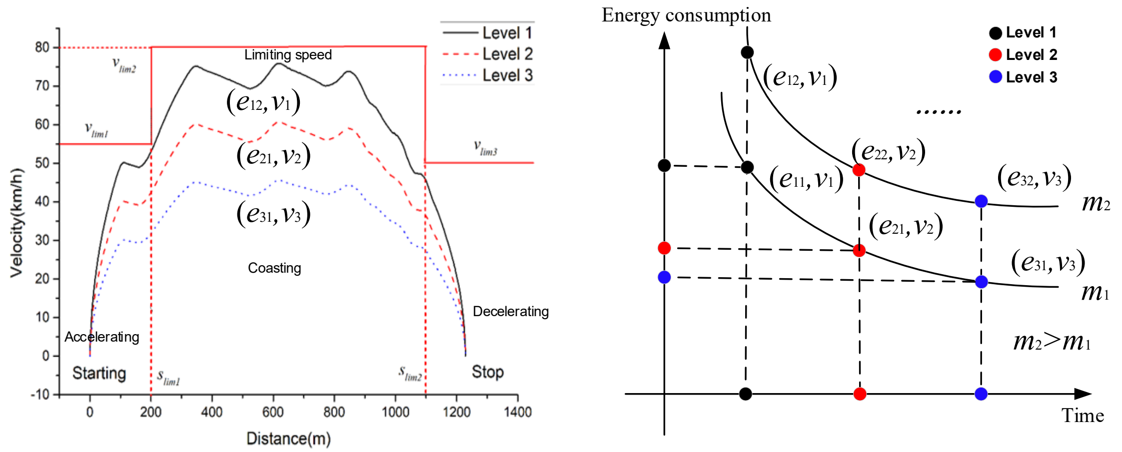
where β is an estimated scaling parameter.

The passenger volume of all trains on the track t of line l is

$$q_{lt} = \sum_{\forall jj' \in J, r \in R_{jj'}, u \in U} q_{jj'u} \cdot \eta_{ur} \cdot \lambda_{rlt} \tag{12}$$

Given average passenger weight τ, the passenger load $m_{l,t}$ on track t is

$$m_{lt} = q_{lt} \cdot \tau \tag{13}$$



(a) Alternative speed profiles. (b) Energy consumption.

Fig. 5. Speed profiles at different levels and energy consumption with different weights.

2.3 Calculation of energy consumption

As explained above, the energy consumption includes two parts: the energy for empty trains and the energy for the passenger loading (passenger weights). The relationship among passenger weight, running time and energy consumption is illustrated in Fig. 5. Running times and energy consumption satisfy $v_1 \leq v_2 \leq v_3$ and $e_{11} \geq e_{21} \geq e_{31}$, respectively. For the same operation level, the heavier the train is, the more energy consumption is involved (i.e., $e_{12} \geq e_{11}; e_{22} \geq e_{21}; e_{32} \geq e_{31}$).

For track t, only one level can be selected, expressed as Eq. (14).

$$\sum_{g \in G_{lt}} \theta_{ltg} = 1, \quad \forall\, l \in L, \forall t \in T_l \tag{14}$$

Then, the total energy consumption of the empty train of line l, E_l^1, equals to

$$E_l^1 = K_l \cdot \sum_{t \in T_l} \sum_{g\, \in\, G_{lt}} \theta_{ltg} \cdot e_{ltg}^0 \tag{15}$$

Operation level, g , will determine the running time, $v_{l,t}$, which can be formulated as

$$v_{lt} = \sum_{g \in G_{lt}} \theta_{ltg} v_{ltg}^0, \qquad \forall\, l \in L, \forall t \in T_l \tag{16}$$

The energy consumption caused by the passenger weight can be formulated as

$$e(m_{lt}, v_{lt}) = \sum_{g \in G_{lt}} \theta_{ltg} \cdot \frac{m_{lt}}{m_l^0} \cdot e_{ltg}^0, \qquad \forall l \in L, \forall\, t \in T_l \tag{17}$$

