Passenger Flow Control at Platforms in Urban Rail Systems

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SHORT SUMMARY

Flow of passengers is the one of main operational aspects for urban rail systems. Over the recent years, passenger flows vary frequently causing disturbances on trains operation, and unsafe overcrowded environments for passengers. To improve the performance of rail systems, this paper presents a control method to regulate disturbed passenger flows and rail system. A traffic model coupling dynamics between staying times at stations and accumulated passengers at platforms is formulated. The suggested control method applies actions both on the train traffic and stations' facilities, using real time measurements and model predictive control optimization method. Actions at each stage are calculated as a solution of quadratic programming problem with regulation objective, and safety, feasibility, and limited platform capacity constraints. Moreover, the objective function accounts passengers at platforms to reach effective flows during the controlled period. Numerical examples are given to demonstrate the rail system performances under the proposed control method.

Keywords: Train regulation, Passengers flow control, Urban rail systems.

1. INTRODUCTION

Urban rail systems, in particular Metro and Urban Rail Transit (URT) systems, are characterized by high frequency service and increasing passenger demand. These systems usually have limited ability of infrastructure development (Hou et al., 2019), and the increased demand may result disturbances, such as schedule deviations, and overloaded train vehicles or platform facilities. Hence, new rail traffic management of rail systems can be developed to increase the capacity by controlling the train operations and passenger demand. Some studies in rail traffic management have been focused on introducing optimized predetermined timetables, e.g., (R. Liu, Li, & Yang, 2020). However, disturbances or changes in the passenger arrival rates may be unpredictable and occur even when the predetermined timetable is considered effective. Other studies, including this research, consider real time regulation of rail systems, i.e. restoring the disturbed system to the normal operation, or adapting operational strategy to the present situation, according to measurements taken in real time.

In order to design control methods for urban rail systems, several rail traffic models were studied in the literature. Characterizing rail system traffic by the trains' positions and velocities over time is studied in some rail systems, e.g., (Y. Liu, Zhou, Su, Xun, & Tang, 2021) for interurban railways, and (Wu, Gao, & Tang, 2021; Luo, Tang, Liu, Zhang, & Li, 2021) for metro. However, these works do not model stopping dynamics at stations, which is a crucial part of urban rail traffic. Therefore, discrete event models were introduced in various real time regulation problems for urban rail systems. A discrete event model can capture the departure times of the trains from the stations.

One of the first discrete models was formulated in (Van Breusegem, Campion, & Bastin, 1991) with linear dynamics and a quadratic cost function; and solved by a linear quadratic regulator (LQR) control. Recent works integrated nonlinear dynamics and applied other control methods, e.g., in (Li, Yang, & Gao, 2018a) a constrained regulation problem with safety and feasibility constraints was considered, and solved using a model predictive control (MPC) method; and in (Li, Yang, & Gao, 2018b), an optimal switched control for time-varying passenger arrival flows was developed.

There are few research works that integrate the passenger flows in the system dynamics. A mixed-integer linear-programming problem was formulated in (R. Liu et al., 2020) with passenger control at stations as variables; however, it was solved offline and used known passenger arrivals without considering unpredictable disturbances. Real time regulation was also utilized in some researches referring passenger control at stations. The number of loaded passengers on the train at departure times of the trains was considered in (Wang, Li, Tang, & Yang, 2020; Li, Dessouky, Yang, & Gao, 2017), as a *joint* discrete event model of train regulation and passenger flow was proposed. In (Li et al., 2017) the MPC was utilized, while the event-triggered MPC was used in (Wang et al., 2020). In both works, manipulating the number of passengers boarding the train was proposed as a control measure. The idea is that restricting some of boarding passengers at platforms can enhance the train traffic operations and improve the passenger flow performance of the rail system. Nevertheless, in (Li et al., 2017; Wang et al., 2020) conservation of passengers at platforms is neglected, as non-boarding passengers are not dealt with in the next steps, implying they are vanished from the system. Additionally, while the train capacity constrained the problem, platform capacity was not considered. To reflect the effect of platform limitations both on train traffic and passenger flow, the current paper improves the joint model by regulating the boarding passengers number while considering oversaturated conditions at platforms.

2. METHODOLOGY

In this section, the joint discrete event model of urban rail systems and the proposed control strategy are presented. First, a mathematical model that evaluates the train departure times from stations and the train load, according to previous departure times and loads of itself and other trains, is developed. The dynamic equations are based on the model presented in (Li et al., 2017; Wang et al., 2020), with some modifications considering non-boarding passengers dynamics at platforms, and limited platform capacity constraints. Second, a new state vector is formed to describe the state of a full rail system at each step according to the real time model (RTM) method proposed in (Van Breusegem et al., 1991) and also utilized in (Li et al., 2017). Finally, an MPC strategy is developed, enabling the control vector to be calculated at each step according to current state, thus allowing a real time control application for urban rail systems.