Eq. (17) consists of the product of binary variable θ_{ltg} and variable $\frac{m_{lt}}{m_l^0}$. Then, a group of constraints are introduced to linearize Eq. (17).

$$\begin{cases} \frac{m_{lt}}{m_l^0} - M\left(1 - \theta_{ltg}\right) \leq \rho_{ltg} \leq \frac{m_{lt}}{m_l^0} + M\left(1 - \theta_{ltg}\right) \\ \rho_{ltg} \leq \theta_{ltg} M \\ \rho_{l,t,g} \geq 0 \\ \forall\; l\; \in L, t \in T_l, g\; \in\; G_{lt} \end{cases} \tag{18}$$

Therefore, total energy caused by the passenger weight of line l, E_l^2, is presented as Eq. (19).

$$E_l^2 = \sum_{t \in T_l} \sum_{g\, \in\, G_{lt}} \rho_{ltg} \cdot e_{ltg}^0 \tag{19}$$

Finally, we can get the total energy consumption of a URT network as follow

$$E_l = E_l^1 + E_l^2 = N_l \cdot \sum_{t \in T_l} \sum_{g\, \in\, G_{lt}} \theta_{ltg} \cdot e_{ltg}^0 + \sum_{t \in T_l} \sum_{g\, \in\, G_{lt}} \rho_{ltg} \cdot e_{ltg}^0 \tag{20}$$

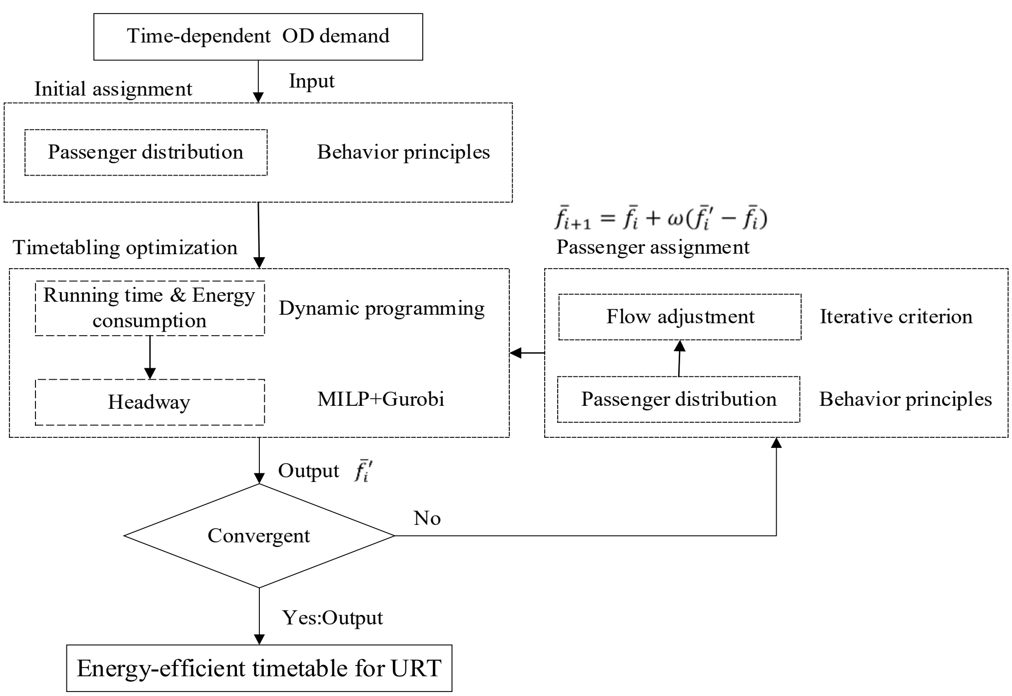


Fig. 6. Flowchart of the solution algorithm.

3. Solution algorithm

To simultaneously consider passenger total travel time and flow adjustment to the energy-efficient timetabling for the URT network, we propose a model framework including a passenger travel time constraint and passenger flow adjustment in Fig. 6. We find that once the running time (speed profile) on each track is given, the travel time is stable in a narrow range. Therefore, we replace the total travel time constraint as the total running time constraint and utilize a two-stage method to solve the timetable optimization. In the first state, we apply dynamic programming to find the optimal speed profile on each track under the allowed total running time constraint. In the second stage, the timetabling problem, given train running times, is formulated as MILP that can be solved by a MILP solver (e.g., Gurobi). Furthermore, passengers may adjust their path choices given a new timetable. Passenger flow adjustment is incorporated and demonstrated to converge after an iterative adjustment process.