The mathematical model is comprised of two main state variables, the train departure time from a station, and the load of the train at the departure time. The departure time of a train from a station can be evaluated according to its departure time from the previous station, as well as the load of the train at the departure time is related to the load of the train at the previous station, as follows

$$t^i_{j+1} = t^i_j + r^i_j + s^i_{j+1} \ , \tag{1}$$
$$l^i_{j+1} = l^i_j + b^i_{j+1} - a^i_{j+1} \ , \tag{2}$$

where t^i_j [s] is the departure time of ith train from the jth station, r^i_j [s] is the running time of train i from station j to station $j+1$, and s^i_{j+1} [s] is the staying time of ith train at station $j+1$; and l^i_j [passenger] is the load of passengers on ith train when it departs from jth station, and a^i_{j+1} [passenger] and b^i_{j+1} [passenger] are respectively numbers of the alighting and boarding passengers to train i at station $j+1$, see also Fig. 1.

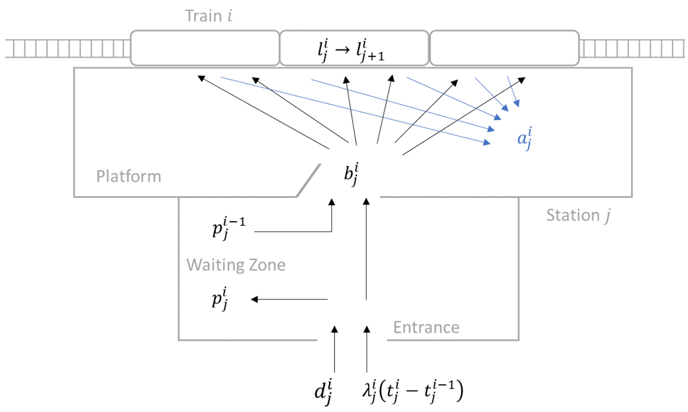

Figure 1: Description of passenger state variables, control inputs, and disturbances at a station.

A timetable, which defines the departure times and running times of the trains, is assumed to be determined a priori. By utilizing a communication system, a command can be given to increase or decrease the train velocity. Thus, the running time R_j^i[s] can be controlled. Moreover, the system can be subjected to running time disturbances, denoted by ${w_r}_j^i$ [s]. The staying time is assumed to have a constant minimal value, and increases linearly according to the number of boarding and alighting passengers. As the passenger demand is growing over the year, it is also assumed that the staying time is subjected to disturbance, denoted by ${w_s}_{j+1}^i$ [s]. This reads as follows

$$r_j^i = R_j^i + u_j^i + {w_r}_j^i \ , \tag{3}$$
$$s_{j+1}^i = {S_0}_{j+1}^i + \alpha_{j+1} a_{j+1}^i + \beta_{j+1} b_{j+1}^i + {w_s}_{j+1}^i \ , \tag{4}$$

where u_j^i [s] is the adjustment time that controls the running time of train i from station j to station $j+1$, i.e., negative (or positive) values of u_j^i are related to increasing (or decreasing) the train speed. α_{j+1} [s/passenger] and β_{j+1} [s/passenger] are coefficients that express the time duration needed for a single passenger to alight or board a train at station j.

During the time interval between departures of two successive trains, it is assumed that passengers arrive to the station at a known rate. Thus, one component of the number of passengers attending to board the train is the number of passengers arrived at the time period between the current train and the previous one. Let us denote p_j^{i-1} [passenger] as the number of the passengers restricted from boarding the previous train. Then, the actual number of boarding passengers is the number of accumulated passengers including the previous controlled passengers p_j^{i-1}, minus the current controlled passengers, denoted by p_j^i [passenger]. Another assumption is that the number of alighting passengers at a station is proportional to the load of the train when it departs the previous station. Hence,

$$b_{j+1}^i = \lambda_{j+1}^i (t_{j+1}^i - t_{j+1}^{i-1}) + p_{j+1}^{i-1} - p_{j+1}^i + d_{j+1}^{i-1} \ , \tag{5}$$
$$a_{j+1}^i = \eta_{j+1}^i l_j^i \ , \tag{6}$$

where λ_{j+1}^i [passenger/s] is the passenger arrival rate to station $j+1$ at the time interval between trains $i-1$ and i, d_{j+1}^{i-1} [passenger] is the disturbance in term of passengers, and $\eta_{j+1}^i \in [0,1]$ is the proportion of passengers alighting at station $j+1$ from the load of the i train when it departs from station j. Equations (2) and (5) present respectively conservation equations of passenger numbers for onboarding the train and at the station platform, also illustrated in Fig. 1.

Note that equation (5) considers the evolution of aggregated non-boarding passengers p_j^i, the departure times, and load evolution through stations and trains. p_{j+1}^{i-1} in (5) denotes the passengers reduced from previous train, i.e., the aggregated non-boarding passengers until departure of train $i-1$; and the *passenger* control input can be defined as the difference $v^i_{j+1} = p^i_{j+1} - p^{i-1}_{j+1} - d^{i-1}_{j+1}$. Substituting equations (3)–(6) into (1) and (2), the departure time and load of train transitions between stations and trains are given as follows

$$t^i_{j+1} = t^i_j + R^i_j + u^i_j + S^i_{j+1} + \beta_{j+1}\left(\lambda_{j+1}(t^i_{j+1} - t^{i-1}_{j+1}) - v^i_{j+1}\right) + \alpha_{j+1}\eta^i_{j+1}l^i_j + w^i_{j+1} \ , \tag{7}$$

$$l^i_{j+1} = (1-\eta^i_{j+1})l^i_j + \lambda_{j+1}(t^i_{j+1} - t^{i-1}_{j+1}) - v^i_{j+1} \ , \tag{8}$$

$$p^i_{j+1} = p^{i-1}_{j+1} + v^i_{j+1} + d^{i-1}_{j+1} \ , \tag{9}$$

where $w^i_{j+1} = {w_r}^i_{j+1} + {w_s}^i_{j+1}$. Rail systems are also subjected to several constraints, ensuring the state and control are feasible and safe. The state and control constraints are given as follows:

$$-u^i_j \leq -u_{\min} \ , \tag{10}$$

$$u^i_j \leq u_{\max} \ , \tag{11}$$

$$p^i_j - p^{i-1}_j - \lambda^i_j(t^i_j - t^{i-1}_j) \leq 0 \ , \tag{12}$$

$$-p^i_j \leq 0 \ , \tag{13}$$

$$-t^i_j + t^{i-1}_j \leq -t_{\min} \ , \tag{14}$$

$$l_j \leq l_{\max} \ , \tag{15}$$

$$p^{i-1}_j - p^i_j + \lambda^i_j(t^i_j - t^{i-1}_j) + \eta^i_j l^i_j \leq p_{\max} \ . \tag{16}$$

Constraints (10)–(13) are feasibility constraints, where constraints (10) and (11) are related to feasible range of running times, and constraints (12) and (13) ensure that the passenger control action of reducing the number of boarding passengers does not exceed the number of passenger attending to board, or negative number of passengers. Constraint (14) and (15) are safety constraints, ensuring the headway between successive trains is larger than the minimal safety headway, and the load of passengers does not exceed the maximum determined load. Last, constraint (16) considers the number of passengers on the platform that can not exceed a platform capacity. This number of passengers takes into account the sum of boarding and alighting passengers when passengers alight from the train.

The predetermined timetable is calculated according to the dynamic equations (7) and (8), without interfering neither disturbances nor control actions, i.e., $w^i_j = u^i_j = 0$ and $p^i_{j+1} = p^{i-1}_{j+1} = d^i_j = 0$. Thus, deviation variables can be assigned as follows $e^i_j = t^i_j - T^i_j$ and $\delta^i_j = l^i_j - L^i_j$, where T^i_j [s] is the designed timetable departure time, and L^i_j [passenger] is the train load induced by the predetermined timetable. By addressing the predetermined timetable equations, the deviation variables equations are transformed into a set of linear equations, as follows

$$T^i_{j+1} = T^i_j + R^i_j + S^i_{j+1} + \beta_{j+1}\lambda_{j+1}(T^i_{j+1} - T^{i-1}_{j+1}) + \alpha_{j+1}\eta^i_{j+1}L_j \ , \tag{17}$$

$$L^i_{j+1} = (1-\eta^i_{j+1})L^i_j + \lambda_{j+1}(T^i_{j+1} - T^{i-1}_{j+1}) \ , \tag{18}$$

$$e^i_{j+1} = \frac{1}{1-\beta_{j+1}\lambda^i_{j+1}}\left(e^i_j + \beta_{j+1}\lambda^i_{j+1}e^{i-1}_{j+1} + \alpha_{j+1}\eta^i_{j+1}\delta^i_j - \beta_{j+1}v^i_{j+1} + u^i_j + w^i_j\right) \ , \tag{19}$$

$$\delta^i_{j+1} = \left(1 - \eta^i_{j+1} + \frac{\alpha_{j+1}\eta^i_{j+1}\lambda^i_{j+1}}{1-\beta_{j+1}\lambda^i_{j+1}}\right)\delta^i_j + \frac{1}{1-\beta_{j+1}\lambda^i_{j+1}}\left(\lambda^i_{j+1}(e^i_j - e^{i-1}_{j+1} + u^i_j + w^i_j) - v^i_{j+1}\right) \ . \tag{20}$$