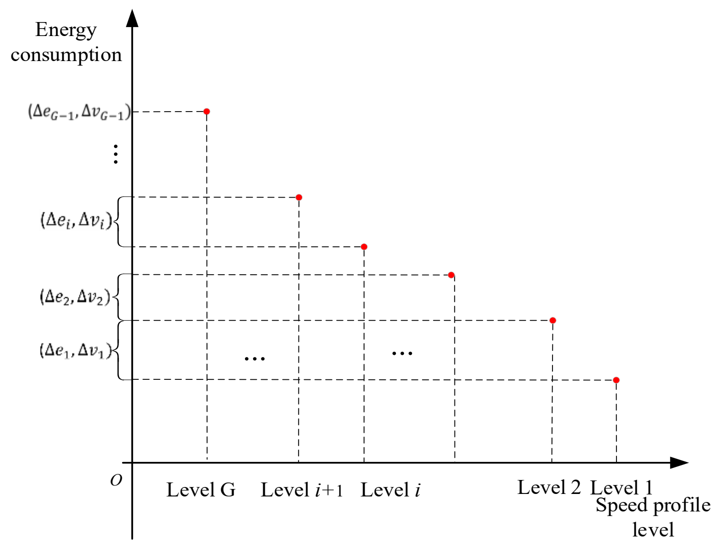


Fig. 7. Energy changes on the adjacent speed profile level.

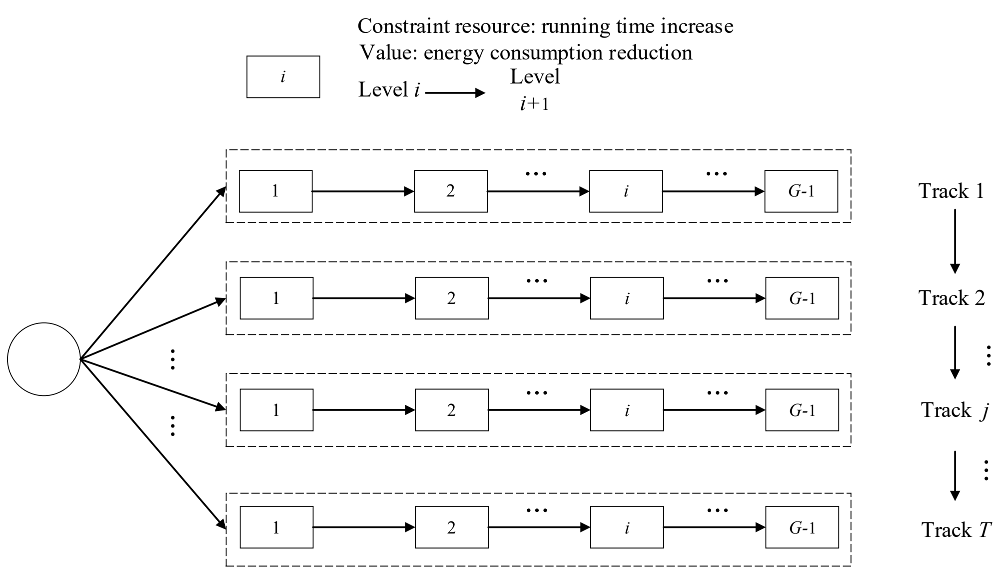


Fig. 8. Knapsack problem in tree type.

In the first step, we innovatively reformulate the relationship between energy consumption and running time as a knapsack problem in a tree structure. As shown in Fig. 7, between two adjacent speed profile levels, there is an energy consumption reduction Δe_i, which also corresponds to running time increment Δv_i.This relationship can be regarded as an item in the knapsack problem. As shown in Fig. 8, in each track, there are in total G speed levels that can be divided into $G-1$ items. We develop a dynamic programming, grouping the items and searching in the group rely on the dependency between the items to find the optimal running times.