Equations (19) and (20), and constraints (10)-(16) are set of equations for train i, for $i = 1, 2, \ldots, M$, at the $j+1$ station, for $j = 0, 1, \ldots, N-1$, where N is the number of stations and M is the number of trains in the controlled time period. The RTM vector formation for trains control in (Van Breusegem et al., 1991), with its extension to joint train and passengers flow control in (Li et al., 2017), allows capturing the state of the full rail system in a vector, and describes the evolution of the next state vector $\bar{x}_{k+1}$ according to the current state vector $\bar{x}_k$, the current control action $\bar{u}_k$, and the disturbance $\bar{w}_k$, i.e.

$$\bar{x}_{k+1} = A_k\bar{x}_k + B_k\bar{u}_k + D_k\bar{w}_k \ , \tag{21}$$

where $\bar{x}_k = \left[e_1^{k-1}, \delta_1^{k-1}, \ldots, e_N^{k-N}, \delta_N^{k-N}\right]^T$, $\bar{u}_k = \left[u_1^{k-1}, v_1^{k-1}, \ldots u_N^{k-N}, v_N^{k-N}\right]^T$, $\bar{w}_k = \left[w_1^{k-1}, \ldots w_N^{k-N}\right]^T$, A_k and B_k are $2N \times 2N$ matrices and D_k is a $2N \times N$ matrix. Matrices A_k, B_k and D_k are constructed according to equations (19) and (20), specified by values of α_{j+1}, β_{j+1}, η_{j+1}^i and λ_{j+1}^i. Since η_{j+1}^i and λ_{j+1}^i are varied according to different trains, the dynamic system (21) is varied in stages.

In this paper, the non-boarding passengers are also modeled at each stage. Hence, one can form non-boarding passengers state $\bar{p}_k = \left[p_1^{k-1}, \ldots p_N^{k-N}\right]^T$, and disturbances $\bar{d}_k = \left[d_1^{k-1}, \ldots d_N^{k-N}\right]^T$. The dynamics of the non-boarding passengers system in RTM vector notation is introduced as

$$\bar{p}_{k+1} = \bar{p}_k + H_2\bar{u}_k + \bar{d}_k \ , \tag{22}$$

where $H_1 = I_N \otimes [1 \ \ 0], H_2 = I_N \otimes [0 \ \ 1]$, I_N is the identity matrix of size $N \times N$, and $\otimes$ is the Kronecker product. Thus, the joint train and passenger flow system in (21), and the non-boarding passengers system in (22) use the same control vector $\bar{u}_k$. To calculate an optimal control for each stage, the suggested objective should consider regulation on the departure time deviations and load deviations, as described in (Li et al., 2017), along with regulating the number of the non-boarding passengers (which was not addressed in (Li et al., 2017)). In addition, the regulation objective should also consider the headway regulation and the control action terms. Then, the optimal control problem for K stages can be formulated as follows:

$$J^* = \min_{\bar{u}_{k_0}, \ldots \bar{u}_{K-1}} \sum_{k=k_0}^{K-1} \bar{x}_k^T Q_1 \bar{x}_k + (\bar{x}_{k+1} - \bar{x}_k)^T Q_2 (\bar{x}_{k+1} - \bar{x}_k) + \bar{p}_k^T Q_3 \bar{p}_k + \bar{u}_k^T Q_4 \bar{u}_k \tag{23}$$

s.t.