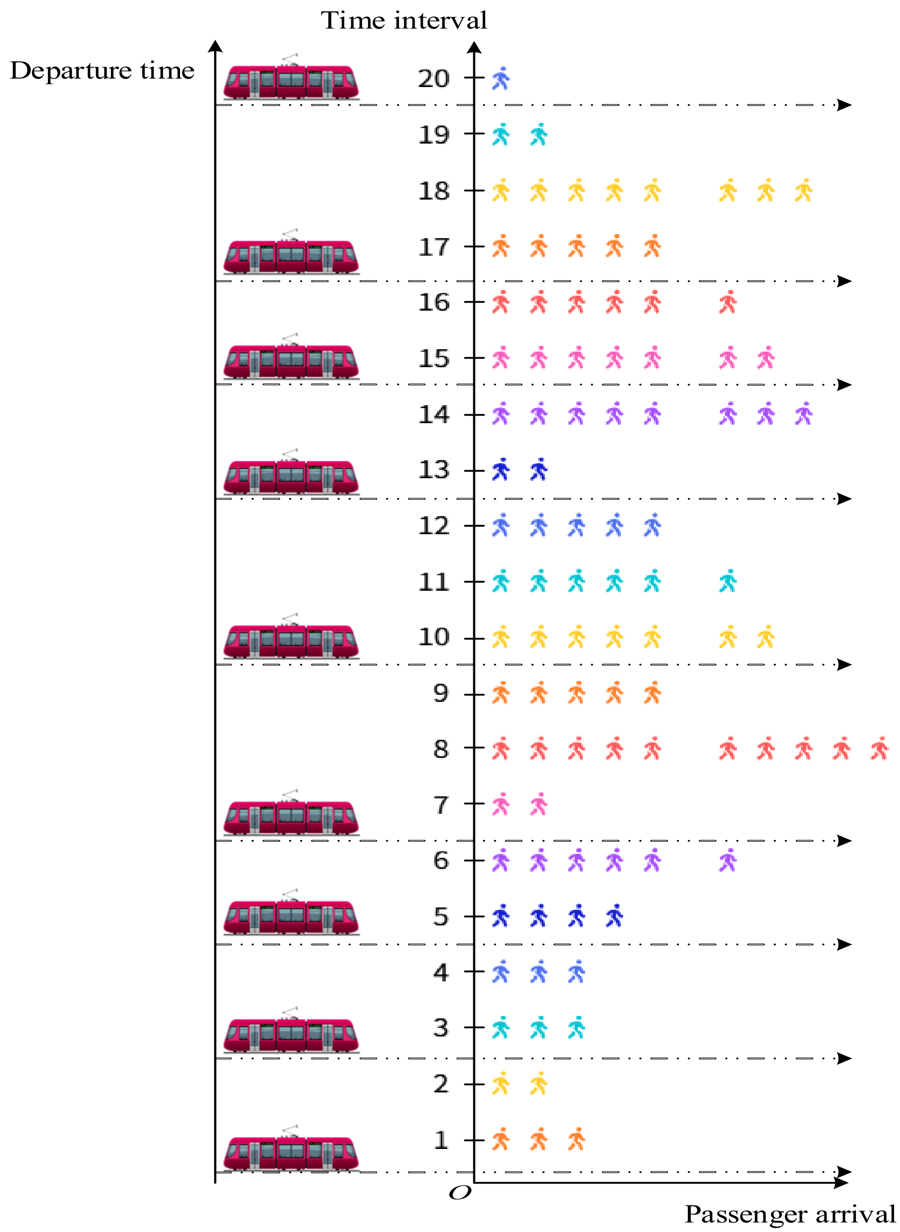


Fig. 9. Illustration of the combination of train departure and passenger arrival.

Since the MILP of timetabling optimization is repeated in the flow adjustment, we propose an approximation algorithm to effectively find a satisfactory solution. The passenger arrival is statistically discrete in a time interval Δu. As shown in Fig. 9, given the train fleet size, there are many order

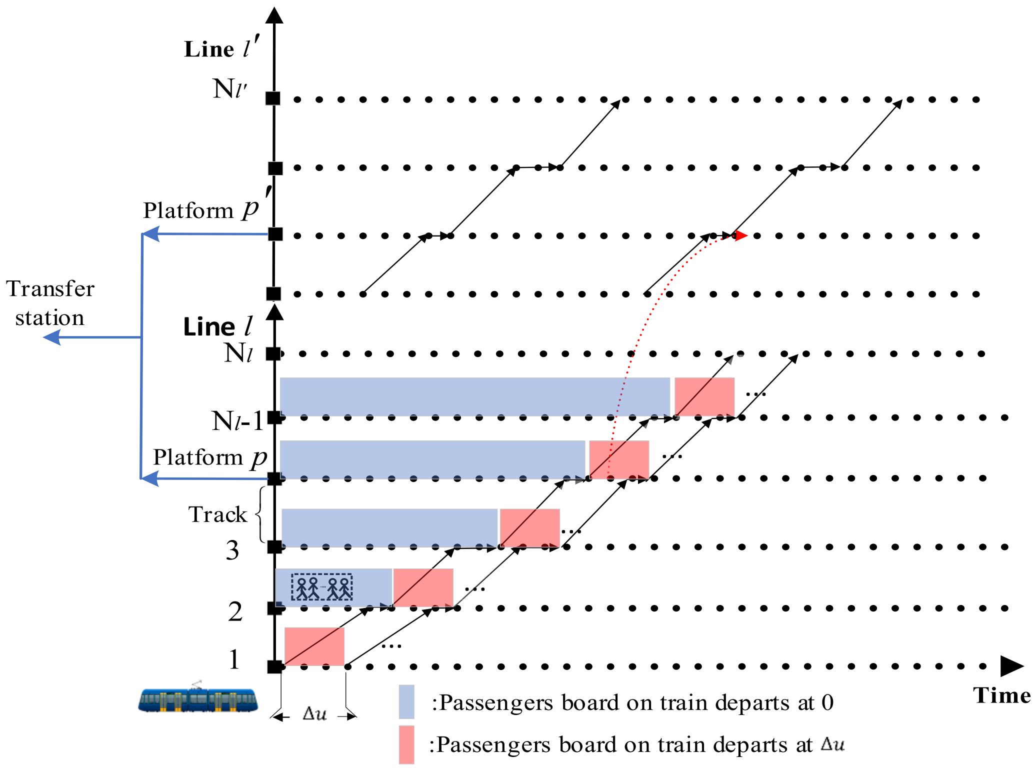


Fig. 10. Equivalent passenger flow in the starting platform at each time interval.

combinations between train departures and discrete passenger arrivals. Two steps are involved to find a satisfactory solution responsively. First, based on the heuristic rule that the train departure prefers to denser passenger arrivals, an order combination between train departure and discrete passenger arrival can be given. Second, we can utilize a solver to obtain the specific departure time. Since the decision variables are reduced to the departure time at the starting platform and dwell time, we should find the equivalent passenger flow in the starting platform at each time interval.

4. Case study

In this section, we demonstrate the effectiveness of the proposed model framework and solution algorithm with the URT network in the City of Xi'an (China), which covers 4 lines and 94 stations in service (Fig. 11). After running the solution algorithm for the three passenger behavioral mechanisms, the proportions of different times are shown in Fig. 12. It shows that the proportions of different times are stable when the elements (specifically, running time) of the timetable are given.

Fig. 11. The Xi'an URT network.