$$\begin{aligned}
\bar{x}_{k+1} &= A_k\bar{x}_k + B_k\bar{u}_k + D_k\bar{w}_k \ , \\
\bar{p}_{k+1} &= \bar{p}_k + H_2\bar{u}_k + \bar{d}_k \ , \\
-H_1\bar{u}_k &\leq -u_{\min}\mathbb{1} \ , \\
H_1\bar{u}_k &\leq u_{\max}\mathbb{1} \ , \\
\bar{p}_k - \bar{p}_{k-1} - \Lambda_k H_1(x_{k+1} - x_k) &\leq \Lambda_k t_h \mathbb{1} \ , \\
-\bar{p}_k &\leq 0 \ , \\
-H_1\left(\bar{x}_k - \bar{x}_{k-1}\right) &\leq (t_h - t_{\min})\mathbb{1} \ , \\
H_2\bar{x}_k &\leq l_{\max}\mathbb{1} - L_k \ , \\
\mathrm{H}_k H_2\bar{x}_k &\leq p_{\max}\mathbb{1} - \mathrm{H}_k L_k \ ,
\end{aligned}$$

where Q_1, Q_2, Q_3 and Q_4 are diagonal weight matrices, $\mathbb{1}$ is a vector of size N with all elements equal to 1, $\Lambda_k = \mathrm{diag}\{\lambda_1^{k-1}, \ldots, \lambda_N^{k-N}\}$, t_h is the fixed headway determined by the timetable, $L_k = \left[L_1^{k-1}, \ldots L_N^{k-N}\right]^T$ is the timetable load at stage k, and $\mathrm{H}_k = \mathrm{diag}\{\eta_1^{k-1}, \ldots, \eta_N^{k-N}\}$.

In this paper, the MPC approach is used with finite prediction horizon H. The model prediction states are composed as $\tilde{X}_k = \left[\bar{x}_{k+1}^T, \ldots, \bar{x}_{k+H}^T\right]^T$ and $\tilde{P}_k = \left[\bar{p}_{k+1}^T, \ldots, \bar{p}_{k+H}^T\right]^T$. The decision variables are sequence of control actions $\bar{U}_k = \left[\bar{u}_k^T, \ldots, \bar{u}_{k+H-1}^T\right]^T$. For the model prediction it is assumed that the current states $\bar{x}_k$ and $\bar{p}_k$ can be measured, and the disturbances $\bar{w}_k$ and $\bar{d}_k$ are considered only through measurements $\bar{x}_k$ and $\bar{p}_k$. Thus, the predicted states are functions of the current states and sequence of controls, i.e.,

$$\tilde{X}_k = F_k\bar{x}_k + \Phi_k\bar{U}_k \ , \tag{24}$$

$$\tilde{P}_k = G\bar{p}_k + \Gamma\bar{U}_k \ , \tag{25}$$

where F_k and Φ_k are constructed from (21), and G and Γ are constructed from (22). The optimal control problem at each step k for the prediction horizon can be rewritten as a quadratic optimization problem with set of linear constraints, i.e.

$$J_k^* = \min_{\bar{U}_k} \left(\bar{U}_k^T \Sigma_k \bar{U}_k + 2\Omega_k \bar{U}_k + \Psi_k \right) \tag{26}$$

s.t.

$$\begin{bmatrix} -I_H \otimes H_1 \\ I_H \otimes H_1 \\ I_H \otimes H_2 - \bar{\Lambda}_k (I_H \otimes H_1) H_4 \Phi_k \\ -\Gamma \\ -(I_H \otimes H_1) H_4 \Phi \\ (I_H \otimes H_2) \Phi_k \\ \mathrm{H}(I_H \otimes H_2) \Phi_k \end{bmatrix} \bar{U}_k \leq \begin{bmatrix} -u_{\min} \mathbb{1}_{NH \times 1} \\ u_{\max} \mathbb{1}_{NH \times 1} \\ \bar{\Lambda}_k \left(t_h \mathbb{1} + (I_H \otimes H_1)(H_4 F_k + H_5) x_k \right) \\ G p_k \\ -(t_{min} - t_h) \mathbb{1} + (I_H \otimes H_1)(H_4 F_k + H_5) x_k \\ l_{\max} \mathbb{1} - \bar{L}_k - (I_H \otimes H_2) F_k x_k \\ p_{\max} \mathbb{1} - \bar{\mathrm{H}}_k \left(\bar{L}_k + (I_H \otimes H_2) F_k x_k \right) \end{bmatrix},$$

where $\Sigma_k, \Omega_k, \Psi_k$ are the objective matrices constructed from $F_k, \Phi_k, G_k, \Gamma, H_4, H_5, Q_1, Q_2, Q_3$ and Q_4. Constraints are formulated according to (23), where $\bar{\Lambda}_k = \mathrm{diag}\{\Lambda_{k+1}, \ldots, \Lambda_{k+H}\}$ and $\bar{\mathrm{H}}_k = \mathrm{diag}\{\mathrm{H}_{k+1}, \ldots, \mathrm{H}_{k+H}\}$ are block diagonal matrices, H_4 and H_5 are constructed such that $H_4 \tilde{X}_k + H_5 x_k = \tilde{X}_k - \tilde{X}_{k-1}$, and $\bar{L}_k = \left[L_{k+1}^T \ldots L_{k+H}^T \right]^T$.

3. RESULTS AND DISCUSSION

The case study in this paper is based on Beijing metro line 9, as described and studied in (Li et al., 2017). A simulation is constructed in MATLAB framework, which solves the optimization problem of the MPC at each stage with (MATLAB Optimization Toolbox, 2018). The metro line consists 12 passenger stations and one end terminal. The predetermined timetable is assigned with a headway of $t_h = 180$ [s], and a minimum allowed safety time-gap $t_{\min} = 160$ [s]. The control action in terms of time adjustment u_j^i is bounded between maximum increase of the running time by 25 seconds, and decrease by 20 seconds.

The passenger stations are characterized by different arrival rates λ_j^i, which also vary in each stage such that the rates increase from stage $k = 1$ to stage $k = 12$, and then decrease back to the initial values. The passenger boarding and alighting coefficients are $\beta_j = \alpha_j = 0.04$ [s/passenger]. The maximum load of the trains is $L_{\max} = 2000$ [passenger] and the nominal loads are varied between 1950 to 1990 passengers. Additionally, it is considered that the maximum number of passengers allowed on each platform is $p_{\max} = 400$ [passenger].

In Example 1, the state deviation variables are assigned with initial values for the departure times $\bar{e}_1 = [0,\ 0,\ 0,\ 0,\ 20,\ 20,\ 35,\ 20,\ 20,\ 0,\ 0,\ 0]^T$, and for the load deviation $\bar{\delta}_1 = [0,\ 0,\ 5,\ 6,\ 40,\ 40,\ 40,\ 30,\ 30,\ 10,\ 0,\ 0]^T$. At stages $k = 5$, $k = 9$, and $k = 13$, there are disturbances in terms of departure time deviations, i.e. $\bar{w}_5 = [0,\ 0,\ 0,\ 0,\ 45,\ 45,\ 55,\ 45,\ 40,\ 0,\ 0,\ 0]^T$, $\bar{w}_9 = [0,\ 0,\ 0,\ 0,\ 25,\ 25,\ 25,\ 25,\ 0,\ 0,\ 0,\ 0]^T$, and $\bar{w}_{13} = [0,\ 0,\ 0,\ 0,\ 10,\ 10,\ 0,\ 20,\ 0,\ 0,\ 0,\ 0]^T$. At the disturbed stations $5 - 9$, it is also assumed that between stages $k = 1$ to $k = 5$ there is a disturbance such that 20 passengers were measured at the platform, in addition to the nominal predicted arrival rate. The prediction horizon for MPC strategy is $H = 3$.

Fig. 2 presents the states and control actions corresponding to stations at each stage k, under the suggested MPC policy. In this example, the objective weight matrices in (23) are $Q_1 = Q_3 = Q_4 = \mathrm{diag}\{1, \ldots, 1\}$ and $Q_2 = \mathrm{diag}\{1, 0, \ldots, 1, 0\}$. This implies no explicit preference between departure time, headway, and load deviations and adjustment of time control, passengers control, and non-boarding passengers. It can be noticed that time disturbances at stages $k = 5$, $k = 9$, and $k = 13$ have immediate effect on the departure time deviations; and passenger disturbances at stations

5 − 9 affect the non-boarding passengers. However, the MPC strategy does converge after some stages. Moreover, using the MPC strategy that considers the full rail system, it is shown that some of the deviations propagate between stations, for example the number of non-boarding passengers at station 10 increases while the disturbance in terms of passengers occurred at previous stations.