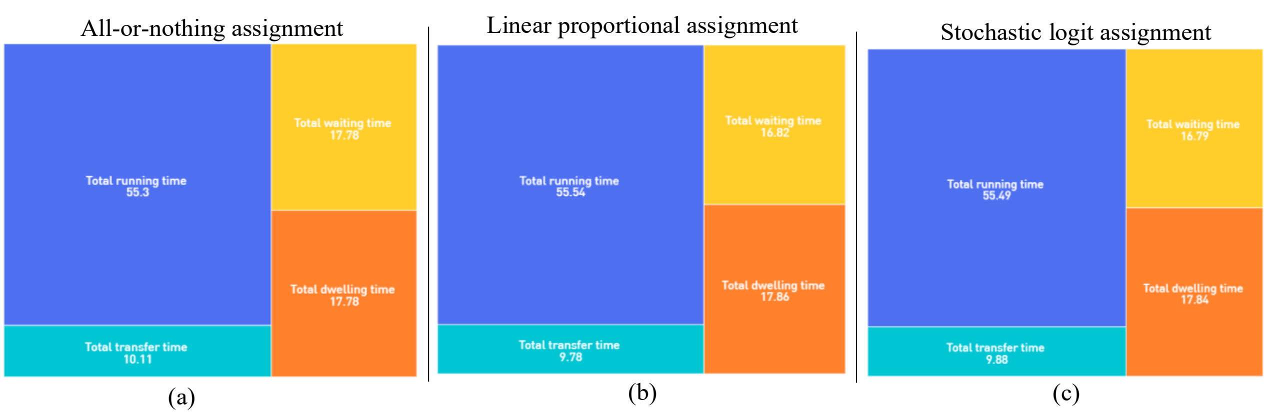
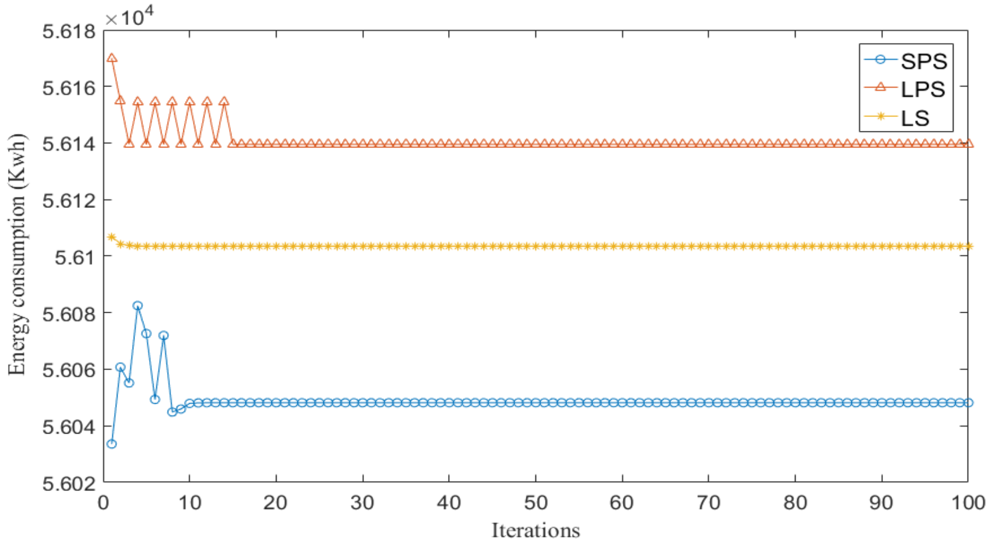


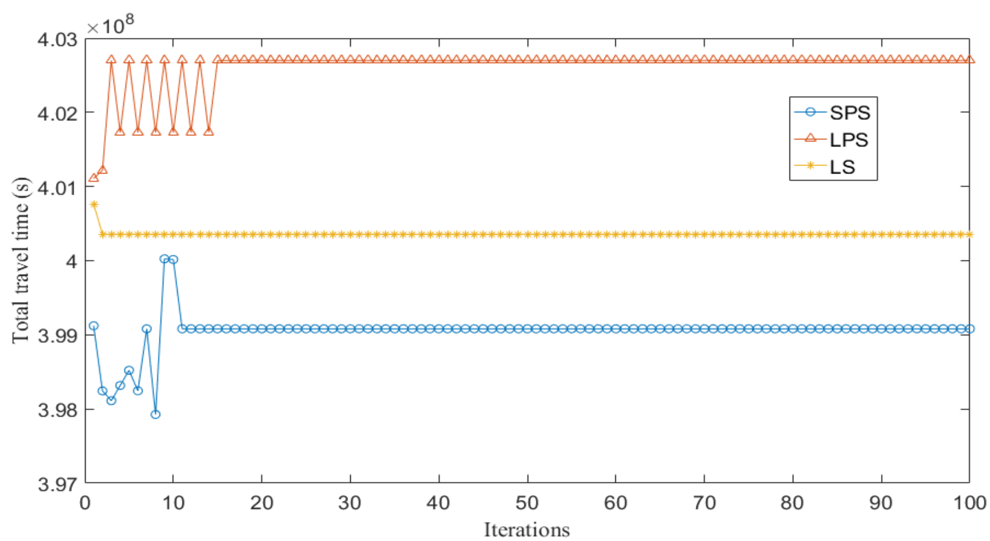
Fig. 12. The proportions of different times for different passenger behavioral mechanisms (%).