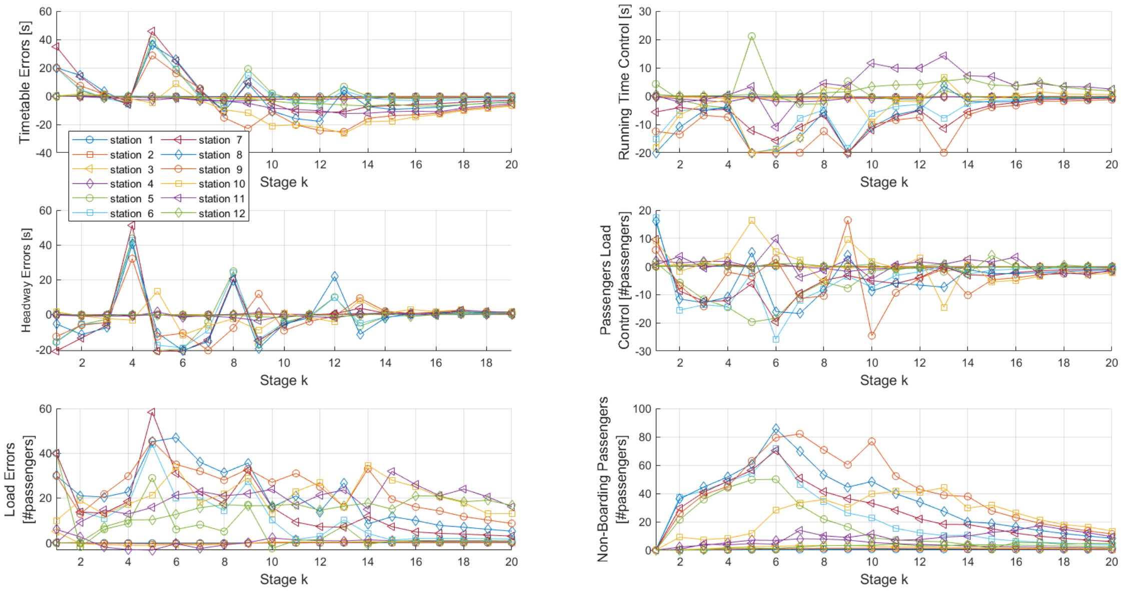

Figure 2: Example 1: State variables and control inputs under equal objective weight matrices.

Considering Example 1, some modifications are made on the objective weight matrices. First, the load of passengers is induced by the timetable, therefore, regulating it does not indicate better performance of the rail system. The corresponding weight matrix is changed to $Q_1 = \mathrm{diag}\{1, 0.1, \ldots, 1, 0.1\}$. Additionally, to give preference to evacuating the platforms from non-boarding passengers, the weight matrix Q_3 is modified to $Q_3 = \mathrm{diag}\{10, \ldots, 10\}$, while the other weight matrices remain the same. Fig. 3 presents the states and control inputs corresponding to stations at each stage k, under the suggested MPC policy with the new assigned weights. Under this strategy, it can be noted that the number of non-boarding passengers converges at all stations. Moreover, the departure time deviations at the disturbed stations reach nearly −60 seconds for three successive trains. In other words, automatic feedback control resulted a real time change in the timetable to overcome the unexpected arrival of passengers.

Example 2 has the same system parameters of Example 1. The initial values are equal to 0, and no disturbances in terms of departure times. Instead, for stages 1 − 5, stations 5 − 9 have disturbances in terms of arrival passengers, that is 40 passengers at each stage at each station, in addition to the known arrival rate. Fig. 4 shows state variables and control inputs for Example 2 under the MPC proposed strategy. The objective matrices are set such that the non-boarding passengers are preferred, i.e., $Q_3 = \mathrm{diag}\{10, \ldots, 10\}$, and the other matrices are $Q_1 = Q_4 = \mathrm{diag}\{0.1, \ldots, 0.1\}$ and $Q_2 = \mathrm{diag}\{0.1, 0, \ldots, 0.1, 0\}$. The preference for non-boarding passengers resulted significant deviation in the timetable, but the number of passengers has converged, i.e. all passengers eventually

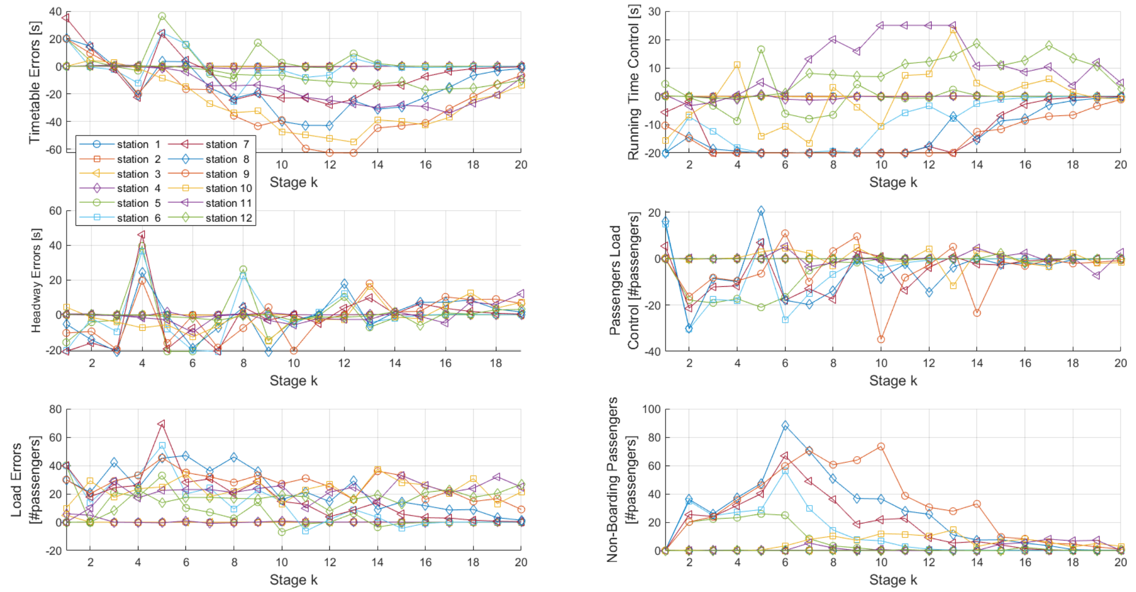

Figure 3: Example 1: State variables and control inputs under boarding preference weights.

board trains. The number of passengers arrived who were not accounted in the predetermined timetable is 1000, equivalent to a half loaded train. The proposed regulation strategy changed in real time the timetable to fit this large number of unexpected arrival passengers. Moreover, the proposed centralized control strategy utilizes coordination between stations. For example, in Fig. 4 at station 4 passengers load control was applied even though the disturbance happened at stations ahead.

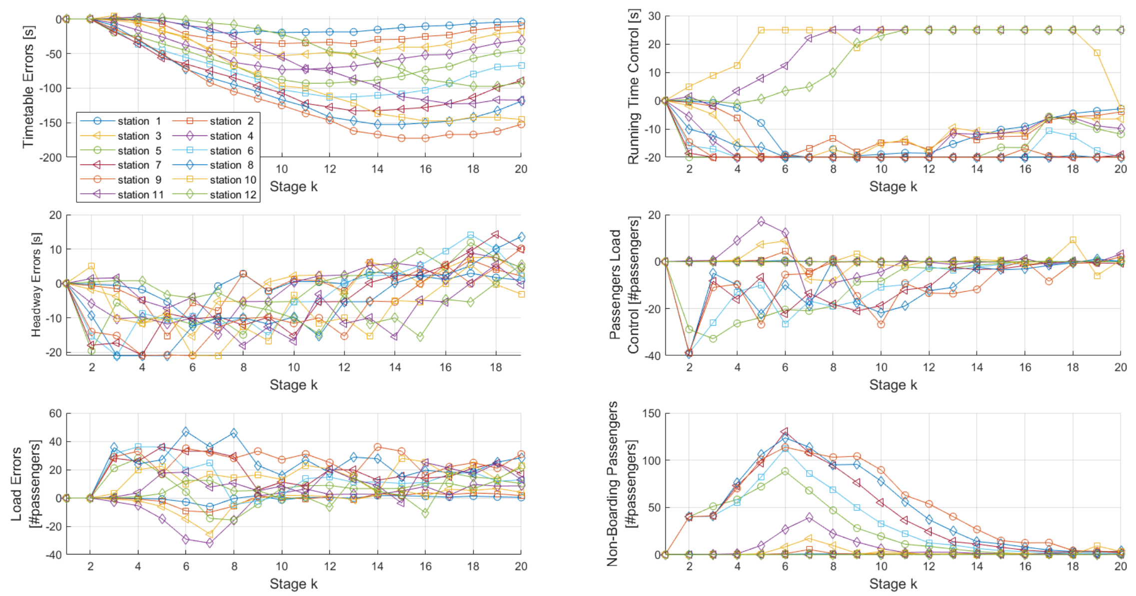

Figure 4: Example 2: State variables and control inputs.

4. CONCLUSIONS

In this paper, a joint discrete event model for urban rail systems, coupling departure times and traveling passenger load evolution, was presented. Compared with previous works, the joint model is extended to address (i) unexpected boarding passengers, (ii) limited platform capacity, and (iii) regulating the number of controlled passengers. Real time control under the MPC strategy solution was developed and implemented in a simulation environment. Two numerical examples, using realistic nominal parameters for a Metro system under unknown disturbances, were presented. The results show that the developed MPC can compensate unexpected disturbances, by regulating the number of accumulating passengers at the stations, while converging the system back to predetermined timetable. The effect of different objective weights was demonstrated, resulting in real time adjustment of the timetable. Moreover, limited platform and load capacities, along with timetable modifications, resulted coordination between trains and stations, where passengers accumulated in some stations that had no disturbances, to handle other over-saturated stations. This kind of coordination confirms the research efforts to control urban rail traffic as a cooperated system.

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