To further optimize the energy consumption, we conduct experiments with different allowed travel time constraints. As shown in Fig. 13(a)-(b), with a 20% increment of the allowed travel time, the energy and travel time can convergence after a dozen of passenger flow adjustment iterations.

The relationship between optimal energy consumption and travel time can be seen in Fig. 14. Energy consumption reduction increases with increasing allowed travel time in Fig. 15(a). The ratio of energy consumption reduction to travel time increment decrease progressively in Fig. 15(b). When the travel time increment is around 10-15%, the ratio decrease becomes slow. Notice that when the allowed travel time increment is over 20%, the ratio of energy consumption reduction to travel time increment tends to be close to 1. This indicates a near equivalent exchange between energy consumption reduction and travel time.



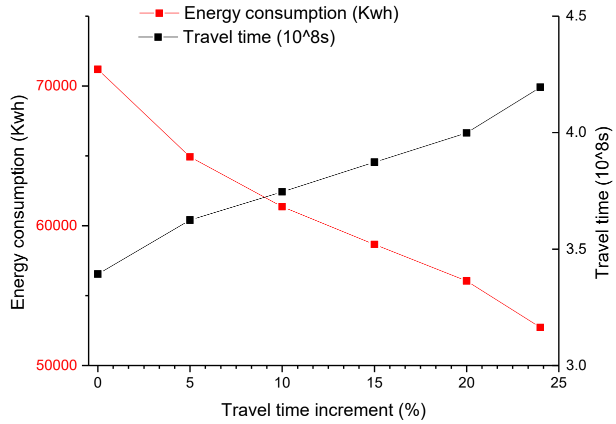
(a) The convergence of energy consumption.



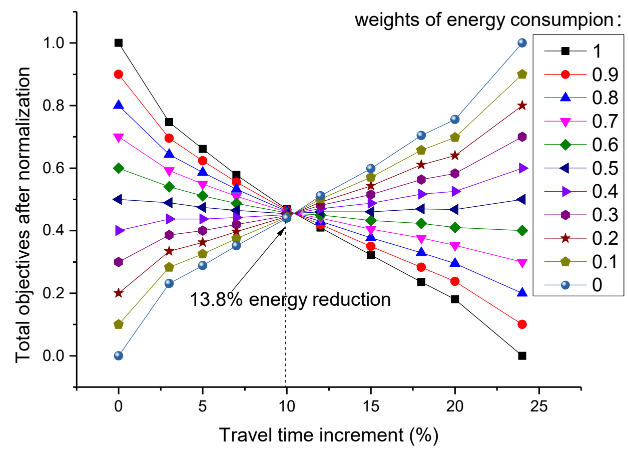
(b) The convergence of travel time.

(ANS: shortest path assignment, LPS: linear proportional assignment,
LS: one-off stochastic logit assignment)

Fig. 13. The convergence of different passenger behaviors.

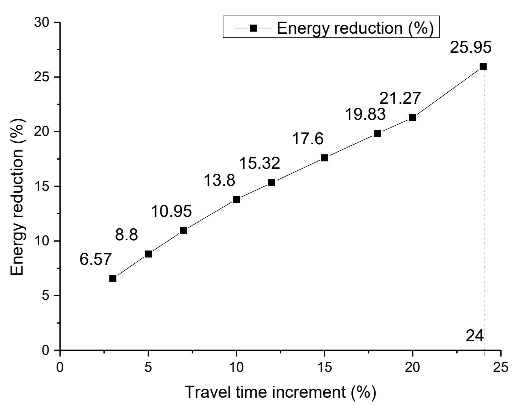


(a) Energy and travel time in different allowed travel time increments.

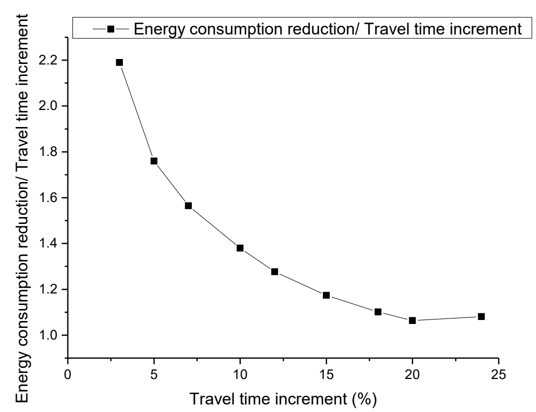


(b) Normalization energy and travel time with different weights.

Fig. 14. Energy and travel time relationship.



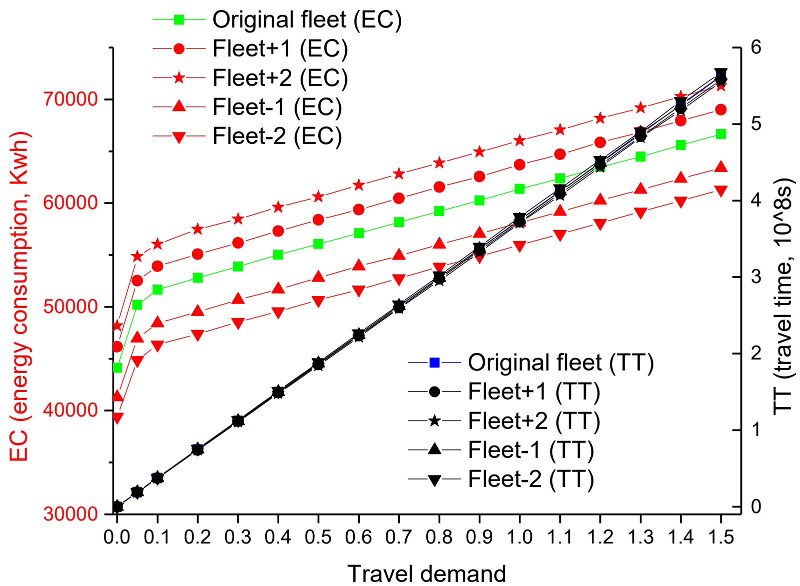
(a) Energy consumption reduction varies from different allowed travel time increments.



(b) The ratio of energy consumption reduction to travel time increment.

Fig. 15. Relationship between energy consumption reduction and travel time increment.

In Fig. 16, both the Energy consumption (EC) and Total Travel time (TT) curves turn upward with the rise of passenger demand. A slight increase in train fleet size causes more changes in energy consumption than those in travel time. It shows that more train fleet size consumes more energy but decreases the travel time in all proportional changes of passenger demand. This finding suggests that the operator may cut down up to two trains in the non-peak hours for more energy consumption reduction on the condition that the passenger travel times are only slightly affected.



Energy consumption: EC (Kwh), Total Travel time: TT (10^8s)

Fig. 16. Influences passenger demand and train fleet on energy consumption and travel time.

5. Conclusions

This paper develops a general model framework including timetabling and passenger assignment considering passenger travel time constraints. Three types of passenger loading mechanisms are considered to capture the path choice behavior on energy-efficient timetabling. A dynamic programming and heuristic search process are incorporated in the solution algorithm to find a satisfactory solution. The efficiency and effectiveness of the model framework and solution algorithm are validated in the numerical experiments. The model can obtain a significant reduction in energy consumption with certain allowed travel time increments with the consideration of different passenger behavioral mechanisms.

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